The 9-th International Algebraic Conference in Ukraine

L'viv, Ukraine, July 8–13, 2013

Abstracts of Reports

Contents

Local Organizing Committee

R. Kushnir – Co-head of the Committee,
M. Zarichnyi – Co-head of the Committee,
M. Komarnytskyi – Vice-head of the Committee,
V. Petrychkovych – Vice-head of the Committee,
V. Shchedryk – Secretary of the Committee.

Program Committee

Yu. Drozd (Kyiv) – Head of the Committee.

Members:

Arnautov V. (Moldova),
Artamonov V. (Russia),
Artemovich O. (Poland),
Baginski C. (Bialystok),
Banakh T. (Ukraine),
Bavula V. (United Kingdom),
Bovdi A. (Hungary),
Coelho F. (Brazilia),
Chernikov M. (Ukraine),
Couchot F. (France),
Dlab V. (Canada),
Dubrovin N. (Russia),
Futorny V. (Brazilia),
Grigorchuk R. (USA),
Ivanov A. (Poland),
Holubowski W. (Poland),
Kashu A. (Moldova),
Kilp M. (Estonia),
Kirichenko V. (Ukraine),
Knauer U. (Germany),
Kozhukhov I. (Russia),
Krempa J. (Poland),
Kurdachenko L. (Ukraine),
Lavrenyuk Ja. (Ukraine),
Lyman F. (Ukraine),
Lyubashenko V. (Ukraine),
McGovern W. (United Kingdom),
Mikhalev A, (Russia),
Novikov B. (Ukraine),
Okninski J. (Poland),
Ol'shanskii A. (USA),
Osin D. (USA),
Petravchuk A. (Ukraine),
Protasov I. (Ukraine),
Puczylowski E. (Poland),
Ruzicka P. (Czech Republic),
Sergejchuk V. (Ukraine),
Sharko V. (Ukraine),
Shestakov I. (Brazilia),
Subbotin I. (USA),
Sushchansky V. (Poland),
Trlifaj J. (Czech Republic),
Tuganbaev A. (Russia),
Varbanets P. (Ukraine),
Wiegand R. (USA),
Yanchevskii V. (Belarus),
Zabavsky B. (Ukraine),
Zaicev M. (Russia),
Zelmanov E. (USA),
Žemlička J. (Czech Republic).

Topical Sections:

1. **Representations and Linear algebra**
2. **Group Theory and Algebraic Dynamics**
3. **Rings and Modules**
4. **Algebraic Geometry and Number Theory**
5. **Algebraic Topology and Topological Algebra**
6. **Model Theory and Applied Algebra**
7. **Semigroups, Quasigroups, S-acts and other Algebraic Systems**

Semigroups of automata transformations

A. Antonenko, E. Berkovich

Odessa I.I. Mechnikov National University
Odessa National Polytechnic University
aantonenko@mail.ru, evg.berkovich@gmail.com

We consider finite automata $A = (X, Q, \pi, \lambda)$ over two-symbol alphabet $X = \{0, 1\}$ with the following two properties: for each state $q \in Q$ of an automaton, there exists not more than one symbol $x \in X$ such that $\pi(x, q) \neq q$ (slowmoving automata); there are no cycles except loops in the Moore diagram of an automaton (automata of the finite type).

Let $p \in Sym(X)$ (i.e. $p : X \to X$ and p is bijective), $x \in X$, and let $f : X^\omega \to X^\omega$ be an arbitrary transformation of infinite words. Detone by $px]f$ a transformation, which acts by permutation p on all the symbols up to the first occurrence of the symbol x inclusive, and on the rest of the word by the transformation f. [1]

Lemma 1. *Let $f : X^\omega \to X^\omega$. Then $px]f = f$ if and only if $f = p$. I.e. the operator $px]$ has the only fixed point $p : X^\omega \to X^\omega$. It is the transformation acting on all the symbols by the permutation p.*

Theorem 1. *Let $f, g : X^\omega \to X^\omega$ be invertible transformations, $p_1, p_2 \in Sym(X)$, $x_1, x_2 \in X$, $f \neq p_1$, $g \neq p_2$. Then $p_1x_1]f \circ p_2x_2]g$ is slowmoving, iff $p_2(x_2) = x_1$ and $f \circ g$ is slowmoving.*

Any invertible slowmoving transformation of the finite type can be represented in the form $f = p_1x_1]p_2x_2] \ldots p_kx_k]p$, where p_i, $p \in Sym(X)$, $x_i \in X$.

Consider the family of transformations:

$$\alpha_0 = \sigma, \quad \alpha_1 = id0]\sigma, \quad \ldots, \quad \alpha_n = id0]^n\sigma, \quad \ldots \tag{1}$$

where $\sigma(x) = 1 - x$ is the inversion and $id(x) = x$ is the identical permutation. We have $\alpha_i^2 = id$. Any slowmoving transformation of finite type can be represented in the form $\alpha_{i_1}\alpha_{i_2} \cdots \alpha_{i_k}\alpha_{j_s}\alpha_{j_{s-1}} \cdots \alpha_{j_1}$, where $k, s \geq 0$, $i_1, i_2, \ldots, i_k$ is a strictly increasing sequence of indexes, and $i_k, j_s, j_{s-1}, \ldots j_1$ is a strictly decreasing one (see [1]).

Consider the group generated by all slowmoving transformations of finite type $GSlC_2 = \langle \alpha_0, \alpha_1, \alpha_2, \ldots \rangle$ and the free product of a countable number of cyclic groups $\mathbb{Z}_2$ with a set of generators $\{\alpha_i\}$: $F_2 = \left\langle \alpha_i, i = \overline{0, \infty} | \alpha_i^2 = 1 \right\rangle$. Let $\phi : F_2 \to GSlC_2$ be a homomorphism which turns a generator $\alpha_i \in F_2$ into the transformation of words $\alpha_i \in GSlC_2$. Each element of F_2 has the unique representation as group word $\alpha_{i_1}\alpha_{i_2} \cdots \alpha_{i_k}$ without equal adjacent symbols. Define $i_0 = i_{k+1} = -1$. Let us call α_{i_p} a peak where $1 \leq p \leq k$ if $i_p > i_{p-1}$ and $i_p > i_{p+1}$.

Theorem 2. *Let $g \in F_2$ be such that $g \neq 1$ but $\phi(g) = id$ then the number of peaks in g is greater than 3.*

1. Antonenko A.S., Berkovich E.L., *Groups and semigroups defined by some classes of Mealy automata*, Acta Cybernetica 2007, 18, pp. 23-46

On the number of metrizable group topologies on countable groups

V.I. Arnautov, G.N. Ermakova

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova
Transnistrean State University
arnautov@math.md, galla0808@yandex.ru

Researches on the possibility of the definition of Hausdorff group topologies on countable groups were started in [1]. In this work also a method to define such group topologies on any countable group was given.

Later, in [2] it was proved that any infinite Abelian group admits a non-discrete, Hausdorff group topology, and in [3] an example of a countable group, which does not allow non-discrete, Hausdorff group topologies, was constructed.

However, the question how many different Hausdorff group topologies can be defined on a countable group, on which at least one non-discrete, Hausdorff group topology can be defined, was remained open.

In the present paper we answer this question for metrizable group topologies.

Theorem 1. *If a countable group G admits a non-discrete, metrizable, group topology τ_0, then G admits a continuum of non-discrete, metrizable, group topologies stronger than τ_0, and any two of these topologies are incomparable with each other.*

Theorem 2. *If the countable group G admits a non-discrete, metrizable, group topology τ_0, then there exists the continuum of non-discrete, metrizable, group topologies on G stronger than τ_0, and any two of these topologies are comparable.*

1. Markov A.A, On absolutely closed sets, Mat.Sb., 18: 3 - 28 (1945) (in Russian).
2. Kertesz A., Szele T., On existence of non-discrete topologies on an infinite Abelian groups, Publ.Math., 3: 187 - 189 (1953).
3. Ol'shansky A.Yu., Remark on countable non-topologizable group, Vestnik MGU. Ser.Mat. i Mech., 3: 103 (1980) (in Russian).

Minimal non-PC-groups

Orest D. Artemovych

Cracov University of Technology, Cracov, Poland
artemo@usk.pk.edu.pl

A group G is called *a PC-group* if the quotient group $G/C_G(\langle x\rangle^G)$ is virtually polycyclic (i.e. polycyclic-by-finite) for all $x \in G$ [1]. This is a natural extension of the notion of an FC-group (i.e., a group with all conjugate classes to be finite). It is known [2] that G is a PC-group if and only if it is central-by-(virtually polycyclic). If $\mathfrak{X}$ is a class of groups, then a group G is called *a minimal non-$\mathfrak{X}$-group* if it is not a $\mathfrak{X}$-group, while every proper subgroup of G is a $\mathfrak{X}$-group. Every minimal non-FC-group is a minimal non-PC-group. V. V. Belyaev and N. F. Sesekin have proved that every minimal non-FC-groups with a non-trivial finite or abelian homomorphic image is a finite cyclic extension of a divisible Černikov p-group. F. Russo and N. Trabelsi [3] have shown that a minimal non-PC-group with a non-trivial finite homomorphic image is an extension of a divisible abelian group of finite rank by a cyclic group. In the following proposition we extend this result and prove that such groups are torsion.

Proposition 1. *Let G be a non-perfect group. Then G is a minimal non-PC-group if and only if it is a minimal non-FC-group.*

Finitely generated torsion-free minimal non-PC-groups there exist. The question about the structure of perfect locally graded minimal non-FC-groups discussed by V. V. Belyaev, M. Kuzucuoğlu and R. E. Phillips, F. Leinen. Every perfect locally graded minimal non-FC-group must be a p-group. In this way we prove the following

Theorem 1. *A perfect locally graded minimal non-PC-group is an indecomposable countable locally finite p-group.*

This theorem answers Question 17.106 from "Kourovka Notebook" [4].

1. S. Franciosi, F. de Giovanni and M.J. Tomkinson, Groups with polycyclic-by-finite conjugacy classes, Boll. Unione Mat. Ital., **7** (1990), 35–55.
2. L.A. Kurdachenko, J. Otal and P. Soules, Groups with polycyclic-by-finite conjugacy classes of subgroups, Comm. Algebra, **32** (2004), 4769–4784.
3. F. Russo and N. Trabelsi, On minimal non-PC-groups, Annales Mathématiques Blase Pascal, **16** (2009), 277–286.
4. The Kourovka notebook. Unsolved problems in group theory. Seventeenth edition. Including archive of solved problems (Russian). V. D. Mazurov and E. I. Khukhro (eds.), Russian Academy of Sciences Siberian Division, Institute of Mathematics, Novosibirsk, 2010.

An example of cyclic diagonal bi-act

T. V. Apraksina, I. B. Kozhukhov

INational Research University of Electronic Technology MIET, Moscow
taya.apraksina@gmail.com, kozhuhov_i_b@mail.ru

A right act [1] over a semigroup S is defined as a set X with a mapping $X \times S \to X$, $(x, s) \mapsto xs$ satisfying the axiom $(xs)s' = x(ss')$ for $x \in X$. $s, s' \in S$. A left act Y over a semigroup S is defined analogously by a mapping $S \times Y \to Y$, $(s, y) \mapsto sy$ and the axiom $s(s'y) = (ss')y$ for $y \in Y$, $s, s' \in S$. Let a set X be a left act over a semigroup S and a right act over a semigroup T, then we call it a bi-act if $(sx)t = s(xt)$ for $x \in X$, $s \in S$, $t \in T$.

The set $S \times S$ will be a right act over a semigroup S if the action is defined by this way $(x, y)s = (xs, ys)$ for $x, y, s \in S$; it is a left act relating the rule $s(x, y) = (sx, sy)$, and the bi-act if both actions are introduced. We will call them diagonal right, left and bi-act respectively. We denote them $(S \times S)_S$, ${}_S(S \times S)$, ${}_S(S \times S)_S$. The diagonal act is cyclic if it is generated by one element.

We say the a semigroup S is right cancellative if $xz = yz \Rightarrow x = y$ for all $x, y, z \in S$, and left cancellative if $zx = zy \Rightarrow x = y$ for $x, y, z \in S$. A semigroup is cancellative if both conditions hold.

It was proved in [2] that if X is an infinite set and S is a semigroup of partial injective transformations of the set X, then the diagonal bi-act ${}_S(S \times S)_S$ is cyclic, but diagonal right $(S \times S)_S$ and left ${}_S(S \times S)$ acts are not finitely generated ([2], Theorems 4.3, 2.5, 3.5). It was proved in [3] (Corollary 5.1) that the diagonal bi-act of S is not cyclic if S is a cancellative nontrivial semigroup. There was an open question if the statement will true in case when the cancellation will be only one-sided.

In this paper we construct an example of an infinite right cancellative semigroup whose diagonal bi-act is cyclic.

1. Kilp M., Knauer U., Mikhalev A.V. Monoids, acts and categories. Berlin - New York: W. de Gruyter, 2000.
2. Gallagher P., Ruškuc N. Generation of diagonal acts of some semigroups of transformations and relations *Bull. Austral. Math. Soc.* V.72. P. 139-146, 2005.
3. Gallagher P. On the finite and non-finite generation of diagonal acts. *Comm. Algebra 34* pp. 3123-3137, 2006.

Simply connected bimodule problems with quasi multiplicative basis

V. Babych, N. Golovashchuk, S. Ovsienko

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
vyacheslav.babych@gmail.com
golovash@gmail.com
ovsiyenko.sergiy@gmail.com

Let $\mathcal{A} = (\mathsf{K}, \mathsf{V})$ be a faithful connected finite dimensional one-sided bimodule problem from the class $\mathcal{C}$ considered in [1] with a quasi multiplicative basis Σ. In [2] we associate a two-dimensional cell complex $\mathfrak{L}$ with $\mathcal{A}$. A fundamental group G of complex $\mathfrak{L}$ is called *fundamental group* of bimodule problem $\mathcal{A}$.

We construct in a standard way the universal covering $\pi : \tilde{\mathcal{A}} \to \tilde{\mathcal{A}}/G = \mathcal{A}$ of the bimodule problem $\mathcal{A}$ ([3]). A bimodule problem is said to be (geometrically) *simply connected* provided it is connected and its fundamental group is trivial. For a bimodule problem $\mathcal{A} \in \mathcal{C}$ the constructed universal covering $\tilde{\mathcal{A}}$ is simply connected.

A bimodule problem $\mathcal{A}$ will be called *shurian* ([4]) provided every indecomposable representation of $\mathcal{A}$ has only scalar endomorphism. Shurian bimodule problem $\mathcal{A}$ is of finite representation type, its Tits form $q_{\mathcal{A}}$ is weakly positive, the set of dimension vectors of indecomposable representations of $\mathcal{A}$ coinsides with the set of positive roots of $q_{\mathcal{A}}$ and two indecomposable representations with the same dimension vector are isomorphic.

Theorem 1. *A geometrically simply connected bimodule problem $\mathcal{A} \in \mathcal{C}$ having weakly positive Tits form is shurian. In particular, $\mathcal{A}$ is of finite representation type.*

The proof of this statement uses the geometric techniques of diagrams and contracting closed walks in quasi multiplicative basis Σ.

1. V. Babych, N. Golovashchuk, S. Ovsienko. *Generalized multiplicative bases for one-sided bimodule problems* // Algebra and Discrete Mathematics, Vol. **12** (2011), No. 2, 1 – 24.
2. V. Babych, N. Golovashchuk. *Bimodule problems and cell complexes* // Algebra and Discrete Mathematics, 2006, No. 3, 16–28.
3. P. Gabriel. *Auslander-Reiten sequences and representation-finite algebras.* — Lecture Notes in Mathematics, 831, 1980, pp. 1-71.
4. A. V. Roiter, M. M. Kleiner. *Representations of differential graded categories.* — Lecture Notes in Mathematics, 488, Springer-Verlag, 1975, pp. 316 - 340.

The Solecki submeasure on groups

Taras Banakh

Ivan Franko National University of Lviv, Lviv, Ukraine
tbanakh@yahoo.com

The *Solecki submeasure* on a group G is the invariant subadditive function $\sigma\colon \mathcal{P}(G) \to [0,1]$ on the power-set $\mathcal{P}(G)$ defined by the formula

$$\sigma(A) = \inf_{\mu} \sup_{x,y\in G} \mu(xAy) \quad \text{for} \quad A \subset G$$

where μ runs over all finitely supported probability measures on G.

The Solecki submeasure σ has left and right modifications called *left and right Solecki densities*:

$$\sigma_L(A) = \inf_{\mu} \sup_{x\in G} \mu(xA) \quad \text{and} \quad \sigma_R(A) = \inf_{\mu} \sup_{y\in G} \mu(Ay) \quad \text{for } A \subset G.$$

We shall discuss some properties of Solecki submeasure and Solecki densities and their applications in Combinatorics of Groups. Also we shall show that for any closed subset A of a compact topological group G its Haar measure $\lambda(A)$ coincides with the Solecki submeasure $\sigma(A)$ of A. This means that the Haar measure of a compact topological group is determined by the Solecki submeasure.

1. T. Banakh, *The Solecki submeasures and densities on groups*, preprint http://arxiv.org/abs/1211.0717.

On arithmetic properties of recurrent sequences on Legendre curves

G. Barabash

Ivan Franko National University of Lviv, Lviv, Ukraine
galynabarabash71@gmail.com

Let E be an elliptic curve over finite field $\mathbb{F}_q$ of characteristic $\neq 2,\ 3$, given in the form of Legendre

$$E: \quad y^2 = x(x-1)(x-\lambda),\ \ \lambda \in \mathbb{F}_q,\ \ \lambda^2 \neq 0,\ 1.$$

The points of the curve E with O form abelian group relatively the operation $\oplus$, which is defined as follows: $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$ different finite points of the curve E, $P_1 \neq -P_2$, then

$$P_1 \oplus P_2 = P_3, \quad P_3 = (x_3, y_3), \quad \begin{cases} x_3 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)^2 - x_1 - x_2 + \lambda + 1, \\ y_3 = \dfrac{y_2 - y_1}{x_2 - x_1} x_3 - \frac{y_1 x_2 - x_1 y_2}{x_2 - x_1}. \end{cases}$$

If $P_1 = P_2$, then $2P_1 = P_1 \oplus P_1 = P_3$ and

$$\begin{cases} x_3 = \left(\dfrac{3x_1^2 - 2(\lambda+1)x_1 + \lambda}{2y_1}\right)^2 - 2x_1 + \lambda + 1, \\ y_3 = \dfrac{3x_1^2 - 2(\lambda+1)x_1 + \lambda}{2y_1} x_3 - \dfrac{x_1^3 - \lambda x_1}{2y_1}. \end{cases}$$

The point I on the curve E is called divisible by 2, if $\exists\ Q \in E: \ Q \oplus Q = I$. The order of the point P on E appoints as m. We examine points $R_n = I + nP$, $1 \leq n \leq m$, $P \neq O$. Let $x_1,\ ...,\ x_m$ be the first coordinates of points R_n.

Theorem 1. *On the field $\mathbb{F}_q$ the following holds:*
if I is divisible by 2 points, then x_i are square of elements of $\mathbb{F}_q$ for all pair n;
if $I + P$ is divisible by 2 point, then the same property holds for all unpaired n;
if I and P are divisible by 2 point, then the property holds for all n.

1. Silverman J.H., The Arithmetic of Elliptic Curves, Graduate Texts in Mathematics, 106, Springer-Verlag, New York, 1986.
2. Тараканов В.Е., Несколько замечаний об арифметических свойствах рекуррентных последовательностей на эллиптических кривых над конечным полем // Матем. заметки – 2007. – Т.82,– Вып. 6. – С.926-933.

On twisted group algebras of SUR-type of finite groups over a local noetherian ring

L. F. Barannyk, D. Klein

Pomeranian University of Słupsk
leonid.barannyk@apsl.edu.pl
darekklein@poczta.onet.pl

In this talk we continue the studies of twisted group rings of SUR type carried out in [1] and [2].

Let $p \geq 2$ be a prime, S a commutative local noetherian ring of characteristic p^k, S^* the unit group of S, G a finite group, G_p a Sylow p-subgroup of G, and $S^\lambda G$ the twisted group algebra of the group G over S with a 2-cocycle $\lambda \in Z^2(G, S^*)$. We assume that S is not a field, and if S is not an integral domain then the residue class field $S/\operatorname{rad} S$ of S modulo the Jacobson radical $\operatorname{rad} S$ is infinite.

By an $S^\lambda G$-module we mean a finitely generated left $S^\lambda G$-module which is S-free. Denote by $\operatorname{Ind}_m\left(S^\lambda G\right)$ the set of all isomorphic classes of indecomposable $S^\lambda G$-modules of S-rank m. We say that $S^\lambda G$ is of strongly unbounded representation type (SUR-type, in short) if there exists a function $f_\lambda\colon \mathbb{N} \to \mathbb{N}$ such that $f_\lambda(n) \geq n$ and $\operatorname{Ind}_{f_\lambda(n)}\left(S^\lambda G\right)$ is an infinite set for every integer $n > 1$. A function f_λ will be called an SUR-dimension-valued function.

We prove the following theorems.

Theorem 1. *Assume that G_p contains a subgroup H such that $|H| > 2$ and the restriction of $\lambda \in Z^2(G, S^*)$ to $H \times H$ is a coboundary.*

(i) *The algebra $S^\lambda G$ is of SUR-type with an SUR-dimension-valued function f_λ satisfying $n|G_p : H| \leq f_\lambda(n) \leq n|G : H|$ for any integer $n > 1$.*

(ii) *If G_p is normal, then there exists an SUR-dimension-valued function f_λ such that $f_\lambda(n) = n|G_p : H|t_n$, where $1 \leq t_n \leq |G : G_p|$ for any integer $n > 1$.*

Theorem 2. *Let $p \neq 2$, $\operatorname{char} S = p$, F be a subfield of S, $\lambda \in Z^2(G, F^*)$, and $d = \dim_F\left(F^\lambda G_p / \operatorname{rad} F^\lambda G_p\right)$. If the algebra $F^\lambda G$ is not semisimple, then the algebra $S^\lambda G$ is of SUR-type with an SUR-dimension-valued function f_λ such that $nd \leq f_\lambda(n) \leq nd|G : G_p|$ for every integer $n > 1$.*

1. L. F. Barannyk and D. Klein, *Twisted group rings of strongly unbounded representation type*, Colloq. Math. 100(2004), 265-287.
2. P. M. Gudivok and I.B. Chukhraj, *On the number of nonequivalent indecomposable matrix representations of the given degree of a finite p-group over commutative local ring of characteristic* p^s, An. Şt. Univ. Ovidius Constanţa Ser. Mat. 8 (2000), 27-36.

On $\mathscr{H}$-complete topological semilattices

S. Bardyla, O. Gutik

Ivan Franko National University of Lviv, Lviv, Ukraine
o_gutik@franko.lviv.ua

We shall follow the terminology of [2, 3], and [4]. We shall call a Hausdorff topological semilattice E is *$\mathscr{H}$-complete* if E is a closed subsemilattice of any Hausdorff topological semilattice which contains E as a subsemilattice and E is *$\mathscr{AH}$-complete* if every continuous homomorphic image of E into a Hausdorff topological semilattice is an $\mathscr{H}$-complete topological semilattice [6]. In [5] there proved that every Hausdorff linearly ordered $\mathscr{H}$-complete topological semilattice is $\mathscr{AH}$-complete and showed that every linearly ordered semilattice is a dense subsemilattice of an $\mathscr{H}$-complete topological semilattice. Also there were given a criterion for a linearly ordered topological semilattice to be $\mathscr{H}$-complete. An example of an $\mathscr{H}$-complete topological semilattice which is not $\mathscr{AH}$-complete is constructed in [1]. Also there constructed an $\mathscr{H}$-complete topological semilattice of the cardinality λ which has 2^{λ} many open-and-closed continuous homomorphic images which are not $\mathscr{H}$-complete topological semilattices. The constructed examples give a negative answer to Question 17 from [6].

In our report we discuss on the structure of Hausdorff topological semilattices with $\mathscr{H}$-complete maximal chains. We showed that such topological semilattices are $\mathscr{H}$-complete and described their algebraic properties.

1. S. Bardyla and O. Gutik, *On $\mathscr{H}$-complete topological semilattices*, Mat. Stud. 38:2 (2012), 118—123.
2. J. H. Carruth, J. A. Hildebrant and R. J. Koch, *The Theory of Topological Semigroups*, Vol. I, Marcel Dekker, Inc., New York and Basel, 1983; Vol. II, Marcel Dekker, Inc., New York and Basel, 1986.
3. R. Engelking, *General Topology*, 2nd ed., Heldermann, Berlin, 1989.
4. G. Gierz, K. H. Hofmann, K. Keimel, J. D. Lawson, M. W. Mislove, and D. S. Scott, *Continuous Lattices and Domains*. Cambridge Univ. Press, Cambridge (2003).
5. O. Gutik and D. Repovš, *On linearly ordered H-closed topological semilattices*, Semigroup Forum **77**:3 (2008), 474—481.
6. J. W. Stepp, *Algebraic maximal semilattices*. Pacific J. Math. **58**:1 (1975), 243—248.

Diagonal ranks of semigroups

I.V. Barkov, I.V. Kozhukhov

National Research University of Electronic Technology, Moscow, Russia
zvord@b64.ru
kozhuhov_i_b@mail.ru

Let S be a semigroup. The Cartesian product $S \times S$ can be considered as a right S-act, a left S-act and a (S, S)-bi-act with operations $(a, b)s = (as, bs)$, $s(a, b) = (sa, sb)$ $(s, a, b \in S)$. Then we call ${}_S(S \times S)$, $(S \times S)_S$ and ${}_S(S, S)_S$ *the left diagonal act, the right diagonal act and the diagonal bi-act over the semigroup S. The left, the right act and the bi-act ${}_S(S^n)$, $(S^n)_S$, ${}_S(S^n)_S$ of order n* are defined analogously where $S^n = \underbrace{S \times \ldots \times S}_{n}$ and $s(a_1, a_2, \ldots, a_n) = (sa_1, sa_2, \ldots, sa_n)$, $(a_1, a_2, \ldots, a_n)s = (a_1s, a_2s, \ldots, a_ns)$ for all $a_1, a_2, \ldots, a_n, s \in S$.

The right diagonal rank of semigroup S (denoted $\mathrm{rdr}S$) is the least cardinality of a generating set of the diagonal right act $(S \times S)_S$. *The left diagonal rank* ldrS and *bidiagonal rank* bdrS are defined analogously

Theorem 1. *If S is an infinite semigroup and* $\mathrm{rdrS} < \infty$, *then* $\mathrm{rdr_nS} \leq (\mathrm{rdrS})^{\mathrm{n}}$ *for odd n and* $\mathrm{rdr_nS} \leq (\mathrm{rdrS})^{\mathrm{n}-1}$ *for even n. An analogous statement holds for* ldrS *and* $\mathrm{ldr_nS}$.

Theorem 2. *Let A be an universal algebra with signature $\Sigma = \{\varphi_i | i \in I\}$, where all operations φ_i are unary. If A is finitely generated, then every irreducible generating set of A is minimal.*

Corollary 1. *For any act over a semigroup any irreducible generating set is minimal.*

Theorem 3. *Let S be an infinite semigroup which satisfies some non-trivial semigroup identity $u = v$ (where u, v are elements of a free semigroup). Then the diagonal right act over S is not finitely generated.*

Theorem 4. *If S is an infinite epigroup, then the diagonal bi-act ${}_S(S \times S)_S$ is not cyclic.*

Also for the semigroups of full transformations, partial transformations and binary relations we find the general form of the generating pairs.

1. Gallagher, P., 'On the finite and non-finite generation of diagonal acts' Comm. Algebra, 2006, 34, pp. 3123-3137.
2. Gallagher, P., Ruškuc, N., 'Finite generation of diagonal acts of some infinite semigroups of transformations and relations' Bull. Austral. Math. Soc., 2005, 72, pp. 139-146.
3. M. Kilp, U. Knauer, A.V. Mikhalev, 'Monoids, acts and categories' // Berlin, New York: W. de Gruyter, 2000 - 529 p.
4. L. Pyber, 'Groups of intermediate subgroup growth and a problem of Grothendieck', Duke Math. J. Volume 121, Number 1 (2004), 169-188.

On groups with classes of the conjugated uncomplemented subgroups

P. P. Baryshovets

National Aviation University, Kyiv, Ukraine
pbar@ukr.net

A subgroup A of a group G is called complemented in G, if in G there is such subgroup B, what $G = AB$ and $A \cap B = 1$. Ph. Hall [1] studied finite groups with complemented subgroups. Description of arbitrary (both finite and infinite) groups with such property (completely factorizable groups), obtained by N. V. Chernikova [2] and S. N. Chernikov [3].

Let K be a class of the conjugated subgroups of group G. If a subgroup A from the class K is complemented (not complemented) in group G, also all subgroups from this class are complemented (non complemented) in G. Not completely factorizable groups (i.e. group, in which not all subgroups are complemented) contain some number of classes of the conjugated noncomplemented subgroups. It would be natural to study influence of number of such classes on a structure of group in the same way as O. J. Shmidt made for property of normality.

Theorem 1. *An infinite locally finite not completely factorizable group has nonfinite number of classes of conjugated noncomplemented subgroups.*

Corollary 2. *Locally finite not completely factorizable group with finite number of classes of conjugated noncomplemented subgroups is finite.*

1. Hall Ph. Complemented groups // J. L. Math. Soc. — 1937. — 12. — P. 201 – 204.
2. Chernikova N. V. Groups with complemented subgroups // Mat. Sb. — 1956. — 39. — P. 273 – 292.
3. Chernikov S. N. Groups with systems complemented subgroups // Mat. Sb. — 1954. — 35. — P. 93 – 128.

New criteria for a ring to have a semisimple left quotient ring

V. V. Bavula

University of Sheffield
v.bavula@sheffield.ac.uk

Goldie's Theorem (1958, 1960), which is one of the most important results in Ring Theory, is a criterion for a ring to have a semisimple left quotient ring. The aim of the talk is to give four new criteria [1] (using a completely different approach and new ideas). The first one is based on the recent fact that for an *arbitrary* ring R the set $\mathcal{M}$ of *maximal* left denominator sets of R is a non-empty set [2]:

Theorem (The First Criterion). *A ring R has a semisimple left quotient ring Q iff $\mathcal{M}$ is a finite set, $\bigcap_{S\in\mathcal{M}} \mathrm{ass}(S) = 0$ and, for each $S \in \mathcal{M}$, the ring $S^{-1}R$ is a simple left Artinian ring. In this case, $Q \simeq \prod_{S\in\mathcal{M}} S^{-1}R$.*

The Second Criterion is given via the minimal primes of R and goes further than the First one in the sense that it describes explicitly the maximal left denominator sets S via the minimal primes of R. The Third Criterion is close to Goldie's Criterion but it is easier to check in applications (basically, it reduces Goldie's Theorem to the prime case). The Fourth Criterion is given via certain left denominator sets.

1. V. V. Bavula, New criteria for a ring to have a semisimple left quotient ring
2. V. V. Bavula, The largest left quotient ring of a ring, Arxiv:math.RA:1101.5107.

Derivations and identities for Kravchuk polynomials

L. Bedratyuk

Khmelnitsky National University, Khmelnitsky, Ukraine
leonid.uk@gmail.com

The binary Kravchuk polynomials $K_n(x,a)$ are defined by the following explicit formula

$$K_n(x,a) := \sum_{i=0}^{n} (-1)^i \binom{x}{i}\binom{a-x}{n-i}, n = 0, 1, \ldots .$$

We are interested in finding polynomial identities satisfied by the polynomials, i.e., identities of the form

$$P(K_0(x,a), K_1(x,a), \ldots, K_n(x,a)) = \text{const.},$$

where $P(x_0, x_1, \ldots, x_n)$ is a polynomial of $n+1$ variables. We introduce the notion of Kravchuk derivations of the polynomial algebra.

Definition 1. Derivations of $\mathbb{Q}[x_0, x_1, x_2, \ldots, x_n]$ defined by

$$D_{\mathcal{K}_1}(x_0) = 0, D_{\mathcal{K}}(x_n) = \sum_{i=1}^{n} \frac{1-(-1)^i}{2i} x_{n-i},$$
$$D_{\mathcal{K}_2}(x_0) = 0, D_{\mathcal{K}_2}(x_n) = \sum_{i=0}^{n-1} \frac{(-1)^{n+1+i}}{n-i} x_i, n = 1, 2, \ldots, n, \ldots,$$

are called **the first Kravchuk derivation** and **the second Kravchuk derivation** respectively.

We prove that any element of the kernel of the derivation gives a polynomial identity satisfied by the Kravchuk polynomials.

Theorem 1. *Let $P(x_0, x_1, \ldots, x_n)$ be a polynomial.*
(i) If $\mathcal{D}_{\mathcal{K}_1}(P(x_0, x_1, \ldots, x_n)) = 0$ then $P(K_0(x,a), K_1(x,a), \ldots, K_n(x,a)) \in \mathbb{Q}[a]$;
(ii) if $\mathcal{D}_{\mathcal{K}_2}(P(x_0, x_1, \ldots, x_n)) = 0$ then $P(K_0(x,a), K_1(x,a), \ldots, K_n(x,a)) \in \mathbb{Q}[x]$.

Also, we prove that any kernel element of the basic Weitzenbök derivations yields a polynomial identity satisfied by the Kravchuk polynomials. We describe the corresponding intertwining maps.

1. *Krawtchouk M.* Sur une generalisation des polynomes d'Hermite// C.R.Acad. Sci.— 1929. —**189**. — №17.— P.620 – 622.
2. *Pryzva G. Y.* Kravchuk orthogonal polynomials// Ukrainian Math. Journal.—1992. — **44**. –№7. — P.792 – 800.
3. *Bedratyuk L.* Derivations and identities for Fibonacci and Lucas polynomials// Fibonacci Quaterly, 2013

Identities with permutations and criteria of linear quasigroups

G. Belyavskaya

Institute of Mathematics and Computer Science, A.S. of Moldova
gbel1@rambler.ru

Let $(Q,\cdot)$ be a quasigroup. In this quasigroup consider the equalities, that we shall call identities with permutations, of the kind

$$\alpha_1(\alpha_2(x \otimes_1 y) \otimes_2 z) = \alpha_3 x \otimes_3 \alpha_4(\alpha_5 y \otimes_4 \alpha_6 z)$$

where x, y, z are variables, α_i, $i \in \overline{1,6}$, are permutations on the set Q, $(\otimes_k)$ are some parastrophies of $(Q,\cdot)$, $k \in \overline{1,4}$. In [3] it were given some identities with permutations providing linearity of distinct types of a quasigroup.

We prove that different types of linear quasigroups can be characterized by identities with permutations (translations) which depend on one fixed element.

A quasigroup $(Q,\cdot)$ is called linear from the left (from the right) if there exist a group $(Q,+)$, its automorphism φ (ψ) and a permutation $\beta\,(\alpha)$, such that $x \cdot y = \varphi x + \beta y$ $(x \cdot y = \alpha x + \psi y)$. If φ and ψ are automorphisms and $x \cdot y = \varphi x + c + \psi y$ where c is some fixed element of Q, then quasigroup $(Q,\cdot)$ is linear [1]. A quasigroup $(Q,\cdot)$ is called alinear if there exist a group $(Q,+)$, its antiautomorphisms $\overline{\varphi}$ and $\overline{\psi}$, an element $c \in Q$, such that $x \cdot y = \overline{\varphi} x + c + \overline{\psi} y$ [2].

Let $R_a x = x \cdot a, L_a x = a \cdot x,\ f_a \cdot a = a \cdot e_a = a,\ x * y = y \cdot x.$

Theorem 1. *A quasigroup $(Q,\cdot)$ is linear from the left (from the right if and only if it satisfies the identity with permutations*

$$(xy) \cdot z = R_{e_a} x \cdot L_a^{-1}(L_a y \cdot z) \;\; (R_a^{-1}(xy) \cdot z = x \cdot (R_a^{-1} y \cdot L_{f_a}^{-1} z))$$

for some element $a \in Q$ and is linear if and only if it satisfies both of these identities.

Theorem 2. *A quasigroup $(Q,\cdot)$ is linear if and only if it satisfies for some element $a \in Q$ the identity with permutations*

$$xy \cdot z = R_a x \cdot (\alpha_a y \cdot L_a^{-1} z) \text{ where } \alpha_a y = R_{e_a}^{-1} L_a^{-1} R_a L_{f_a} y.$$

Theorem 3. *A quasigroup $(Q,\cdot)$ is alinear if and only if it satisfies for some element $a \in Q$ the identity with permutations*

$$(x * y) \cdot z = L_a x \cdot (\beta_a^{-1} y * R_a^{-1} z) \text{ where } \beta_a y = R_{e_a}^{-1} R_a^{-1} L_a L_{f_a} y.$$

1. В.Д. Белоусов, Уравновешенные тождества в квазигруппах. Мат. сборник, 1966, **70**(112), no. 1, 55–97.
2. Г.Б. Белявская, А.Х. Табаров, Характеристика линейных и алинейных квазигрупп. Дискретная математика, 1992, **4**, вып. 2, 142–147.
3. Г.Б. Белявская, А.Х. Табаров, Тождества с подстановками, приводящие к линейности квазигруппы. Дискретная математика, 2009, **21**, вып. 1, стр. 36–52.

On representation and character varieties of Baumslag-Solitar groups

V. Beniash-Kryvets, I. Govorushko

Belarusian State University
benyash@bsu.by

A group G is said to be Hopfian if it is not isomorphic to one of its proper quotients. The family of Baumslag-Solitar groups $BS(p,q) = \langle a, b \mid ab^p a^{-1} = b^q \rangle$, where p and q are nonzero integers, was defined in [1]. The group $BS(p,q)$ is Hopfian if p and q are meshed, i.e. $p \mid q$ or $q \mid p$, or p and q have precisely the same prime divisors. If p and q are not meshed then $BS(p,q)$ is non-Hopfian.

We consider the case relatively prime integers p and q, $p, q \neq \pm 1$. In [2] all irreducible representations of Baumslag-Solitar groups are described. Goodman [3] shows the existence of some particular representations of $BS(p,q)$ and characterizes the geometry of the variety of n-dimensional representations of $BS(p,q)$ at these points.

Given a finitely generated group G, the set $R_n(G)$ of its representations over $GL_n(\mathbb{C})$ can be endowed with the structure of an affine algebraic variety (see [4]). The same holds for the set $X_n(G)$ of characters of representations over $GL_n(\mathbb{C})$. Let $X_n^s(BS(p,q))$ be the character variety of irreducible representations of $BS(p,q)$. Following theorems hold.

Theorem 1. *Every irreducible component W of the representation variety $R_n(BS(p,q))$ has dimension n^2 and W is a $\mathbb{Q}$-unirational variety. The number of irreducible components of $R_n(BS(p,q))$ is equal to the number of conjugacy classes of matrices B such that B^p and B^q are conjugated.*

Theorem 2. *Every irreducible component W of the variety $X_n^s(BS(p,q))$ has dimension 1 and W is a $\mathbb{Q}$-rational variety.*

Theorem 3. *The number of irreducible components of $X_n(BS(p,q))$ is equal to the number of conjugacy classes of diagonal matrices B such that B^p and B^q are conjugated.*

Theorem 4. *Let W be an irreducible component of the variety $X_n(BS(p,q))$ and $\chi_\varrho \in W$, where $\varrho = \varrho_1 \oplus \cdots \oplus \varrho_s$ is a sum of s irreducible representations ϱ_i, $i = 1, \ldots, s$. Then $\dim W = s$.*

1. G. Baumslag, D. Solitar. Some two-generator one-relator non-hopfian groups. Bull. AMS. 68:3 (1962), 199–201.
2. D. McLaury. Irreducible representations of Baumslag-Solitar groups. J. Group Theory 15:4 (2012), 543–552.
3. R.E. Goodman. Deformations of simple representations of two-generator HNN extensions. PhD thesis, The University of Oklahoma, Norman, OK, 2002.
4. A. Lubotzky, A. Magid. Varieties of representations of finitely generated groups. Memoirs AMS. 58 (1985), 1-116.

On verbally poor groups

A. Bier

Silesian University of Technology
agnieszka.bier@polsl.pl

Let G be a group and let V be a (finite) set of words. A *verbal subgroup* $V(G)$ generated by the set V is a subgroup generated by all values of words $v \in V$ in group G. Well known examples of verbal subgroups in a group G include the terms of the derived series of G, in particular the derived subgroup G', generated by a commutator word $[x, y]$, as well as the terms of the lower central series of G, power subgroups, etc.

In the talk we consider the verbal structure in nilpotent groups, i.e. groups with the lower central series terminating on the trivial subgroup. We say the a (residually) nilpotent group G is *verbally poor* if it has no verbal subgroups different from the terms of its lower central series, which is equivalent to saying that every verbal subgroup coincides with a term of the lower central series. One may think of a verbally simple group as a specific example of a verbally poor group (a group is said to be verbally simple if it has no nontrivial proper verbal subgroups). In the talk we will give further examples of verbally poor groups and discuss some conditions that guarantee a group to have this property.

1. Bier A., "On verbal subgroups of finitely generated nilpotent groups", A&DM 2 (2009) 1-10
2. Bier A., Fernandez-Alcober G. A., Sushchanskyy V., "A family of verbally poor groups" — to appear

Zero adequate ring

S.I. Bilavska

Ivan Franko National University of Lviv, Lviv, Ukraine
zosia_meliss@yahoo.co.uk

Let R be a commutative ring with $1 \neq 0$, $minR$ is the set of all minimal prime ideals of a ring R.

A ring R is called *clean*, if every element of R can be represented as sum of idempotent and invertible element [1]. A ring R is an *exchange ring* if the following condition holds: for any element $a \in R$ there exists an idempotent $e \in aR$ such that $1 - e \in (1 - a)R$ [1]. A ring R is *a ring of idempotent stable range 1* if the condition $aR + bR = R$ for every elements $a, b \in R$, implies that there exists an idempotent $e \in R$ such that $a + be$ is an invertible element of the ring R [2, 3].

Theorem 1. *[1, 2] Let R be a commutative ring. The following properties are equivalent:*
1) R is an exchange ring;
2) R is a clean ring;
3) R is a ring of idempotent stable range 1.

Theorem 2. *Let R be a commutative Bezout ring in which 0 is an adequate element. Then R is a ring of idempotent stable range 1.*

Theorem 3. *A zero adequate ring is an exchange ring.*

Theorem 4. *A zero adequate ring is a PM-ring.*

Theorem 5. *Let R be a commutative Bezout ring in which 0 is an adequate element and $minR$ is a finite set. Then R is a zero adequate ring.*

Theorem 6. *Let R be a commutative Bezout domain. If $\overline{0}$ is an adequate element of a factor-ring R/aR, then a is an adequate element of the domain R.*

Theorem 7. *Let R be a commutative Bezout domain in which for any non invertible and non zero element a the factor-ring R/aR is a ring in which 0 is an adequate element. Then R is an adequate domain.*

Theorem 8. *Let R be a commutative Bezout domain in which for any non zero and non invertible element a the $min(R/aR)$ is finite. Then R is an adequate domain if and only if R is a PM^*-ring.*

1. Nicholson W. K. Lifting idempotents and exchange rings // Trans. Amer. Math. Soc.–1977.–**229**.–P.269–278.
2. Camilo V. Yu H.P. Exchange rings, units and idempotents // Commun. Algebra.–1994.–**22**.–12.–P.4737–4749.
3. Zabavsky B. V. Elementary divisors of adequate rings with finite minimal prime ideals // Algebra and Topology, Lviv, LGU.–1996.–P.74–78.(Ukr)

On axiomatizibility of tosion classes of $S-$acts

Y. Bilyak

Ivan Franko National University of Lviv, Lviv, Ukraine
jbilyak@ukr.net

Let S be a semigroup with identity 1 and zero 0. Let M be a right $S-$act which is always unitary and centered. We denote the category of all centered right $S-$acts by $S-Act$.

We are interested in some properties of torsion theories. The notion of a torsion theory for $S-$acts has been introduced by J.K.Ludeman in [1].

Definition 1. We define torsion theory σ by setting pair of classes.

There are two classes:
$\mathcal{T}_\sigma = \{M \in S - Act | \sigma(M) = M\}$ (the class of $\sigma-$torsion acts)
$\mathcal{F}_\sigma = \{M \in S - Act | \sigma(M) = 0\}$ (the class of $\sigma-$torsion free acts).

Definition 2. Torsion theory σ is named hereditary if $\mathcal{T}_\sigma$ is closed under subacts.

In this talk we investigate hereditary torsion theories. Let us note that the relationship between hereditary torsion theories and quasi-filters of right congruences defined on the $S-$act was discussed in [3].

Definition 3. Hereditary torsion theory σ is said to be torsion-torsion-free (TTF) if the corresponding quasi-filter has a basis consisting of a single ideal.

Definition 4. A nonempty collection Σ of right ideals of S is a right quotient filter if Σ satisfies

1. if $A, B \in \Sigma$ and $f \in Hom(A, S)$ then $f^{-1}(B) \in \Sigma$.
2. if $I \subseteq S$ and $J \in \Sigma$ and for each $a \in J$ there is $T_a \in \Sigma$ with $aT_a \subseteq I$, then $I \in \Sigma$.

Let L_S be appropriate $S-$acts first order language.

Theorem 1. *Class $\mathcal{T}_\sigma$ is axiomatized if and only if $\mathcal{T}_\sigma$ is the variety defined by equalities of the form $\lambda x = \mu x$ for all $(\lambda, \mu) \in \Gamma \subseteq S \times S$.*

The main result of our talk is to establish a connection between the TTF hereditary torsion theory and right quotient filter .

1. J.K.Luedeman, Torsion theories and semigroups of quotient, Lecture notes in Math 998 (1983), 350-373.
2. R.Z.Zhang, K.P.Shum, Hereditary Torsion Classes of $S-$systems, Semigroup Forum Vol.52 (1996) 253-270, Springer-Verlag New York Inc.
3. R.Z. Zang, W.M. Gao, F.Y. Xi, Torsion theories and quasi-filters, Algebra Colloquium, (Academy of Sciences, Beijing) 1 (1994), 273-280.

On some functors between categories of rings and graphs

Y. T. Bilyak, M. Ya. Komarnytskyy

Ivan Franko National University of Lviv, Lviv, Ukraine
`jbilyak@ukr.net, mykola_komarnytsky@yahoo.com`

The concept of a zero-divisor graph $G(R)$ of commutative ring R was introduced by Beck [2]. He lets all elements of R be vertices of graph and works mostly with coloring of rings. Later D.F. Anderson and P.S. Livingston in [3] studied subgraph of $G(R)$ whose vertices re the nonzero zero-divisors of R. The zero-divisor graph of commutative ring has been studied extensively by R. Levy, J. Shapiro, A. Frazier, A. Lauve,R. Belshoff, J. Chapman, H. R. Maimani, M. R. Pournaki, S. Yassemi and etc.
In this talk we are interested in the following question: when such set correspondences are functorial? Among these correspondences we consider the map from a commutative ring to the zero-divisor graph of this ring, the composition of maps from a commutative ring to the commutative semiring of ideals of this ring and the map from a semiring to the corresponding zero-divisor graph.

In addition we construct the ideal zero-divisor graph of a commutative ring and establish functoriality of it. The functor constructed that way coincides with the composition of maps that was mentioned above. We prove that the ideal zero-divisor graph coincides with the zero-divisor graph of a ring if and only if the ring under consideration is a principal ideal domain.

We also consider several other correspondences between the categories of commutative rings and undirected graphs and establish their functoriality.

1. R.T.Bumby, D.M. Latch Categorical constructions in graph theory. //Internat. J. Math. and Math. Sci., - Vol.9, № 1, 1986, 1-16.
2. I.Beck Coloring of Commutative Rings. //J. Algebra, 116, 1988, 208-226. Lecture Notes in Mathematics 998, Springer-Verlag, Berlin, New York. 1983, P.350-373.
3. D.F.Anderson, P.S.Livingston The Zero-Divisor Graph of Commutative ring. //J. Algebra, 217, 1999, 434-447.
4. M.Aliradeh, A.K.Das, H.R.Maimani, M.R.Pournaki, S.Yassemi On the diamether and girth of zero-divisor graphs of posets. //Discrete Applied Mathematics, 160, 2012, 1319-1324.

A new physical approach to probability

Enzo Bonacci

Liceo Scientifico Statale "G.B. Grassi" in Latina
enzo.bonacci@liceograssilatina.org

The incompatibility between the Gambling Mathematics [5] and the Standard Probability Theory [2] leads to reciprocal accusations of fallacies: professional gamblers are considered cognitively biased aposteriorists using extra-scientific tools, while randomness theorists are regarded as rigid apriorists developing schemes inadequate for the complexity of reality. We can investigate the question in the tradition of the Physical Probability [3] by analyzing a binary event game on a semi-empirical basis and by exploring the physical roots of each predictive pattern employed. It emerges that the standard probability axioms, independently if motivated by the "Principle of Indifference" or originated by symmetry reasons, require the linear time, a physical assumption with paradoxical consequences when applied to chances [4] and recently challenged by alternative theories [1]. A non-linear temporal frame is instead necessary for the gambling strategies, whose common belief is the criticized "Law of Small Numbers" [6], but it is too manifold to be effectively modeled. Beyond such epistemological achievements there is also a heuristic result consisting of a "third way" model based on a recursive non-linear time hypothesis and on a new geometric distribution.

1. Bonacci E., Beyond Relativity, Vol. 43, Physical Sciences, Aracne, Rome, 2008. ISBN 9788854814844.
2. Feller W., An Introduction to Probability Theory and Its Applications, Vol. 2, 2nd ed., Wiley, New York, 1971. ISBN 9780471257097.
3. Gillies D.A., Philosophical Theories of Probability, Routledge, London, 2000. ISBN 9780415182751.
4. Handfield T., A Philosophical Guide to Chance: Physical Probability, Cambridge University Press, Cambridge, 2012. ISBN 9781107607354.
5. Thorp E.O., The Mathematics of Gambling, Lyle Stuart Inc., Secaucus (New Jersey), 1984. ISBN 9780897460194.
6. Tversky A., Kahneman D., Belief in the Law of Small Numbers, Psychological Bulletin 76 (2) (1971) pp. 105-110.

$\mathscr{L}$-cross-sections of the finite symmetric semigroup

E. A. Bondar

Luhansk Taras Shevchenko National University, Luhansk, Ukraine
bondareug@gmail.com

Let ρ be an equivalence relation on a semigroup S. A subsemigroup S' of S is called a ρ-cross-section S provided that S' contains exactly one representative from each equivalence class. It is well known that there exist always five equivalence relations $\mathscr{L}, \mathscr{R}, \mathscr{H}, \mathscr{D}, \mathscr{J}$ called Green's relations on any semigroup.

The purpose of our investigation is to describe $\mathscr{L}$-cross-sections in the finite symmetric semigroup $\mathscr{T}_n$. We define the family of subsets of n-element set X, so-called L-family and construct a semigroup L_X^Γ such that L_X^Γ is an $\mathscr{L}$-cross-section of the semigroup $\mathscr{T}_n$. Conversely, we find that any $\mathscr{L}$-cross-section of the symmetric semigroup $\mathscr{T}_n$ is given by L_X^Γ for a suitable L-family on X. We classify all $\mathscr{L}$-cross-sections in $\mathscr{T}_n$ up to an isomorphism.

It should be noted that $\mathscr{H}$- and $\mathscr{R}$-cross-sections of $\mathscr{T}_n$ were described in [1]. Examples of $\mathscr{L}$-cross-sections of the symmetric semigroup $\mathscr{T}_n$ can be found in [2].

1. *Pekhterev V.* $\mathscr{H}$- and $\mathscr{R}$-cross-sections of the full finite semigroup $\mathscr{T}_n$ // Alg. Discrete Math. — 2003. — 2(3). — P. 82–88.
2. *Kozhuhov I. B.* On transversals of the semigroup T_n for the relation $\mathscr{L}$ // Kamyanets-Podolsky, July, 1–7, 2007. — P. 110.

Expanding graphs from self-similar actions

I. Bondarenko

National Taras Shevchenko University of Kyiv, Kyiv, Ukraine
ibond.univ@gmail.com

A sequence of d-regular finite connected graphs forms an expanding family if the graphs have increasing number of vertices and there is a uniform bound on the spectral gap away from zero. Expanding graphs have numerous applications in theoretical computer science and pure mathematics.

Self-similar group actions are specific actions of groups on words over an alphabet, which preserve the length of words. Considering the action of group generators on words of fixed length n, we get the action (Schreier) graph Γ_n of the group.

Can the sequence Γ_n form a family of expanders? How to characterize this property in terms of self-similar actions? How to estimate the spectral gap of these graphs? All of these questions seem exciting.

Look how easy one can define the sequence of graphs Γ_n – just specify a generating set of a group, which is a finite automaton or, more generally, a finite wreath recursion. At the same time, the spectral properties of these graphs are not clear at all. The complete spectrum of graphs Γ_n was calculated only for a few self-similar groups, which model Sierpinski graphs, Hanoi graphs, etc. However, these graphs possess properties orthogonal to the expanding property. As for expanding graphs, there are several candidates among automata with three states over binary alphabet [2, Section 10] that may produce expanding family, but this remains unknown.

Finally, there exists a self-similar group whose sequence of action graphs forms a family of expanders! This follows from the result of Glasner and Mozes [1], who realized some groups with property (T) by finite automata. (For groups with property (T) every increasing sequence of finite connected action graphs forms an expanding family.) And if you are wondering about realization of Ramanujan graphs, check [1, Section 3.1] about realization of free groups by finite automata – another interesting problem in self-similar groups – and compare it with the original construction of Ramanujan graphs [3].

In my talk I will show a combinatorial construction of self-similar groups (wreath recursions) whose actions on finite words form a sequence of expanders.

1. Y. Glasner and S. Mozes. *Automata and square complexes*, Geom. Dedicata, Volume 111, 43–64 (2005).
2. R. I. Grigorchuk. *Some topics in the dynamics of group actions on rooted trees*, Modern problems of mathematics, Collected papers. In honor of the 75th anniversary of the Institute, Tr. Mat. Inst. Steklova, 273, MAIK Nauka/Interperiodica, Moscow, 72–191 (2011).
3. A. Lubotzky, R. Phillips, P. Sarnak. *Ramanujan graphs*, Combinatorica, Volume 8, No. 3, 261–277 (1988).

Characteristic subalgebras of Lie algebra associated with the Sylow p-subgroup of symmetric group

N. Bondarenko

Kyiv National University of Construction and Architecture, Kyiv, Ukraine
natvbond@gmail.com

Let p be a prime number and m be a fixed integer, $m > 2$. The Sylow p-subgroup P_m of the symmetric group S_{p^m} is isomorphic to the wreath product of cyclic groups of order p (see [3]). In [2] L.A. Kaloujnine constructed a special tableau representation of the group P_m and used this representation to study the structure of the group. In particular, he described all characteristic subgroups in terms of parallelotopic subgroups. In [4] similar tableau representation was constructed for the Lie algebra L_m associated with the lower central series of the group P_m and tableau technic was used to study the structure of L_m and its finitary analog in [1]. The elements of L_m can be identified with the tableaux of the form:

$$u = [c_1, c_2(x_1), \ldots, c_m(x_1, \ldots, x_{m-1})],$$

where $c_1 \in \mathbb{F}_p$, $c_i(\overline{x}_{i-1}) \in \mathbb{F}_p^{(0)}[x_1, \ldots, x_{i-1}]$ for $i = 2, 3, \ldots, m$ are reduced polynomials of degree $\leq p-1$ with respect to the ideal I_i generated by polynomials $x_1^p, x_2^p, \ldots, x_{i-1}^p$. The parallelotopic subalgebra of the Lie algebra L_m of characteristic $h =< h_1, h_2, \ldots, h_m >$ is defined as the subalgebra of all tableaux with characteristic less or equal than h. The characteristic subalgebra A of L_m is a subalgebra which is invariant under the action of every automorphism of L_m, i.e., $\varphi(A) \subset A$ for all $\varphi \in Aut(L_m)$.

In the following statement we prove the analog of Kaloujnin's result for the characteristics subalgebras of L_m.

Theorem 1. *A subalgebra of the Lie algebra L_m is characteristic if and only if it is a parallelotopic ideal.*

1. Bondarenko N.V., Gupta C.K., Sushchansky V.I. Lie algebra associated the group of finitary automorphisms of p-adic tree // Journal of Algebra. — 2010. — Vol. 324. — P. 2198-2218.
2. Kaloujnine L. La structure des p-groupes de Sylow des groupes Symmetriques finis. Ann. Sci l'Ecole Normal Superior, 1948, Vol 65, P. 239-276.
3. Sushchansky V.I., Sikora V.S. Operations on the permutation groups. Chernivci, "Ruta", 2003.
4. Sushchansky V.I, Netreba N.V. Lie algebras associated with Sylow p-subgroups of finite symmetric groups // Matematychni Studii. — 2005. — Vol. 24. — P. 127–138.

On local deformations of positive quadratic forms

V.M. Bondarenko, V.V. Bondarenko, Yu.M. Pereguda

Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv, Ukraine
Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv, Ukraine
S.P. Korolyov Zhytomyr Military Institute, National Aviation University
vitalij.bond@gmail.com, vitaliy.bondarenko@gmail.com, pereguda.juli@rambler.ru

We introduce the concepts of local deformations of quadratic forms over the field of real numbers and P-limit numbers for variables of any such positive quadratic form. Our main results are the complete descriptions of P-limit numbers for the quadratic Tits forms of graphs and posets, and of all the real numbers that can be P-limit for unit integer positive quadratic forms.

The representation type of groups with respect to the representations of constant Jordan type

V. M. Bondarenko, I. V. Lytvynchuk

Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv, Ukraine
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
vitalij.bond@gmail.com
iryna.l@ukr.net

Let $G = \langle g_1, \dots g_r\rangle \cong (\mathbb{Z}/p)^r$ be an elementary abelian p-group and k an algebraically closed field of characteristic p. A matrix representation λ of G is said to be of *constant Jordan type* if the Jordan canonical form of the nilpotent matrix $a_1\lambda(g_1 - 1) + \cdots + a_r\lambda(g_r - 1)$ is independent of $a_1, \dots, a_r \in k$, not all of which are equal to zero. If this Jordan canonical form consists of Jordan blocks of size $t_1, \dots, t_s$, then one says that the representation λ has *Jordan type* $JT(\lambda) = [t_1] \dots [t_s]$.

We call G of *cJ-finite representation type over k* if there are, up to equivalence, only finitely many indecomposable representations of constant Jordan type, and of *cJ-infinite representation type* if otherwise. In the last case G is called of *cJ-semiinfinite representation type* if there are only finitely many indecomposable representations in each dimension.

Modifying Drozd's definition, introduce the notion of a wild group with respect to the representations of constant Jordan type. We say that a matrix representation γ of G over the free associative k-algebra $\Sigma = k\langle x, y\rangle$ is *cJ-perfect* if, for any matrix representations φ and φ' of Σ over k, the representations $\gamma \otimes \varphi$ and $\gamma \otimes \varphi'$ of G over k satisfy the next conditions:

1) $\gamma \otimes \varphi$ is of constant Jordan type;

2) $\gamma \otimes \varphi$ and $\gamma \otimes \varphi'$ are equivalent implies φ and φ' are equivalent;

3) $\gamma \otimes \varphi$ is indecomposable if φ is indecomposable.

A cJ-perfect representation γ is said to be *proportional* if $JT(\gamma\otimes\varphi') = [JT(\gamma\otimes\varphi)]^q$ whenever $\dim \varphi' = q \dim \varphi$.

We call the group G of *cJ-wild representation type* (*over k*) if it has a proportional cJ-perfect representation over Σ.

We prove the following theorem (see [1]).

Theorem. *An elementary abelian p-group $G = (\mathbb{Z}/p)^r$ is of*
cJ-finite representation type if $r = 1$ (for any p),
cJ-semiinfinite representation type if $r = p = 2$,
cJ-wild representation type if otherwise.

1. V. M Bondarenko, I. V. Litvinchuk. The representation type of elementary abelian p-groups with respect to the modules of constant Jordan type, Algebra Discrete Math., 2012, 14, N1, pp. 29–36.

The Center of the Lattice of Factorization Structures

Dumitru Botnaru, Elena Baeş

State University of Tiraspol
Technical University of Moldova
dumitru.botnaru@gmail.com
baeselena@yahoo.it

The center of the lattice of factorization structures of the category of locally convex topological vector spaces is studied. Regarding the notions of category's theory see [1],[2], regarding left and right products see [3]. The center $\mathbb{C}(\mathbb{B})$ of the lattice $\mathbb{B}$ is defined by $\mathbb{C}(\mathbb{B}) = \{(\mathcal{P}, \mathcal{I}) \in \mathbb{B} \mid \mathcal{E}_p \subset \mathcal{P} \subset \mathcal{E}_u\}$ (see [2]).

Theorem 1. *Consider $(\mathcal{P}, \mathcal{I}) \in \mathbb{C}(\mathbb{B})$ and let $\mathcal{K}$ and $\mathcal{R}$ be those subcategories of the category $\mathcal{C}_2\mathcal{V}$ for which $(\mathcal{P}, \mathcal{I}) \in \mathbb{L}_\kappa(\mathcal{K}) \cap \mathbb{L}_\rho(\mathcal{R})$. Then the coreflector functor $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and the reflector functor $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ switch: $k \cdot r = r \cdot k$.*

Theorem 2. *Consider $(\mathcal{P}, \mathcal{I}) \in \mathbb{C}(\mathbb{B})$, $(\mathcal{P}, \mathcal{I}) \in \mathbb{L}_\kappa(\mathcal{K}) \cap \mathbb{L}_\rho(\mathcal{R})$ where the class $\mathcal{P}$ (respectively: the class $\mathcal{I}$) is $\mathcal{M}_u$-hereditary (respectively: $\mathcal{E}_u$-cohereditary). Then:*

1. $(\mathcal{P}, \mathcal{I}) = (\mathcal{E}'(\mathcal{K}), \mathcal{M}'(\mathcal{K})) = (\mathcal{P}''(\mathcal{R}), \mathcal{I}''(\mathcal{R}))$.

2. For any object $X \in \mid \mathcal{C}_2\mathcal{V} \mid$ the square $r^X \cdot k^X = k^{rX} \cdot r^{kX}$ is pushout and pullback.

3. $\mathcal{K} = \mathcal{C} *_s \mathcal{R}$ and $\mathcal{R} = \mathcal{K} *_d \mathcal{C}_1$, for any coreflective subcategory $\mathcal{C}$, with the condition $\mathcal{K} \subset \mathcal{C} \subset \widetilde{\mathcal{M}}$ and any reflective subcategory $\mathcal{C}_1$ with the condition $\mathcal{S} \subset \mathcal{C}_1 \subset \mathcal{R}$,

4. The pair $(\mathcal{K}, \mathcal{R})$ is a relative torsion theory (RTT).

5. The subcategory $\mathcal{K}$ is closed in relation to $(\varepsilon\mathcal{R})$-subobjects, and the subcategory $\mathcal{R}$ is closed in relation to $(\mu\mathcal{K})$-factorobjects.

Example 1. For the reflective subcategory $\mathcal{S}$ of spaces with weak topology, we have $\mathcal{P}''(\mathcal{S}) = (\varepsilon\mathcal{S}) \cdot \mathcal{E}_p = (\mathcal{E}_u \cap \mathcal{M}_u) \cdot \mathcal{E}_p = \mathcal{E}_u$.

Hence $\mathcal{I}''(\mathcal{S}) = \mathcal{M}_p$. Then $Q_{\mathcal{E}_u}(\widetilde{\mathcal{M}}) = \mathcal{C}_2\mathcal{V}$. For the pair $(\mathcal{C}_2\mathcal{V}, \mathcal{S})$, the coreflector functor $i : \mathcal{C}_2\mathcal{V} \to \mathcal{C}_2\mathcal{V}$ and the reflector functor $s : \mathcal{C}_2\mathcal{V} \to \mathcal{S}$ switch $s \cdot i = i \cdot s$.

Example 2. Let $\mathcal{R}$ be a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$, $\mathcal{S} \subset \mathcal{R}$ and $\mathcal{S} \neq \mathcal{R}$. Then $Q_{\mathcal{P}''(\mathcal{R})}(\widetilde{\mathcal{M}}) \neq \mathcal{C}_2\mathcal{V}$.

Theorem 3. *Let $\mathcal{R}$ be a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ $\mathcal{S} \subset \mathcal{R}$ and $\mathcal{S} \neq \mathcal{R}$. Then:*

1. $\widetilde{\mathcal{M}} *_s \mathcal{R} \neq \mathcal{C}_2\mathcal{V}$.

2. The coreflector functor $l : \mathcal{C}_2\mathcal{V} \to \widetilde{\mathcal{M}} *_s \mathcal{R}$ and the reflector functor $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ switch: $l \cdot r = r \cdot l$.

3. For any object $(E, u) \in |\mathcal{C}_2\mathcal{V}|$ the $\mathcal{L}$-coreplica's $(E, l(u))$ has the property $l(u) = max(u, rm(u))$, where $rm(u)$ is $\mathcal{R}$-replica of the object $(E, m(u))$.

1. Adamek J., Herrlich H., Strecker G. S., Abstract and concrete categories, Boston, 2005, 524 p.
2. Botnaru D., Structures bicatégorielles complémentaires, ROMAI J., 2009 v.5, N2, 2009, p.5-27.
3. Botnaru D., Ţurcanu A., The factorization of the right product of two subcategories, ROMAI J., 2010 v.VI, N2, p.41-53.

Semireflexive subcategories and the pairs of conjugate subcategories

D. Botnaru, O. Cerbu

Tiraspol State Universitaty
State Universitaty of Moldova
dumitru.botnaru@gmail.com
olga.cerbu@gmail.com

We examine some relations between semireflexive subcategories, semireflexive product, the right product of two subcategories and pairs of conjugate subcategories.

Concerning to terminology and notations see [1], to the semireflexive subcategories see [2], to the right product see [3].

Theorem 1. *Let $(\mathcal{K}, \mathcal{R})$ be a pair of conjugate subcategories and $\Gamma \in \mathbb{R}(\mathcal{M}_p)$. The followings affirmations are equivalent:*

1. $\mathcal{R} \times_{sr} \Gamma = \mathcal{K} \times_d (\mathcal{R} \cap \Gamma)$.
2. $v \cdot r = r \cdot v = l$.
3. $v \cdot k = k \cdot l \cdot k$.

Theorem 2. *For the functor $k, g \colon \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{C}_2\mathcal{V}$ the followings affirmations are equivalent:*

1. $k \cdot g = g \cdot k$.
2. *The subcategory $\mathcal{R} \times_{sr} \Gamma$ is $\mathcal{P}''(\Gamma)$-reflective.*

Theorem 3. *The followings affirmations are equivalent:*

1. $k(\varepsilon\mathcal{L}) \subset \mathcal{M}_p$.
2. *The subcategory $\mathcal{R} \times_{sr} \Gamma$ is $\mathcal{M}_p$-reflective.*

Theorem 4. *Let $(\mathcal{K}, \mathcal{R})$ be a pair of conjugate subcategories and $\Gamma \in \mathbb{R}(\mathcal{M}_p)$. Further, let that exists the object $X \in |\mathcal{S}|$, so that $X \in |\mathcal{K}|$ and it Γ-replique coincide with Γ_0-replique. Than $\mathcal{R} \times_{sr} \Gamma$ is a semireflexive subcategory.*

1. Botnaru D., *Structures bicatégorielles complémentaires,* ROMAI J., 2009, v.5, nr.2, p.5-27.
2. Botnaru D., Cerbu O., *Semireflexive product of two subcategories,* Proceedings of the Sixth Congress of Romanian Mathematicians, Bucharest, 2007, v.1, p.5-19.
3. Botnaru D., Ţurcanu A. *The factorization of the right product of two subcategories,* ROMAI Journal, 2010, v.6, nr.2, p.41-53.

On right product of two categories

Dumitru Botnaru, Alina Turcanu

Universitatea Tehnică din Moldova
dumitru.botnaru@gmail.com, alina_turcanu@yahoo.com

In this work we study the properties of the right product of two subcategories of the category of all local convex topological vectorial Hausdorff spaces $\mathcal{C}_2\mathcal{V}$.

About the terminology and notations see [1], about the right product see [2]; $\widetilde{\mathcal{M}}$ is the subcategory of the space with the Mackey topology; $\mathcal{S}$ is the subcategory of the space with the weak topology.

Theorem 1. *Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ be a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$, both non-zero. Then* 1. $\mathcal{K} *_d \mathcal{R} = \mathcal{S}_{\mu\mathcal{K}}(\mathcal{R})$.

2. *The subcategory $\mathcal{K} *_d \mathcal{R}$ is multiplicative (closed by the products).*

Theorem 2. *Let $\mathcal{V} = \mathcal{K} *_d \mathcal{R}$ be a reflective subcategory of category $\mathcal{C}_2\mathcal{V}$.*

1. *For any object A of subcategory $\mathcal{V} = \mathcal{K} *_d \mathcal{R}$ is true equality: $krA = kA$. Thus in the subcategory $\mathcal{V}$ the functors k and r verify the equality $k \cdot r = k$.*

2. *Let A be an object of the subcategory $\mathcal{V} = \mathcal{K} *_d \mathcal{R}$, $b_1 : kA \to Z$ and $b_2 : Z \to rA$ be two bimorphisms that verify the equality: $b_2 \cdot b_1 = r^A \cdot k^A$. Then: $Z \in |\mathcal{V}|$.*

3. *Class objects of the subcategory $\mathcal{K} *_d \mathcal{R}$ coincides with the class $\{X \in |\mathcal{C}_2\mathcal{V}|, krX = kX\}$.*

4. *The subcategory $\mathcal{V}$ is closed by the relation of $(\varepsilon\mathcal{R})$-factor objects.*

Theorem 3. *1. Let $\mathcal{K}$ be a $\mathcal{M}_u$-coreflective subcategory of the category $\mathcal{C}$. Then for any reflective subcategory $\mathcal{R}$ of the category $\mathcal{C}$ we have:*

a) *the left product $\mathcal{K} *_s \mathcal{R}$ is a $\mathcal{M}_u$-coreflective subcategory of the category $\mathcal{C}$;*

b) *the right product $\mathcal{K} *_d \mathcal{R}$ is a reflective subcategory of the category $\mathcal{C}$.*

2. Let $\mathcal{R}$ be a $\mathcal{E}_u$-reflective subcategory of the category $\mathcal{C}$. Then for any coreflective subcategory $\mathcal{K}$ of the category $\mathcal{C}$ we have:

a) *the right product $\mathcal{K} *_d \mathcal{R}$ is a $\mathcal{E}_u$-reflective subcategory of the category $\mathcal{C}$;*

b) *the left product $\mathcal{K} *_s \mathcal{R}$ is a coreflective subcategory of the category $\mathcal{C}$.*

Theorem 4. *1. Let $\mathcal{K}$ be a coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $\widetilde{\mathcal{M}} \subset \mathcal{K}$. Then for any reflective subcategory $\mathcal{R}$ of the category $\mathcal{C}_2\mathcal{V}$ we have:*

a) the left product $\mathcal{K} *_s \mathcal{R}$ is a coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $\widetilde{\mathcal{M}} \subset \mathcal{K} *_s \mathcal{R}$;

b) the right product $\mathcal{K} *_d \mathcal{R}$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.

2. Let $\mathcal{R}$ be a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $\mathcal{S} \subset \mathcal{R}$. Then for any coreflective subcategory $\mathcal{K}$ of the category $\mathcal{C}_2\mathcal{V}$ we have:

a) the right product $\mathcal{K} *_d \mathcal{R}$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $\mathcal{S} \subset \mathcal{K} *_d \mathcal{R}$;

b) the left product $\mathcal{K} *_s \mathcal{R}$ is a coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.

Let $\mathcal{M}$ be a class of monomorphisms, and $\mathcal{A}$ a subcategory of category $\mathcal{C}$. We view $\mathcal{S}_{\mathcal{M}}(\mathcal{A})$ the full subcategory of all $\mathcal{M}$-subobjects of objects of subcategory $\mathcal{A}$.

Theorem 5. *Let $\mathcal{K}$ and $\mathcal{R}$ be two non-zero subcategories of the category $\mathcal{C}_2\mathcal{V}$, (the first one is coreflective, the second is reflective). The following conditions are equivalent:*

1. $\mathcal{K} *_d \mathcal{R} = \mathcal{R}$.
2. *The subcategory $\mathcal{R}$ is closed by the relation of $(\mu\mathcal{K})$-subobjects.*

1. Botnaru D., Structures bicatégorielles complemmentaires, ROMAI Journal, v.5, Nr2, 2009, p.5-27.
2. Botnaru D., Cerbu O., Semireflexive product of two subcategories, Proc. of the Sixth Congress of Romanian Mathematicians, Bucharest, 2007, V.1, p.5-19.

On partial augmentations in a group ring

V. Bovdi

Ferenc Rákóczi II Transcarpathian Hungarian Institute, Beregovo
vbovdi@gmail.com

Let $V(\mathbb{Z}G)$ be the group of units of augmentation one in the integral group ring $\mathbb{Z}G$ of a not necessarily finite group G. Let $u = \sum_{g\in G} \alpha_g g \in V(\mathbb{Z}G)$ be a torsion unit of order $|u|$. The following conjectures are well known:

(i) A conjecture of H. Zassenhaus states that, in case G is finite, u is conjugate in $\mathbb{Q}G$ to an element of G.

(ii) A conjecture of A. Bovdi states that if $|u| = p^n$, then

$$T^{p^n}(u) = 1 \quad \text{and} \quad T^{p^i}(u) = 0 \quad \text{for} \quad i \neq n,$$

where $T^i(u) = \sum_{|g|=i} \alpha_g$ is the generalized trace of u.

It is well known (see for example [1, 5]) that these conjectures (as several other problems in group ring theory) can be reformulated in terms of the partial augmentations of u.

Note that the literature on (i) is vast, however (ii) has only been treated in a couple of papers [1, 2, 3, 4].

1. V. A. Artamonov and A. A. Bovdi. Integral group rings: groups of invertible elements and classical K-theory. In *Algebra. Topology. Geometry, Vol. 27 (Russian)*, Itogi Nauki i Tekhniki, pages 3–43, 232. Akad. Nauk SSSR Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1989. Translated in J. Soviet Math. **57** (1991), no. 2, 2931–2958.
2. M. Dokuchaev. Torsion units in integral group rings of nilpotent metabelian groups. *Comm. Algebra*, 20(2):423–435, 1992.
3. M. A. Dokuchaev and S. K. Sehgal. Torsion units in integral group rings of solvable groups. *Comm. Algebra*, 22(12):5005–5020, 1994.
4. S. Juriaans. Torsion units in integral group rings. *Comm. Algebra*, 22(12):4905–4913, 1994.
5. I. B. S. Passi. Group algebras. *Indian J. Pure Appl. Math.*, 43(2):89–106, 2012.

O-theorems for special class multiplicative functions

V. Breide

Odessa National University named by I.I.Mechnickov, Odessa, Ukraine
vika_brd@mail.ru

An asymptotic formula is constructed for a mean value of the sum-function $\Sigma_{n\leq x}\, f(n)$, where $f(n)$ is a multiplicative function that can be described in the next way: for any p – prime and nonnegative we have

$f(p) = a, a\epsilon N$

$f(p^k) = b + cp$, where $k = 2$ or 3, $c\epsilon N$

$f(p^k) = O(p^{k-2})$, where $k \geq 4$

And a, b, c – some constants, satisfied the condition

$ac + (b - a^2)c > 1$

The Dirichlet series for the sum-function is

$F(s) = \sum_{n=1}^{\infty} \dfrac{f(n)}{n^s} = \frac{\varsigma^a(s)\varsigma^c(2s-1)\varsigma^{b-a^2}(2s)\varsigma^{c-ac}(3s-1)\varsigma^{b-a}(3s)}{\varsigma^{ac+(b-a^2)c}(4s-1)} G(s)$

where $G(s) = \Pi_p(1 + O(1/p^{4s-2}))$ is regular function in the area Re s$>3/4$

Using Perron's formula and the estimates of $\varsigma(s)$ in the strip $0 \leq Res \leq 1+\varepsilon$, we obtain the following result

$$\Sigma_{n\leq x} f(n) = \frac{x}{2^c} * \frac{(\varsigma^{b-a^2}(2)\varsigma^{c-ac}(2)\varsigma^{b-a}(3))}{(\varsigma^{ac+(b-a^2)c}(3))} + O(x^{2/3+\varepsilon}) + \frac{(2x + logx + 1)}{3^{c-ac+1}} * \frac{(\varsigma^{a+c}(2/3)\pi^(- 1/6)\Gamma(1/3)\varsigma^{b-a^2}(4/3)\varsigma^{b-a}(2))}{(\Gamma(1/6)\varsigma^{ac+(b-a^2)c}(5/3)} + O(x^{\frac{(1-\sigma)}{3}(4c+2b-a^2-2ac)-\frac{\sigma}{3}(3b-a^2-2a)+3)}$$

1. K.Ramachandra (1976), Some Problems of analytic number theory, ACTA ARITHMETICA XXXI.

Quivers of semiperfect rings

N. Bronickaya, V. Darmosiuk

Mykolayiv National Sukhomlynsky University
natbron@mail.ru, darmos@gmail.com

We give a review of the results on the quivers of semiperfect rings, in particular we consider quiver of right Noetherian semiperfect ring and prime quiver of semiperfect ring [1]. If A be a semiperfect ring with prime radical $Pr(A)$ and $Pr(A)$ is a nilpotent ideal then the prime quiver $PQ(A)$ is connected if and only if the ring A is an indecomposable ring.

If A is a right Artinian ring the prime quiver is obtained from the quiver $Q(A)$ by changing all arrows going from one vertex to another one to one arrow.

Let A be a semiperfect ring and $e^2 = e \in A$, $e \neq 0$. We consider the quiver $Q(eAe)$ and the relations of this quiver with the corresponding subquiver of $Q(A)$.

1. M. Hazewinkel, N. Gubareni and V.V. Kirichenko, *Algebras, rings and modules.* Vol. **1**. Mathematics and Its Applications (Springer), 575. Springer, Dordrecht, 2004.

The correspondence between classes of objects of the category of all acts over different monoids and radical functors

N.Yu. Burban, O.L. Horbachuk

Ivan Franko National University of Lviv, Lviv, Ukraine
Pidstryhach Institute for Applied Problems of Mechanics and Mathematics
n.burban@mail.ru, o_horbachuk@yahoo.com

We construct the category $\bigcup \mathcal{S} - \mathcal{A}ct$. The objects of the category $\bigcup \mathcal{S} - \mathcal{A}ct$ will be the pairs $(S, A) =_S A$, where S is a monoid, A is an S-act. The morphisms of the category $\bigcup \mathcal{S} - \mathcal{A}ct$ will be semilinear transformations $(\varphi, \psi)\colon (S_1, A_1) \to (S_2, A_2)$, where $\psi\colon A_1 \to A_2$ is a map, and $\varphi\colon S_1 \to S_2$ is a homomorphism of monoids, and $\forall s \in S_1$, $\forall a \in A_1 \quad \psi(sa) = \varphi(s)\psi(a)$. Product of morphisms is defined naturally.

Definition 1. A preradical functor on $\bigcup \mathcal{S} - \mathcal{A}ct$ is a subfunctor of the identity functor on $\bigcup \mathcal{S} - \mathcal{A}ct$. A preradical functor T on $\bigcup \mathcal{S} - \mathcal{A}ct$ is called a radical functor if $T(I/T) = 0$, where I is an identity functor.

To a preradical functor T one can associate the class of objects of $\bigcup \mathcal{S} - \mathcal{A}ct$, namely $\mathcal{T}_T = \{(S, A) \mid T(S, A) = (S, A)\}$.

Throughout the whole text, all preradical functors on the category $\bigcup \mathcal{S} - \mathcal{A}ct$ are considered to be such that theirs restrictions on every category $S-$Act are preradical functors, i. e. $T(S, A) = (S, T_S(A))$, where T_S is the restriction of the functor T on the category $S-$Act.

Definition 2. A class $\mathcal{P}$ of objects of the category $\bigcup \mathcal{S} - \mathcal{A}ct$ is called a torsion class if it is closed under quotient objects, direct sums (if they exist) and normal extensions.

Theorem 1. *There is a bijective correspondence between idempotent radical functors of $\bigcup \mathcal{S} - \mathcal{A}ct$ and torsion classes of objects of $\bigcup \mathcal{S} - \mathcal{A}ct$.*

Definition 3. Let T be an idempotent preradical functor of the category $\bigcup \mathcal{S} - \mathcal{A}ct$, $(S, A) \in \mathcal{T}_T$. If every normal subobject of (S, A) belongs to $\mathcal{T}_T$, then T is called a pretorsion functor.

Definition 4. A pretorsion functor is called a torsion functor if it is a radical one.

Theorem 2. *There is a bijective correspondence between torsion functors of $\bigcup \mathcal{S} - \mathcal{A}ct$ and torsion classes of objects of $\bigcup \mathcal{S} - \mathcal{A}ct$, closed under normal subobjects.*

1. Kashu A.I. *Radicals and torsions in modules.* Kishinev: Stiinca, 1983, 152p. (in Russian).
2. Stenström B. *Rings of Quotients.* Berlin, Springer, 1975, 309p.
3. Mitchel B. *Theory of categories.* New York, London, Acad. Press, 1965, 273p.
4. N. Yu. Burban, O. L. Horbachuk. *Preradical and radical functors in different categories.* Matematychni Studii. V.35, No.2, 2011, pp. 121-127.

About one class of minimal ω-composition non-$\mathfrak{H}$-formations

L. Buyakevich

Gomel Engineering Institute
Ludab@rambler.ru

All groups considered are finite and τ is a subgroup functor such that for any group G all subgroups in $\tau(G)$ are subnormal in G. We use standard terminology of [1, 2]. Let $\mathfrak{H}$ be some class of groups. A formation $\mathfrak{F}$ is called a minimal τ-closed ω-composition non-$\mathfrak{H}$-formation [3] if $\mathfrak{F} \nsubseteq \mathfrak{H}$ but $\mathfrak{F}_1 \subseteq \mathfrak{H}$ for all proper τ-closed ω-composition subformations $\mathfrak{F}_1$ of $\mathfrak{F}$. In the work [4] the structure of the minimal τ-closed ω-composition non-$\mathfrak{H}$-formations, where $\mathfrak{H}$ is an arbitrary formation of classical type have been described.

In this paper describe the minimal τ-closed ω-composition non-$\mathfrak{N}^n$-formations.

Theorem 1. *A formation $\mathfrak{F}$ is a minimal τ-closed ω-composition non-$\mathfrak{N}^n$-formation $(n \geq 2)$ if and only if $\mathfrak{F} = c^{\tau}_{\omega} form G$ where G is a monolithic τ-minimal non-$\mathfrak{N}^n$-group and $P = G^{\mathfrak{N}^n}$ is the socle of G, where $P \nsubseteq \Phi(G)$ and either $\pi = \pi(Com(P)) \cap \omega = \varnothing$ or $\pi \neq \varnothing$ and $G = [P]H$, $H = [Q]N$, where $P = C_G(P)$ is an abelian p-group, $Q = C_H(Q) = H^{\mathfrak{N}^{n-1}}$ is a minimal normal subgroup of H, where $p \notin \pi(Com(Q))$.*

1. Skiba A.N., Shemetkov L.A., Multiply $\mathfrak{L}$-composition formations of finite groups, Ukrainsk. Math. Zh., 52 (6),2000, pp.783-797.
2. Skiba A.N., Algebra of formations, Minsk, Belaruskaja Navuka, 1997.
3. Shemetov L.A., Screens of gratduated formations, Proc. VI All-Union Symposium on the theory of groups, Kiev, Nauk. Dumka, 1980.
4. Belous L.I, Sel'kin V.M., Skiba A.N., About one class of critical ω-composition formations, Doklady of the National Academy of Sciences of Belarus, 50 (6), 2006, pp.36-40.

On some combinatorical problems for finite posets

I. V. Chervyakov

Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv, Ukraine
deathcalculus@gmail.com

In [1] V. M. Bondarenko introduced the concept of minimax equivalence of posets, which played an important role in solving a number of combinatorial problems of the theory of posets and their quadratic Tits forms (see, e.g., [2]–[4]). We study posets of special type up to the minimax equivalence.

1. V. M. Bondarenko, *On (min, max)-equivalence of posets and applications to the Tits form*, Bull. of the University of Kiev (series: Physics and Mathematics), N1 (2005), pp. 24-25.
2. V. M. Bondarenko, M. V. Stepochkina, *(Min, max)-equivalence partially ordered sets and quadratic Tits form*, Zb. Pr. Inst. Mat. NAN Ukr., 2, N3 (2005), pp. 18-58 (in Russian).
3. V. M. Bondarenko, M. V. Stepochkina, *(Min, max)-equivalence of posets, and nonnegative Tits forms*, Ukr. Mat. Zh., 60, N9 (2008), pp. 1157-1167 (in Russian); translation in Ukrainian Math. J.,60, N9 (2008), pp. 1349-1359.
4. V. M. Bondarenko, M. V. Stepochkina, *Description of partially ordered sets that are critical with respect to the nonnegativity of the quadratic Tits form*, Ukr. Mat. Zh., 61, N5 (2009), pp. 611-624 (in Russian); translation in Ukrainian Math. J.,61, N5 (2009), pp. 734-746.

Groups with minimal conditions on abelian subgroups

N.S. Chernikov

Institute of Mathematics of Nat. Acad. Sci. of Ukraine, Kyiv, Ukraine
chern@imath.kiev.ua

I plan to give a survey of results relating to groups above.The well-known S.N. Chernikov's I-groups are among them. S.N.Chernikov has disclosed the structure of non-periodic I-groups and also the important properties of periodic I-groups withoui min-ab (see Theorems 4.6, 4.10, 4.11 [1]). The following new theorem of the author completely describes such periodic groups. Below p is a prime, A_p is the Sylow p-subgroups of A. (Remind: an infinite non-abelian group with the minimal condition on abelian non-normal subgroups is called an I-group; min-ab is the minimal condition on abelian subgroups).

Theorem 1. *For the periodic group G the following statements are equivalent:*

(i) G is an I-group without min-ab.

(ii) Either G is Hamiltonian non-Chernikov, or for some $b \in G$ and Dedekind non-Chernikov subgroup A with Chernikov A_2 and with $1 < m = |G : A| < \infty$, $G = A < b >$ and $A \cap < b > \subseteq Z(A)$ and b induces on any A_p an automorphism of certain order $n_p | m$, raising each element of A_p to certain fixed power l_p, and also: $n_p = sp^k$ with some $s | p-1$ and $k \geq 0$; $1 \leq l_p \leq p-1$, if $p \nmid n_p$; $p \nmid n_p$, if $p \neq 2$ and A_p is infinite; n_2 is a power of 2; $n_2 = 1$, if A is Hamiltonian; $l_2 = \pm 1$, if A_2 is infinite; the subgroup $< A_p, 1 : n_p \neq m >$ is Chernikov.

Note: $p \nmid n_p$ and $n_p = m$, if A_p is non-Chernikov.

1. S.N. Chernikov. Groups with prescribed properties of the system of subgroups. — Moskow: Nauka, 1980. – 384 p. (in Russian)

On generalized soluble AFF–groups

O.Yu. Dashkova

Dnepropetrovsk National University
odashkova@yandex.ru

Antifinitary linear groups have been investigated in [1]. In [2, 3, 4, 5] the author has considered the analogues of antifinitary linear groups in the theory of modules over group rings. In this paper we continue the investigation of such analogues of antifinitary linear groups.

Let A be an $\mathbf{R}G$–module where $\mathbf{R}$ is an associative ring, G is a group. We say that a group G is an AFF–group if each proper subgroup H of G for which $A/C_A(H)$ is infinite, is finitely generated.

Let $FFD(G)$ be a set of all elements $x \in G$, such that $A/C_A(x)$ is a finite $\mathbf{R}$–module. $FFD(G)$ is a normal subgroup of G. Later on it is considered $\mathbf{R}G$–module A such that $\mathbf{R}$ is any associative ring, $C_G(A) = 1$. The main results are theorems 1–3.

Theorem 1. *Let A be an $\mathbf{R}G$–module, G be a locally soluble* AFF*–group. Then G is hyperabelian.*

Theorem 2. *Let A be an $\mathbf{R}G$–module, G be a finitely generated soluble* AFF*–group. If $A/C_A(G)$ is an infinite $\mathbf{R}$–module, then the following conditions holds: (1) $A/C_A(FFD(G))$ is a finite $\mathbf{R}$–module; (2) G has a normal abelian subgroup U such that $U \leq FFD(G)$ and G/U is a polycyclic quotient group.*

A group G is called a hyper(locally soluble) if G has an ascending series of normal subgroups $\langle 1\rangle = G_0 \leq G_1 \leq G_2 \leq \cdots \leq G_\gamma \leq \cdots \leq G_\delta = G$, such that each factor $G_{\gamma+1}/G_\gamma, \gamma < \delta$, is locally soluble (ch.1 [6]).

Theorem 3. *Let A be an $\mathbf{R}G$–module, G be a hyper(locally soluble)* AFF*–group. Then G has an ascending series of normal subgroups $\langle 1\rangle = G_0 \leq G_1 \leq G_2 \leq \cdots \leq G_\gamma \leq \cdots \leq G_\delta = G$, such that each factor $G_{\gamma+1}/G_\gamma, \gamma < \delta$, is hyperabelian.*

1. Kurdachenko L.A., Muñoz-Escolano J.M., Otal J. *Antifinitary linear groups. Forum Math.* **20**, no. 1, (2008), p. 7-44.
2. Dashkova O.Yu. Locally soluble AFN-groups. Problems of physics, mathematics and technics, no. 3(12), (2012), p. 58-64.
3. Dashkova O.Yu. On locally soluble AFN-groups. Algebra Discrete Math. **14**, no. 1, (2012), p. 37-48.
4. Dashkova O.Yu. On module analogues of antifinitary linear groups. Review of science. The South of Russia. Mathematical Forum. **6**, "Groups and graphs", Vladikavkaz, 2012, p. 18-24.
5. Dashkova O.Yu. Locally soluble AFA-groups. Ukr. Math. J. **65**, no. 4, (2013), p. 459-469.
6. Robinson D.J.S. Finiteness conditions and generalized soluble groups (1972). Ergebnisse Math. und ihrer Grenzgebiete. Vol. 1.

On certain class of a permutable inverse monoids

V. Derech

Vinnytsia National Technical University, Vinnytsia, Ukraine
derechvd@gmail.com

Let $N = \{1, 2, \ldots, n\}$. Denote by S_n and I_n respectively the symmetric group and the symmetric inverse semigroup on N. Let G be an arbitrary subgroup of the group S_n. Denote by $I(G)$ the set $\{\varphi \in I_n \colon \varphi \subseteq \eta \, for\, some\, \eta \in S_n\}$. It is easy to verify that $I(G)$ is an inverse subsemigroup of the semigroup I_n. It is clear that $I(S_n) = I_n$.

A semigroup S is called a permutable if any two congruences on S commute with respect to composition.

Definition 1. A subgroup G of the symmetric group S_n is called globally-transitive if for any A and B ($A \subseteq N, B \subseteq N$) such that $|A| = |B|$ there exists $\xi \in G$ such that $(A)\xi = B$.

Example 3. The alternating group A_n is a globally-transitive group.

Example 4. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{pmatrix}$. Denote by $\langle \alpha, \beta \rangle$ a subgroup of S_5 generated by $\{\alpha, \beta\}$. The group $\langle \alpha, \beta \rangle$ contains 20 elements. It is easy to check that $\langle \alpha, \beta \rangle$ is a globally-transitive group.

Theorem 1. *Inverse monoid $I(G)$ is a permutable if and only if a group G is globally-transitive group.*

Next, let G be a globally-transitive subgroup of S_n. We describe congruences on $I(G)$. We also describe stable orders on $I(A_n)$, where A_n is the alternating group.

IP-loops generalization

Ivan Deriyenko

Kremenchuk National University, Kremenchuk, Ukraine
ivan.deriyenko@gmail.com

Generalized inverse identities are introduced. All the loops meeting these identities will be called generalized IP-loops. It is shown that all the loops of the order $n < 7$ are generalized IP-loops, only for $n \geq 7$ there exist loops which are not generalized IP-loops, the so-called rigid loops. Let $Q(\cdot)$ be a quasigroup on the set $Q = \{1, 2, 3, ..., n\}$ and L_i, R_i, ϕ_i left, right and middle translations [1, 2, 3]. Establish a connection between the formulas

$$\alpha(x) \cdot \beta(x \cdot y) = \gamma(y),$$

$$\beta(y \cdot x) \cdot \alpha(x) = \gamma(y),$$

$$\alpha(x) \cdot \beta(y \cdot x) = \gamma(y),$$

$$\beta(x \cdot y) \cdot \alpha(x) = \gamma(y),$$

$$\rho(x \cdot y) = \sigma(y) \cdot \omega(x),$$

where $\alpha, \beta, \gamma, \rho, \sigma, \omega$ are permutations on the set Q and the following relations:

$$L_i = \beta^{-1}\phi_{\gamma(i)}\alpha,$$

$$L_i = \beta^{-1}\phi^{-1}_{\gamma(i)}\alpha,$$

$$R_i = \beta^{-1}\phi_{\gamma(i)}\alpha,$$

$$R_i = \beta^{-1}\phi^{-1}_{\gamma(i)}\alpha,$$

$$L_i = \rho^{-1}R_{\omega(i)}\sigma.$$

1. Belousov V.D. Foundations of the theory of quasigroups and loops, (Russian), Moscow, 1967.
2. Denes I. and Keedwell R.D. Latin squares and their applications, Akademai Kiado, Budapest, 1974.
3. Deriyenko I.I. Configurations of conjugated permutations, Quasigroups and Related Systems 18 (2010), 17-24.

On properties of monomial matrices over commutative rings

R. F. Dinis

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ruslanadinis@ukr.net

We continue to study monomial matrices of the form

$$M(t,k,n) = \begin{array}{c} \overbrace{\hphantom{0 \quad \dots \quad 0}}^{k} \hphantom{\quad 0 \quad \dots \quad 0 \quad t} \\ \begin{pmatrix} 0 & \dots & 0 & 0 & \dots & 0 & t \\ 1 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & t & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & \dots & t & 0 \end{pmatrix} \end{array}$$

over a commutative ring K, where $t \in Rad\, K$ (see [1]).

In particular, the following theorems are proved:

Theorem 1. *Let K be a commutative local principal ideal ring of length 2 and t a generator of its Jacobson radical. If $n > 5$, then the matrix $M(t, n-4, n)$ is reducible over K.*

Theorem 2. *Let K and t be as in Theorem 1. If $n > 6$ is odd, then there exists $1 < k < n$ such that $(k, n) = 1$ and the matrix $M(t, k, n)$ is reducible over K.*

1. R. F. Dinis, O. A. Tylyshchak. On reducibility of matrices of some form over local principal ideal domains // Nauk. Visn. Uzhgorod. Univ. Ser. Mat. Inform., 2012, vol. 23, N1, P. 57-62 (in Ukrainian).

On some variants of Schur and Baer theorems

M.R. Dixon, L.A. Kurdachenko, A.A. Pypka

University of Alabama
Dnipropetrovsk National University
Dnipropetrovsk National University
mdixon@as.ua.edu, lkurdachenko@i.ua, pypka@ua.fm

I. Schur was first who started to consider the relationships between the properties of central factor-group $G/\zeta(G)$ of a group G and its derived subgroup [1]. In particular, from his results follows that if $G/\zeta(G)$ is finite then $[G,G]$ is also finite. On the other hand, it is well known a following fact: if $G = \zeta_k(G)$, then $\gamma_{k+1}(G) = \langle 1\rangle$. Based on Schur's theorem R. Baer [2] has considered a following generalization of Schur's theorem. He proved that if the factor-group $G/\zeta_k(G)$ is finite, then $\gamma_{k+1}(G)$ is likewise finite. This time there are many generalizations of Schur's theorem. Among them and a following automorphic variant. Let G be a group and A a subgroup of $\mathbf{Aut}(G)$. As usual we put

$$C_G(A) = \{g \in G | \alpha(g) = g \text{ for each } \alpha \in A\},\ [G,A] = \langle g^{-1}\alpha(g) | g \in G\rangle.$$

Theorem 1. *Let G be a group and A a subgroup of $\mathbf{Aut}(G)$. Suppose that $\mathbf{Inn}(G) \le A$ and index $|A : \mathbf{Inn}(G)| = \mathbf{k}$ is finite. If $G/C_G(A)$ is finite, then $[G,A]$ is finite. Moreover, $|[G,A]| \le \mathbf{kt}^d$, where $d = \frac{1}{2}(log_p\mathbf{t} + 1)$, $|G/C_G(A)| = \mathbf{t}$.*

Corollary 1. *[3] Let G be a group and suppose that $G/C_G(\mathbf{Aut}(A))$ is finite. Then $[G, \mathbf{Aut}(A)]$ is finite.*

Theorem 2. *Let G be a group and A a subgroup of $\mathbf{Aut}(G)$. Suppose that $\mathbf{Inn}(G) \le A$ and index $|A : \mathbf{Inn}(G)| = \mathbf{k}$ is finite. Let Z be an upper A-hypercenter of G. Suppose that $\mathbf{zl}(G,A) = \mathbf{m}$ is finite and G/Z is finite, $|G/Z| = \mathbf{t}$. Then there exists the function β_1 such that $|\gamma_\infty(G,A)| \le \beta_1(\mathbf{k},\mathbf{m},\mathbf{t})$.*

1. Schur I. Über die Darstellungen der endlichen Gruppen durch gebrochene lineare substitutionen. J. Reine Angew. Math. - 127(1904), 20-50.
2. Baer R. Endlichkeitskriterien für Kommutatorgruppen. Math. Annalen - 124(1952), 161-177.
3. Hegarty P. The absolute centre of a group. Proc. Journal of Algebra - 169(1994), 929-935.

Partial cohomology of groups

M. Dokuchaev[1], M. Khrypchenko[2]

Institute of Mathematics and Statistics, University of São Paulo
dokucha@gmail.com, nskhripchenko@gmail.com

Let G be a group. By *a partial G-module* we mean a commutative monoid $A$and a partial action [1, 3] $\theta = \{\theta_x : A_{x^{-1}} \to A_x\}_{x\in G}$ of G on A such that each ideal A_x is generated by a central idempotent 1_x. A morphism of partial G-modules $\varphi : (A,\theta) \to (B,\tau)$ is a morphism of partial actions [2] such that its restriction on each A_x is a homomorphism of monoids $A_x \to B_x$. The category of partial G-modules will be denoted by $\mathsf{pMod}(G)$.

Let $(A,\theta) \in \mathsf{pMod}(G)$ and $n \in \mathbb{N}$. Denoteby $C^n(G,A)$ the abelian group of functions $f : G^n \to A$ such that $f(x_1,\dots,x_n) \in (A_{x_1}A_{x_1x_2}\dots A_{x_1\dots x_n})^*$. By $C^0(G,A)$ we shall mean A^*. For any $f \in C^n(G,A)$ and $x_1,\dots,x_{n+1} \in G$ define $(\delta^n f)(x_1,\dots,x_{n+1})$ to be $\theta_{x_1}(1_{x_1^{-1}}f(x_2,\dots,x_{n+1}))\prod_{i=1}^n f(x_1,\dots,x_ix_{i+1},\dots,x_{n+1})^{(-1)^i} f(x_1,\dots,x_n)^{(-1)^{n+1}}$. Here the inverse elements are taken in the corresponding ideals.If $n = 0$ and $a \in A^*$, we set $(\delta^0 a)(x) = \theta_x(1_{x^{-1}}a)a^{-1}$. The map δ^n is a homomorphism $C^n(G,A) \to C^{n+1}(G,A)$ such that $\delta^{n+1}\delta^n f$ is the identity of $C^{n+2}(G,A)$ for any $f \in C^n(G,A)$. One naturally defines the group $H^n(G,A) = \ker\delta^n/\mathrm{im}\,\delta^{n-1}$ of *partial n-cohomologies* of G with values in A, $n \geqslant 1$ ($H^0(G,A) = \ker\delta^0$).

Proposition 1. *For any $n \geqslant 0$ the map $(A,\theta) \mapsto H^n(G,A)$ is a functor from $\mathsf{pMod}(G)$ to the category of abelian groups.*

We give a description of the partial Schur Multiplier of G in terms of $H^n(G,A)$.

A partial G-module (A,θ) is called *inverse* if A isinverse and $E(A)$ is generated by 1_x $(x \in G)$. For any $(A,\theta) \in \mathsf{pMod}(G)$ we can restrict θ to the inverse subsemigroup $\tilde{A}$ generated by invertible elements of all ideals of A. Then $(\tilde{A},\theta)$ is inverse and $H^n(G,A) \cong H^n(G,\tilde{A})$, so it is sufficient to study cohomology groups with values in inverse partial G-modules.

Theorem 1. *Let (A,θ) be an inverse partial G-module. Then there is a (faithful) partial representation [3] Γ of G in a monoid S satisfying $S = \langle\Gamma(x) \mid x \in G\rangle$ and an S-module [4] structure on A such that $H^n(G,A) \cong H^n_S(A)$ for all $n \geqslant 0$.*

1. M. Dokuchaev, B. Novikov, Partial projective representations and partial actions, *Journal of Pure and Applied Algebra*, **214** (2010), 251–268.
2. M. Dokuchaev, B. Novikov, Partial projective representations and partial actions II, *Journal of Pure and Applied Algebra*, **216** (2012), 438–455.
3. R. Exel, Partial actions of groups and actions of inverse semigroups, *Proc. Amer. Math. Soc.*, **126 (12)** (1998), 3481–3494.
4. H. Lausch, Cohomology of inverse semigroups, *J. Algebra*, **35** (1975), 273–303.

[1]Partially supported by CNPq of Brazil: 307659/2009-7
[2]Supported by FAPESP of Brazil: 2012/01554-7

Canonical form of triangular $(0,1)-$matrices

T.S. Dovbniak

Vasyl Stefanyk Precarpathian National University
iradis07@gmail.com

Let $G=(V,E)$ be a directed graph with the order $|V|=n$. It is the triangular incidence $(0,1)-$matrix with units on its diagonal for each graph. Let $\hbar$ be the function of $(0,1)-$matrix A

$$\hbar=\sum_{j=1}^{i}\sum_{i=j}^{n}a_{ij}(i-j)$$

which shows a distance to the diagonal.

$\hbar(k)$ is a distance of k vertex to the diagonal

$$\hbar(k)=\sum_{j=1}^{k}a_{ik}(i-k)+\sum_{j=k}^{n}a_{kj}(k-j).$$

Lemma 1. *For each graph G we have the equality*

$$\hbar=\frac{1}{2}\sum_{i=1}^{n}\hbar(i)$$

.

If value of the function $\hbar$ is minimal, we have the canonical form of a triangular $(0,1)-$matrix.

1. Zatorsky R.A. Triangular matrix calculus and its application. – Ivano-Frankivs'k: Publishing house "Simyk", 2010. – 456 p.
2. Харари Ф. Теория графов. – М.: Мир, 1973. – 304 с.

Tilting theory and unitary representations of linear groups over algebras

Y. A. Drozd

Institute of Mathematics, National Academy of Sciences of Ukraine
y.a.drozd@gmail.com

It is a report on a common work with V. I. Bekkert and V. M. Futorny.

Let A be a finite dimensional algebra over the field $\mathbb{C}$ of complex numbers. We say that the algebra A is *Dynkinian, Euclidean* or *tubular* if it is derived equivalent, respectively, to the path algebra of a Dynkin or Euclidean quiver or to a weighted projective line of genus 1 [3]. (Note that our definition of tubular algebras is more general then that of Ringel [5].) A *linear group over* A is, by definition, the group $\mathrm{GL}(P, A)$ of automorphisms of a finitely generated projective A-module P.

For a Lie group G we denote by $\hat{G}$ its *dual space*, i.e. the space of irreducible unitary representations of G [4]. A subset $U \subseteq \hat{G}$ is said to be *thick* if it is open, dense and $\mu(\hat{G} \setminus U) = 0$, where μ is the Plancherel measure on $\hat{G}$.

Theorem 1. *Let $G = \mathrm{GL}(P, A)$, where A is tame piecewise hereditary algebra. Then $\hat{G}$ contains a thick subset U isomorphic to*

$\prod_{i=1}^{k} \widehat{\mathrm{GL}}(n_i, \mathbb{C})$ *if A is Dynkinian;*

$\prod_{i=1}^{k} \widehat{\mathrm{GL}}(n_i, \mathbb{C}) \times X(m)$ *if A is Euclidean;*

$\prod_{i=1}^{k} \widehat{\mathrm{GL}}(n_i, \mathbb{C}) \times X(m) \times X'(m')$ *if A is tubular,*

for some k, n_i, m, m' (possibly $k = 0$, or $m = 0$, or $m' = 0$).

(For the first two cases see [2, 1].)

Here X and X' are some open subsets of $\mathbb{P}^1_{\mathbb{C}}$, $X(m)$ is the orbit space $X^{[m]}/S_m$, where $X^{[m]} = \{(a_1, a_2, \dots, a_m) \mid a_i \neq a_j \text{ if } i \neq j\}$, S_m is the symmetric group.

1. V. Bekkert, Y. Drozd and V. Futorny. Tilting, deformations and representations of linear groups over Euclidean algebras. arXiv:0810.2037 [math.RT].
2. Y. A. Drozd. Matrix problems, small reduction and representations of a class of mixed Lie groups. Representations of Algebras and Related Topics. Cambridge Univ. Press, 1992, 225-249.
3. W. Geigle and H. Lenzing. A class of weighted projective curves arising in representation theory of finite dimensional algebras. Singularities, Representations of Algebras and Vector Bundles. Lecture Notes in Math. 1273, Springer, 1987, 265–297.
4. A. A. Kirillov. Elements of the Theory of Representations. Nauka, Moscow, 1978.
5. C. M. Ringel. Tame Algebras and Integral Quadratic Forms. Lecture Notes in Math. 1099, Springer, 1984.

Atoms in p-localizations of the stable homotopy category

Y. Drozd, P. Kolesnyk

Institute of Mathematics, National Academy of Sciences of Ukraine
`y.a.drozd@gmail.com, iskorosk@gmail.com`

We study p-localizations $\mathscr{S}_p^n$, where p is an odd prime, of the full subcategories $\mathscr{S}^n$ of the stable homotopy category $\mathscr{S}$ [2] consisting of polyhedra, i.e. finite CW-complexes, having cells in n successive dimensions. Following Baues [1], we call a polyhedron $X \in \mathscr{S}_p^n$ an *atom* if it is indecomposable into a wedge of non-contractable polyhedra and does not belong to $\mathscr{S}_p^{n-1}$. We classify atoms (indecomposable objects) in $\mathscr{S}_p^n$ for $n \leqslant 4(p-1)$ and obtain the following result.

Theorem 1.

1. *If $n \leqslant 2p-1$, the classification of atoms in $\mathscr{S}_p^n$ is* essentially finite. *Namely, every atom from $\mathscr{S}_p^n$ has at most 4 cells.*
2. *If $2p \leqslant n \leqslant 4(p-1)$, the classification of atoms in $\mathscr{S}_p^n$ is* tame, *i.e. every atom is given by a discrete (combinatorial) invariant and an irreducible polynomial over the residue field $\mathbb{Z}/p$.*
3. *If $n > 4(p-1)$, the classification of atoms in $\mathscr{S}_p^n$ is* wild, *i.e. contains classification of representations of any finitely generated algebra over $\mathbb{Z}/p$.*

The proofs are based on the technique of triangulated categories and matrix problems, as in [3]. For details see [5].

We also describe *genera* of p-local polyhedra in $\mathscr{S}^n$ for $n \leqslant 4(p-1)$, i.e. polyhedra having isomorphic p-localizations [4]. Namely, we prove that the number of polyhedra belonging to the genus of an atom is either 1 or $(p-1)/2$ depending of its local structure.

1. Baues H.-J. Atoms of Topology. Jahresber. Dtsch. Math.-Ver. 104, 4 (2002), 147-164.
2. Cohen J. M. Stable Homotopy. Lecture Notes in Math. 165 (1970).
3. Drozd Y. Matrix problems, triangulated categories and stable homotopy types. São Paulo J. Math. Sci. 4 (2010), 209-249.
4. Drozd Y. and Kolesnyk P. On genera of polyhedra. Cent. Eur. J. Math. 10 (2012), 401-410.
5. Drozd Y. and Kolesnyk P. Atoms in the p-localization of stable homotopy category. arXiv:1305.2589 [math.AT].

Menger algebras of n-place interior operations

W.A. Dudek, V.S. Trokhimenko

Wroclaw University of Technology, Wroclaw, Poland
Pedagogical University, Vinnitsa, Ukraine
wieslaw.dudek@pwr.wroc.pl
vtrokhim@gmail.com

Let A be a nonempty set, $\mathfrak{P}(A)$ the family of all subsets of A, $\mathcal{T}_n(\mathfrak{P}(A))$ the set of all n-place transformations of $\mathfrak{P}(A)$, i.e., maps $f\colon\ (\mathfrak{P}(A))^n \to \mathfrak{P}(A)$, where $(\mathfrak{P}(A))^n$ denotes the n-th Cartesian power of the set $\mathfrak{P}(A)$. For arbitrary $f, g_1, \ldots, g_n \in \mathcal{T}_n(\mathfrak{P}(A))$ we define the $(n+1)$-ary composition $f[g_1 \ldots g_n]$ by putting:

$$f[g_1 \ldots g_n](X_1, \ldots, X_n) = f(g_1(X_1, \ldots, X_n), \ldots, g_n(X_1, \ldots, X_n))$$

for all $X_1, \ldots, X_n \in \mathfrak{P}(A)$.

The $(n+1)$-ary operation $\mathcal{O}\colon (f, g_1, \ldots, g_n) \mapsto f[g_1 \ldots g_n]$ is called the *Menger superposition* of n-place functions. The algebra $(\mathcal{T}_n(\mathfrak{P}(A)), \mathcal{O})$ is a *Menger algebra*, i.e., the operation $\mathcal{O}$ satisfies the so-called *superassociative law*:

$$f[g_1 \ldots g_n][h_1 \ldots h_n] = f[g_1[h_1 \ldots h_n] \ldots g_n[h_1 \ldots h_n]],$$

where $f, g_i, h_i \in \mathcal{T}_n(\mathfrak{P}(A))$, $i = 1, \ldots, n$.

An n-place transformation f of $\mathfrak{P}(A)$ is called an *n-place interior operation* or an *n-place interior operator* on the set A if $f[f \ldots f] = f$, $f(X_1, \ldots, X_n) \subseteq X_1 \cap \ldots \cap X_n$ for $X_1, \ldots, X_n \in \mathfrak{P}(A)$, and $f(X_1, \ldots, X_n) \subseteq f(Y_1, \ldots, Y_n)$ for $X_1 \subseteq Y_1, \ldots, X_n \subseteq Y_n$.

Theorem 1. *An n-place transformation f of $\mathfrak{P}(A)$ is an n-place interior operation on A if and only if for all $X_1, \ldots, X_n, Y_1, \ldots, Y_n \in \mathfrak{P}(A)$ we have*

$$f(X_1 \cap Y_1, \ldots, X_n \cap Y_n) \subseteq f(f(X_1^n), \ldots, f(X_1^n)) \cap f(Y_1^n) \cap Y_1 \cap \ldots \cap Y_n,$$

where X_1^n means $X_1, X_2, \ldots, X_n$.

Theorem 2. *Every n-place transformation f on $\mathfrak{P}(A)$ such that*

$$f(X_1^n) = f(A, \ldots, A) \cap X_1 \cap \ldots \cap X_n.$$

holds for all $X_1, \ldots, X_n \in \mathfrak{P}(A)$, is an n-place interior operation on A.

Theorem 3. *The Menger superposition of given n-place interior operations $f, g_1, \ldots, g_n$ defined on the set A is an n-place interior operation on A if and only if for each $i = 1, \ldots, n$ we have*

$$g_i[f \ldots f][g_1 \ldots g_n] = f[g_1 \ldots g_n].$$

Theorem 4. *A Menger algebra (G, o) of rank n is isomorphic to a Menger algebra of n-place interior operations on some set if and only if it satisfies the following three identities*

$$x[x \ldots x] = x,$$

$$x[y \ldots y] = y[x \ldots x],$$

$$x[y_1 \ldots y_n] = x[y_1 \ldots y_1] \ldots [y_n \ldots y_n].$$

More results on Menger algebras of multiplace functions one can find in the book [1].

1. W. A. Dudek, V. S. Trokhimenko V. S. (2012). Algebras of multiplace functions, De Gruyter, Versita. ISBN 978-3-11-026928-4.
2. W. A. Dudek, V. S. Trokhimenko V. S. (2013). Menger algebras of n-place interior operations, Algebra Universalis (in print).

Semiscalar equivalence of polynomial matrices and solutions of the matrix linear polynomial equations

N. Dzhaliuk, V. Petrychkovych

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the NAS of Ukraine
nataliya.dzhalyuk@gmail.com
vas_petrych@yahoo.com

The matrix linear polynomial equations $A(\lambda)X(\lambda)+Y(\lambda)B(\lambda)=C(\lambda)$ (the Sylvester equation), $A(\lambda)X(\lambda)+B(\lambda)Y(\lambda)=C(\lambda)$ (the Diophantine equation) play a fundamental role in many problems in control and dynamical systems theory. The solutions of the matrix polynomial Sylvester equations were described only for the cases where the matrices $A(\lambda), B(\lambda)$ are regular or at least one from them is regular.

On the basis of standard forms of polynomial matrices with respect to semiscalar equivalence the description of solutions of this equations reduces to the description of solutions of equivalent equations:

$$T^A(\lambda)\widetilde{X}(\lambda)+\widetilde{Y}(\lambda)T^B(\lambda)=T^C(\lambda) \quad \text{and} \quad T^A(\lambda)\widetilde{X}(\lambda)+T^B(\lambda)\widetilde{Y}(\lambda)=\widetilde{C}(\lambda),$$

where $T^A(\lambda)$, $T^B(\lambda)$, $T^C(\lambda)$ are the standard forms of matrices $A(\lambda)$, $B(\lambda)$, $C(\lambda)$. For this equations we established the method of constructing of their solutions, obtained the minimal degree of solutions and found the conditions of uniqueness of such solutions.

Theorem 1. *Let the pair of matrices $(A(\lambda), B(\lambda))$ from matrix Sylvester equation be diagonalizable and its standard pair is the pair of matrices $(\Phi(\lambda), \Psi(\lambda))$, where $\Phi(\lambda)=Q(\lambda)A(\lambda)R_A(\lambda)=\operatorname{diag}(\varphi_1,\ldots,\varphi_n)$, $\Psi(\lambda)=Q(\lambda)B(\lambda)R_B(\lambda)=\operatorname{diag}(\psi_1,\ldots,\psi_n)$, $\varphi_i \mid \varphi_{i+1}$, $\psi_i \mid \psi_{i+1}$, $i=1,\ldots,n-1$; $Q(\lambda), R_A(\lambda), R_B(\lambda)\in GL(n,P[\lambda])$. Let $\widetilde{X}_0(\lambda)$, $\widetilde{Y}_0(\lambda)$ be a particular solution of the corresponding equivalent matrix equation. Then the general solution of this equation is*

$$\widetilde{X}(\lambda)=\widetilde{X}_0(\lambda)+W_\Psi(\lambda)+K(\lambda)\Psi(\lambda),\ \widetilde{Y}(\lambda)=\widetilde{Y}_0(\lambda)-W_\Phi(\lambda)-K(\lambda)\Phi(\lambda),$$

where $W_\Psi(\lambda)=\|\frac{\psi_j}{d_{ij}}w_{ij}\|_1^n$, $W_\Phi(\lambda)=\|\frac{\varphi_j}{d_{ij}}w_{ij}\|_1^n$, $d_{ij}=(\varphi_i,\psi_j)$, w_{ij} are arbitrary elements of a complete set $\mathcal{P}[\lambda]_{d_{ij}}$ of residues modulo d_{ij}, $K(\lambda)=\|k_{ij}\|_1^n$, k_{ij} are arbitrary elements of $\mathcal{P}[\lambda]$. The general solution of matrix Sylvester equation has the form $X(\lambda)=R_A(\lambda)\widetilde{X}(\lambda)R_B^{-1}(\lambda)$, $Y(\lambda)=Q^{-1}(\lambda)\widetilde{Y}(\lambda)Q(\lambda)$.

The general solution of matrix Diophantine equation can be written similarly. Some of this results can be found in [1, 2].

1. N. Dzhaliuk, V. Petrychkovych, The semiscalar equivalence of polynomial matrices and the solution of the Sylvester matrix polynomial equations // Mathematical Bulletin of the Shevchenko Scientific Society. – 2012. – **9**. – P. 81-88. (in Ukrainian)
2. N.S. Dzhaliuk, V.M. Petrychkovych, The solutions of matrix polynomial Diophantine equation // Appl. Probl. of Mechan. and Mathem. – 2012. – **10**. – P. 55-61. (in Ukrainian)

Classification of low-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$

V.M. Fedorchuk, V.I. Fedorchuk

Pedagogical University of Cracow, Poland
Pidstryhach IAPMM of NAS of Ukraine, Lviv, Ukraine
vasfed@gmail.com, volfed@gmail.com

Non-conjugate subalgebras of Lie algebras of local Lie groups of the point transformations play an important role for solving of different tasks of theoretical and mathematical physics, mechanics, gas dynamics etc. (see, for example,[1, 2, 3]).

However, it turned out that the possibilities, of the above mention applications, as well as, the results obtained essentially depend on structural properties of non-conjugate subalgebras of Lie algebras. One way for study the structural properties of non-conjugate subalgebras of the Lie algebras consists in classification of these subalgebras into isomorphism classes.

The present report is devoted to the classification of low-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$ into isomorphism classes. The group $P(1,4)$ is a group of rotations and translations of the five-dimensional Minkowski space $M(1,4)$. The results of classification for non-conjugate subalgebras of the Lie algebra of the group $P(1,4)$ with dimensions up to four can be found in[4, 5].

We plan to give a short review of the results of classification for non-conjugate subalgebras of the Lie algebra of the group $P(1,4)$ with dimensions up to five.

1. Ovsiannikov L.V., Group Analysis of Differential Equations, Academic Press, New York, 1982.
2. Fushchych W.I, Barannyk L.F., Barannyk A.F., Subgroup analysis of the Galilei and Poincaré groups and reductions of nonlinear equations, Kiev, Naukova Dumka, 1991.
3. Olver P.J., Applications of Lie Groups to Differential Equations, Springer-Verlag, New York, 1986.
4. Fedorchuk V.M., Fedorchuk V.I., On classification of the low-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$, Proceedings of Institute of Mathematics of NAS of Ukraine, 2006, Vol. 3, No. 2, 302–308.
5. Fedorchuk V. M., Fedorchuk V. I., Invariant operators of four-dimensional nonconjugate subalgebras of the Lie algebra of the Poincaré group P(1,4), Mat. metody ta fiz.-mekh. polya, 2010, Vol. 53, No. 4, 17–27; translated in Journal of Mathematical Sciences, 2012, Vol. 181, No. 3, 305–319.

Orthogonality of n-ary operations

Iryna Fryz

Vinnytsia Higher Professional College of Lviv State University of Life Safety
friz_irina@ukr.net

A set $\{g_1, \dots, g_n\}$ of n-ary operations defined on a set Q is called *orthogonal*, iff every system of equations

$$\{g_1(x_1, \dots, x_n) = a_1, \quad \dots, \quad g_n(x_1, \dots, x_n) = a_n$$

has a unique solution for all $a_1, \dots, a_n$ from Q.

The recursive method of construction of orthogonal n-ary operations is well-known [1, 2]. Another construction method of orthogonal multiary operations, that gives possibility to construct greater number of sets of orthogonal operations, is given here.

Let $\delta := \{i_1, \dots, i_s\} \subseteq \overline{1,n} := \{1, \dots, n\}$ and f be an n-ary operation on Q. s-ary operation g, which is obtained from $f(x_1, \dots, x_n)$ by replacing variables from $\{x_1, \dots, x_n\} \setminus \{x_{i_1}, \dots, x_{i_s}\}$ with some elements from Q, will be called *δ-retract* of f. δ-retracts of operations $f_1, \dots, f_n$ will be called *similar* if the same variables are replaced with the same elements. If all similar δ-retracts of $f_1, \dots, f_n$ are orthogonal, then the operations $f_1, \dots, f_n$ will be called *δ-recract orthogonal.*

Let $\pi := \{\pi_1, \pi_2, \dots, \pi_s\}$ be a partition of $\overline{1,n}$ and $f_1, \dots, f_n$ be n-ary operations, where $\pi_i := \{n_{i-1}+1, \dots, n_i\}$ for all $i = 1, \dots, s$ and $n_0 := 1$, $n_s := n$. A *π-step algorithm* of constructing of operations $g_1, \dots, g_n$ is defined by the following way

$$\begin{cases} g_1(x_1,\dots,x_n) := f_1(x_1,\dots,x_n), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ g_{n_1}(x_1,\dots,x_n) := f_{n_1}(x_1,\dots,x_n), \\ g_{n_1+1}(x_1,\dots,x_n) := f_{n_1+1}(g_1(x_1,\dots,x_n),\dots,g_{n_1}(x_1,\dots,x_n),x_{n_1+1},\dots,x_n), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ g_{n_2}(x_1,\dots,x_n) := f_{n_2}(g_1(x_1,\dots,x_n),\dots,g_{n_1}(x_1,\dots,x_n),x_{n_1+1},\dots,x_n), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ g_{n_{s-1}+1}(x_1,\dots,x_n) := f_{n_{s-1}+1}(g_1(x_1,\dots,x_n),\dots,g_{n_{s-1}}(x_1,\dots,x_n),x_{n_{s-1}+1},\dots,x_n), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ g_n(x_1,\dots,x_n) := f_n(g_1(x_1,\dots,x_n),\dots,g_{n_{s-1}}(x_1,\dots,x_n),x_{n_{s-1}+1},\dots,x_n). \end{cases}$$

Theorem. *Let $\{f_1, \dots, f_n\}$ be a set of n-ary operations such that for any $i \in \overline{1,s}$ the set of operations $\{f_{n_{i-1}+1}, \dots, f_{n_i}\}$ is π_i-retract orthogonal. Then the set of operations $\{g_1, \dots, g_n\}$, obtained by a π-step algorithm, is orthogonal.*

1. Gonsales S., Cosuelo E., Markov V.T., Nechaev A.A. Recursive MDS-codes and recursively differentiable quasigroups. *Discrete Mathematics* (1998), 10, **2**, 3–29. (Russian)
2. Belyavskaya G., Mullen G.L. Orthogonal hypercubes and n-ary operations. *Quasigroups and Related Systems*(2005), 13, **1**, 73–86.

A multidimensional limit theorem for twists of L-functions of elliptic curves

V. Garbaliauskienė

Siauliai University, Šiauliai State College
virginija@fm.su.lt

Let q be a prime number, χ denote a Dirichlet character modulo q, for $Q \geq 2$,

$$M_Q = \sum_{q \leq Q} \sum_{\substack{\chi = \chi(\mathrm{mod} q) \\ \chi \neq \chi_0}} 1$$

and

$$\mu_Q(...) = M_Q^{-1} \sum_{q \leq Q} \sum_{\substack{\chi = \chi(\mathrm{mod} q) \\ \chi \neq \chi_0 \\ ...}} 1,$$

where χ_0 is the principal character, and in place of dots a condition satisfied by a pair $(q, \chi(\mathrm{mod} q))$ is to be written.

In this report, we present a limit theorem on the weak convergence, as $Q \to \infty$, of the probability measure

$$\mu_Q((|L_{E_1}(s,\chi)|, ..., |L_{E_r}(s,\chi)|) \in A), \;\; A \in \mathcal{B}(\mathbb{R}^r),$$

where $L_{E_j}(s,\chi)$, $s = \sigma + it$, is the twist with the character χ of the L-function of a non-singular elliptic curve E_j over the field of rational numbers, $j = 1, ..., r$, and $\mathcal{B}(\mathbb{R}^r)$ denotes the Borel σ-field of the space $\mathbb{R}^r$. Assuming that $\sigma > \frac{3}{2}$, we obtain that the above probability measure, as $Q \to \infty$, converges weakly to a certain probability measure on $(\mathbb{R}^r, \mathcal{B}(\mathbb{R}^r))$ defined by the characteristic transforms. The theorem obtained generalizes a one-dimensional theorem of [1].

1. V. Garbaliauskienė, A. Laurinčikas, E. Stankus, Limit theorems for twist of L-functions of elliptic curves, *Lith. Math. J.*, **50**(2) (2010), 187–197.

Fractionally IF Bezout rings

A. Gatalevych

Ivan Franko National University of Lviv, Lviv, Ukraine
gatalevych@ukr.net

Throughout this note R is assumed to be a commutative ring with $1 \neq 0$. Let P be a ring property. Following Vamos [1], a ring R is fractionally P provided that the classical quotient ring $Q(R/I)$ of a ring R/I satisfies P for every ideal I of R. For example, any Noetherian ring is a fractionally semilocal. In this paper given the answer to the question posed in [2] on fractionally regular IF-rings.

A ring R is called fractionally regular if for every nonzero element $a \in R$ the classical quotient ring $Q(R/rad(a))$ is regular ring, where $rad(a)$ is the radical of aR [3]. A commutative Bezout ring R with identity is said to be adequate if it satisfies such conditions: for every $a, b \in R$, with $a \neq 0$, there exist $a_i, d \in R$ such that

(i) $a = a_i d$,

(ii) $(a_i, b) = (1)$, and

(iii) for every nonunit divisor d' of d, we have $(d', b) \neq (1)$. [4]

Theorem 1. *A fractionally Bezout IF-ring is fractionally regular.*

Theorem 2. *Let R be fractionally Bezout IF-ring with nonzero Jacobson radical (nilradical). Then stable range of $R \leq 2$.*

Theorem 3. *An adequate ring is finite fractionally IF-ring.*

Theorem 4. *Fractionally regular Bezout ring of stable range 2 is an elementary divisor ring.*

1. Vamos,P. The decomposition of finitely generated modules and fractionally self-injective rings. J. London Math.Soc. 16(2), 1977, 209-220.
2. Facchini A., Faith C. FP-injective quotient rings and elementary divisor rings // Commut. Ring Theory Proc. // Int. conf.,-1996,-185, 293-302.
3. Zabavsky B.V. Fractionally regular Bezout rings, Math. Stud. 32, 2009, 76-80.
4. O. Helmer, The elementary divisor theorem for certain rings without chain condition, Bull. Amer. Math. Soc. vol. 49 (1943) pp. 225-236.
5. Couchot F. The λ-dimension of commutative arithmetic ring, Comm. Algebra 31, 2003, 3143-3158.
6. Larsen M., Lewis W., Shores T. Elementary divisor rings and finitely presented modules, Trans. Amer. Math. Soc. 187, 1974, 231-248.
7. Забавский Б.В., Белявская С.И. Адекватное в нуле кольцо является кольцом со свойством замены // Фундаментальная и прикладная математика,-2011/2012.-17.-№3.-61-66.

Categorical resolutions of curves with quadratic singularities

V. Gavran

Institute of Mathematics NAS of Ukraine
v.gavran@yahoo.com

This is report on a joint work with Yuriy Drozd.

Let X be a singular reduced projective curve of arbitrary geometric genus and suppose that it only has quadratic singularities (i.e. the singularities of type A). We construct a *non-commutative* (or *categorical*) resolution of singularities of X in the spirit of the paper of Kuznetsov Namely, we introduce a certain sheaf of $\mathcal{O}_X$-orders (called the Auslander sheaf) $\mathcal{A} = \mathcal{A}_X$ and study the category $\mathrm{Coh}(\mathcal{A})$ of coherent left modules on the ringed space $(X, \mathcal{A}_X)$. We prove that the global dimension of $\mathrm{Coh}(\mathcal{A})$ is equal to two and there is a pair of functors $\mathbb{F}, \mathbb{I} : \mathrm{Coh}(X) \to \mathrm{Coh}(\mathcal{A})$, where $\mathbb{F}$ is left adjoint and $\mathbb{I}$ is a right adjoint of an exact functor $\mathbb{G}$ $\mathrm{Coh}(\mathcal{A}) \to \mathrm{Coh}(X)$. Moreover, $\mathbb{G}\mathbb{F} \simeq \mathbb{1}_{\mathrm{Coh}(X)}$, and the restrictions of $\mathbb{F}$ and $\mathbb{I}$ on the full subcategory of locally free sheaves coincide. It implies that the derived functors $\mathbb{L}\mathbb{F}$ and $\mathbb{R}\mathbb{G}$ give a *weakly crepant categorical resolution* of X in the sense of [2].

We obtain a description of the bounded derived category $D^b(\mathrm{Coh}(\mathcal{A}))$ via semiorthogonal decomposition for this category. When X is a rational curve, we prove that there is a formal A_∞-algebra structure on the Ext-algebra E of the direct sum of objects from a full exceptional collection and obtain a triangle equivalence $D^b(\mathrm{Coh}(\mathcal{A})) \simeq \mathsf{Perf}(E)$.

Our observations generalize the results from [1] where the curves with only nodal and cuspidal singularities were considered.

1. I.Burban, Y.Drozd, Tilting on non-commutative rational projective curves, Math. Ann. 351 (2011), 665-709.
2. A. Kuznetsov, Lefschetz decompositions and categorical resolutions of singularities, Selecta Math. 13 (2008), no. 4, 661–696.

Superextensions of cyclic semigroups

V. Gavrylkiv

Vasyl Stefanyk Precarpathian National University
vgavrylkiv@yahoo.com, vgavrylkiv@gmail.com

Given a cyclic semigroup S we study right and left zeros, singleton left ideals, the minimal ideal, left cancelable and right cancelable elements of superextensions $\lambda(S)$ and characterize cyclic semigroups whose superextensions are commutative.

1. T. Banakh, V. Gavrylkiv, O. Nykyforchyn, *Algebra in superextensions of groups, I: zeros and commutativity*, Algebra Discrete Math. (2008), No.3, 1–29.
2. T. Banakh, V. Gavrylkiv, *Algebra in superextension of groups, II: cancelativity and centers*, Algebra Discrete Math. (2008), No.4, 1–14.
3. T. Banakh, V. Gavrylkiv, *Algebra in superextension of groups: minimal left ideals*, Mat. Stud. **31** (2009), 142–148.
4. T. Banakh, V. Gavrylkiv, *Algebra in the superextensions of twinic groups*, Dissert. Math. **473** (2010), 74pp.
5. V. Gavrylkiv, *The spaces of inclusion hyperspaces over noncompact spaces*, Mat. Stud. **28**:1 (2007), 92–110.
6. V. Gavrylkiv, *Right-topological semigroup operations on inclusion hyperspaces*, Mat. Stud. **29**:1 (2008), 18–34.
7. V. Gavrylkiv, *Superextensions of cyclic semigroups*, Carpathian Mathematical Publication **5**:1 (2013), 8 p.

Linear non-homogeneous differential equations in a module over a ring

S. Gefter

V.N.Karazin Kharkiv National University
gefter@univer.kharkov.ua

Let R be an arbitrary commutative ring with identity and V be a R-module. By $V[[x,\frac{1}{x}]]$ we denote the R-module of all formal Laurent series with coefficients from V. For a R-linear operator $T: V \to V$ and $f(x) = \sum\limits_{n=-\infty}^{+\infty} v_n x^n \in V[[x,\frac{1}{x}]]$ we set $(Tf)(x) = \sum\limits_{n=-\infty}^{+\infty} (Tv_n)x^n$.

Let now E be a submodule of $V[[x,\frac{1}{x}]]$, which is invariant with respect to the operator $\frac{d}{dx}$, and $f \in E$. Consider the following differential equation in the module E:

$$Ty' + f(x) = y \quad (1)$$

Theorem 1. *Let E be the module of all formal power series with coefficients from V, i.e. $E = V[[x]]$. If the operator T is nilpotent then Equation (1) has a unique solution from $V[[x]]$, $y(x) = \sum\limits_{m=0}^{\infty} T^m f^{(m)}(x)$.*

Theorem 2. *Let E be the module of all formal Laurent series of the form $\sum\limits_{n=-k}^{\infty} \frac{c_n}{x^n}$, $k = 0,1,2,...$, and T be an arbitrary R-linear operator on V. Then the series $y(x) = \sum\limits_{m=0}^{\infty} T^m f^{(m)}(x)$ is well defined as an element from E and is a unique solution of Equation (1) from the module E.*

Moreover, in the case, when $E = V[x]$, a notion of fundamental solution of Equation (1) is considered, and a representation of the Cauchy type for the solution of Equation (1) is obtained.

Some properties of realizations of permutation groups with n-semimetric spaces

O. Gerdiy

National University of Kyiv-Mohyla Academy
lotuseater24@gmail.com

Definition 1. [1] Let X be a set, n some natural number. A function $d_n : \; X^{n+1} \to [0; +\infty)$ is called *n-symmetric* on set X, if:

1. d_n is fully simetric, i.e. for any $x_1, x_2, \dots, x_{n+1} \in X$ and every permutation $\pi \in S_{n+1}$ of numbers $1, \dots, n+1$ we have
$$d_n(x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n+1)}) = d_n(x_1, x_2, \dots, x_{n+1}).$$
2. d_n obeys the simplex inequality, i.e. for arbitrary $x_1, x_2, \dots, x_{n+2} \in X^{n+1}$:
$$d_n(x_1, x_2, \dots, x_{n+1}) \leqslant \sum_{i=1}^{n+1} d_n(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_{n+2}).$$

Set X with n-semimetric d_n is called *n-semimetric space* and is denoted as (X, d_n).

Definition 2. n-semimetric space (X, d_n) is a *realization* of permuation group (G, X), if isometry group $(IsomX, X)$ is isomorphic as permutaion group to (G, X).

For permutation group (G, X) let's denote as $gn(\ (G, X)\ )$ the smallest $n \in N$ that there exists some n-semimetric space (X, d_n) that is a realization of (G, X).

Theorem 1. *For any finite permutation groups (G_1, X_1), (G_2, X_2)*
$$gn(\ (G_1, X_1) \oplus (G_2, X_2)\ ) = max(\ gn((G_1, X_1)), gn((G_2, X_2))\ )$$

Theorem 2. *For any finite permutation groups (G_1, X), (G_2, X)*
$$gn(\ (G_1, X) \bigcap (G_2, X)\ ) \leq max(\ gn((G_1, X)), gn((G_2, X))\ )$$

Theorem 3. $\forall n \in N, n \geq 4$
$$gn(\ C_n\ ) = 5,$$
where C_n is the cyclic group of degree n.

Theorem 4. $\forall n \in N, n \geq 2$
$$gn(\ A_n\ ) = \frac{n*(n-1)}{2},$$
where A_n is the alternating group of degree n.

1. M.Deza n-semimetrics /M.Deza, I.G.Rosenberg//European Journal of Combinatorics, Special Issue «Discrete Metric Spaces» 21-5 — 2000. — p.797–806.
2. *Сущанський В. І., Сікора В. С.* Операції на групах підстановок. Теорія та застусування "— Чернівці: Рута, 2003. "— 255 с.

Variants of free monoids and free semigroups

A. Gorbatkov

Lugansk Taras Shevchenko National University, Lugansk, Ukraine
gorbatkov_a@mail.ru

Let $x \in S$, then a variant of S is a semigroup on S with multiplication $*_x$ defined by $a *_x b = axb$. Operation $*_x$ is called a deformed multiplication and $(S; *_x)$ is called a variant of S. Denote $(S; *_x)$ by S_x. Variants of abstract semigroups were first studied by J. Hickey [3] and variants of some semigroups of relations had earlier been considered by Magill [4].

We describe isomorphism criteria for variants of free semigroups and free monoids. For the similar results on finite generated free commutative semigroups see [1].

Let X be a nonempty set. We denote by X^+ the free semigroup over X and by X^* the free monoid over X with the empty word θ. Now let $x \in X^+$ and $\deg(x)$ be the maximal integer such that $x = a(ba)^{\deg(x)-1}$ for some $a \in X^+, b \in X^*$.

For $\deg(x) > 1$, put

$$\Delta(x) = \{(ba)^k \,:\, a \in X^+,\, b \in X^*, x = a(ba)^{n-1} \,\&\, 1 \le k < n \le \deg(x)\},$$

and $\Delta(x) = \emptyset$ otherwise.

A word $w \in X^+$ is called primitive if $w = v^k$ implies $k = 1$, where $v \in X^+, k \in \mathbb{N}$. Denote the set of all primitive words by $P(X)$.

Theorem 1. *Let X, Y be nonempty sets with $|X| > 1$ and $|Y| > 1$. For all $x \in X^+$ and $y \in Y^+$, variants X^+_x and Y^+_y are isomorphic iff the following conditions are satisfied:*

(i) $|\Delta(x)| = |\Delta(y)|$,

(ii) $\deg(x) = \deg(y)$,

(iii) $x \in P(X) \Leftrightarrow y \in P(Y)$.

Variants X^*_x and Y^*_y are isomorphic iff equalities (i) and (ii) hold.

If $|X| = 1$, then variants of X^* are commutative and pairwise nonisomorphic [2, Theorem 1.1]. Hence $\{x\}^*_{x^n} \cong Y^*_y$ iff $Y = \{z\}$ and $y = z^n$.

1. Givens B., N., K.A.Linton, A. Rosin and L. Dishman, *Interassociates of the Free Commutative Semigroup on n Generators* // Semigroup Forum — 2007. — 74. — PP. 370–378.
2. Gould M., Linton K. A. and Nelson A. W., *Interassociates of monogenic semigroups* // Semigroup Forum — 2004. — 68. — PP. 186–201.
3. Hickey J. B., *Semigroups under a sandwich operation* // Proceeding of Edinburgh Mathematical Society. — 1983. — 26. — PP. 371–382.
4. Magill K. D., *Semigroup structures for families of functions I. Some homomorphism theorems* // J. Austral. Math. Soc. — 1967. — 7. — PP. 81–94.

On growth of random groups of intermediate growth

R. I. Grigorchuk

Texas A&M University, USA
grigorch@math.tamu.edu

By 1968 it became apparent that all known classes of groups have either *polynomial* or *exponential growth* and John Milnor formally asked whether groups of *intermediate growth* exist. In 1984 the speaker provided a construction of a continuum $\mathcal{G}$ of such groups with different rates of growth [2]. All rates of growth in this family are larger than $e^{n^{1/2}}$ (n is the argument running natural numbers), in many cases growth is bounded by a function of type $e^{n^{\alpha}}$, where $1/2 < \alpha < 1$ depends on a group, and for any function $f(n)$ growing subexponentially there is a group of intermediate growth in $\mathcal{G}$ whose growth is not less than the growth of the function $f(n)$. The latter fact shows that intermediate rate of growth can approach exponential growth arbitrary close. One more specific feature of the contsruction is existence of groups with the so-called *oscillating growth* when, given two functions satisfying inequalities $e^{n^{\theta_0}} \prec \gamma_1(n) \prec \gamma_2(n) \prec e^n$, where $\theta_0 \approx 0.7675...$, there is a group G in $\mathcal{G}$ whose growth function $\gamma_G(n)$ infinitely many times becomes greater than $\gamma_2(n)$ and infinitely many times becomes smaller than $\gamma_1(n)$. This fact was used in [2] to show that there are groups with incomparable growth.

Another important aspect of the construction was introduction of the space $\mathcal{M}$ of finitely generated marked groups consisting of pairs (G, S), where G is a group and S is an ordered generating set. In this space the continuum $\mathcal{G}$ can be viewed as a Cantor subset X with a continuous map $T : X \to X$ which preserves basic group properties. In fact, the dynamical system (X, T) is topologically conjugate to the one-sided shift (τ, Ω), where $\Omega = \{0, 1, 2\}^{\mathbb{N}}$ and groups from $\mathcal{G}$ are parametrized by sequences $\omega \in \Omega$. It will be explained why for any reasonable T-invariant probability measure μ on X, a typical (i.e., μ-almost sure) property of a group from the family X is to have growth bounded from above by a function of the type $e^{n^{\alpha}}$, where $\alpha < 1$ is a constant.

On the other hand, from the categorical point of view a generic group exhibits totally different growth behavior, namely, it has *oscillating growth*.

At the beginning of the talk I will discuss a general approach to randomness in group theory based on the use of the space of marked groups.

The above results are based on joint work with M. Benli and Y. Vorobets presented in [1].

1. Mustafa G. Benli, Rostislav Grigorchuk, and Yaroslav Vorobets. On growth of random groups of intermediate growth. (available at *http://arxiv.org/abs/1303.1560*), 2013.
2. R. I.Grigorchuk. Degrees of growth of finitely generated groups and the theory of invariant means. *Izv. Akad. Nauk SSSR Ser. Mat.*, 48(5):939-985, 1984.

On characterization of the hypercenters of fuzzy groups

K. O. Grin

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
catherine.grin@gmail.com

In the recent paper [1] it was carried out the constructing of the upper central series in arbitrary fuzzy group that haven't been done before. We continue the study of the properties of the upper central series for arbitrary fuzzy group defined on a group G. More precisely, a quite explicit description of the members of the upper central series has been obtained.

Let L be a subgroup of a group G and γ be a fuzzy subgroup on G. Denote the function $L|^{\gamma}$ by the following rule:

$$L|^{\gamma}(x) = \begin{cases} \gamma(x), & \text{if } x \in L; \\ 0, & \text{if } x \notin L. \end{cases}$$

Theorem 1. *Let G be a group and γ be a fuzzy subgroup on G. Then*

$$\mathfrak{z}_2(\gamma) = \zeta_2(\mathbf{Supp}(\gamma))|^{\gamma}.$$

Theorem 2. *Let G be a group and γ be a fuzzy subgroup on G. Let*

$$\chi(e, \gamma(e)) = \mathfrak{z}_0(\gamma) \preccurlyeq \mathfrak{z}_1(\gamma) \preccurlyeq \ldots \preccurlyeq \mathfrak{z}_\beta(\gamma) \preccurlyeq \mathfrak{z}_{\beta+1}(\gamma) \preccurlyeq \ldots \preccurlyeq \mathfrak{z}_\sigma(\gamma)$$

be the upper central series of γ. Then

$$\mathfrak{z}_\beta(\gamma) = \zeta_\beta(\mathbf{Supp}(\gamma))|^{\gamma}$$

for every ordinal β.

Corollary 1. *[1] Let G be a group, γ be a fuzzy subgroup on G. Then γ is hypercentral if and only if $\mathbf{Supp}(\gamma)$ is hypercentral.*

1. Kurdachenko L.A., Grin K.O., Turbay N.A. *On hypercentral fuzzy groups.* Algebra and Discrete Mathematics—13(2012). Number 1, pp. 92-106.

On π-solvable group in which some maximal subgroup of π-Hall subgroup is Schmidt group

D.V. Gritsuk, V.S. Monakhov

Gomel Francisk Skorina State University, Gomel
Dmitry.Gritsuk@gmail.com, Victor.Monakhov@gmail.com

All groups considered in this paper will be finite. All notation and definitions correspond to [1], [2].

Let G be a π-solvable group. Then G has a subnormal series

$$G = G_0 \supseteq G_1 \supseteq \ldots \supseteq G_{n-1} \supseteq G_n = 1,$$

whose factors G_{i-1}/G_i are π'-groups or abelian π-groups. The least number of abelian π-factors of all such subnormal series of a group G is called the derived π-length of a π-solvable group G and is denoted by $l_\pi^a(G)$. Clearly, if $\pi = \pi(G)$ then $l_\pi^a(G)$ coincides with the derived length of G. The initial properties of the derived π-length established in [3].

Recall that a group is called a Schmidt group if it is a non-nilpotent group all of whose proper subgroups are nilpotent. A whole paragraph from Huppert's monography is dedicated to Schmidt groups, (see [1, III.5]).

Theorem 1. *Let G be a π-solvable group. If some maximal subgroup M of π-Hall subgroup of G is Schmidt group, then $l_\pi^a(G) \leq 5$. In particular, if M is a Hall subgroup, then $l_\pi^a(G) \leq 4$.*

Corollary 1. *If some maximal subgroup of a solvable group G is Schmidt group, then $d(G) \leq 5$.*

Example 1. The SmallGroup(1944,2293) in databases GAP has the following structure: $G = SR$, $R = [Z_3 \times Z_3]Z_3$ is a normal subgroup of order 27, $S = [Q_8]Z_{27}$ is a maximal subgroup of G, and S is a Schmidt group, $|S \cap R| = 3$. Here Q_8 is a quaternion group of order 8, Z_n is a cyclic of order n. The group G has the derived length equal to 5.

This example shows that the obtained estimate of the derived π-length accurate.

1. Huppert B. Endliche Gruppen I. Berlin; Heidelberg; New York: Springer-Verlag, 1967.
2. Monakhov V. S. Introduction to the theory of finite groups and their classes. Minsk: Higher School, 2006 (in Russian).
3. Gritsuk D. V., Monakhov V. S., Shpyrko O. A. On derived π-length of a π-solvable group // BSU Vestnik, Series 1. 2012. № 3. P. 90–95 (in Russian).

Algorithm of undirected graph exploration by a collective of agents

I.S. Grunsky, A.V. Stepkin

Institute of applied mathematics and mechanics of the NASU
stepkin.andrey@rambler.ru

Nowadays, there are many unknown environments asking to be explored. Therefore the problem of exploration of an environment is widely studied in the literature in various contexts [1, 2]. The problem of the finite undirected graphs exploration [3] by a collective of agents is considered. An exploration algorithm with linear time, quadratic space and $O(n^2 \cdot log(n))$ communication complexity in the number of graph nodes is proposed.

Two agents-researchers are simultaneously moving on the graph, they read and change marks of graph elements, then transfer the information to the agent-experimenter (it builds explored graph representation). Two agents (which move on graph) use two different colors (in total three colors) for graph exploration. An algorithm is based on depth-first traversal method.

Functions of agents:

1. agent-researcher (agent which is moving on the graph; it has growing memory, that is limited at each step):
   - perceives marks of all elements in the neighborhood of the node;
   - moves on graph from node v to node u by edge (v, u);
   - can change color of nodes, edges and incidentors;
   - can write numbers to the nodes memory, as well as read them.
2. agent-experimenter (stationary agent with unlimited growing internal memory):
   - conveys, receives, identifies messages from agents-researchers;
   - builds a graph representation based on messages from agents-researchers.

Theorem 1. *Three agents, performing algorithm of exploration on the graph, explore it up to isomorphism*

Theorem 2. *An exploration algorithm has linear time, quadratic space and $O(n^2 \cdot log(n))$ communication complexity in the number of graph nodes. Need three colors for graph exploration.*

1. S. Albers and M.R. Henzinger. Exploring unknown environments. SIAM Journal on Computing, 29(4): 1164 – 1188, 2000.
2. X. Deng and C.H. Papadimitriou. Exploring an unknown graph. Journal of Graph Theory, 32(3): 265 –297, 1999.
3. V. Kudryavcev. Introduction to Automata Theory / V. Kudryavcev, S. Aleshin, A. Podkozlin. – M.: The science, 1985. – P. 320. (Russian)

Algebras of quantum Laurent polynomials

A. Gupta

ashishg@iiserb.ac.in

We shall discuss the recent advances in the theory of noncommutative polynomial algebras generally known as $\mathbb{Z}^n$-quantum tori. These are crossed products of free abelian groups by fields. Quantum torus algebras arise in various fields throughout mathematics. including C^*-algebras, Topology and Noncommutative geometry. We shall discuss the various aspects of this theory.

On $\mathscr{H}$-complete topological pospaces

O. Gutik

Ivan Franko National University of Lviv, Lviv, Ukraine
o_gutik@franko.lviv.ua

We follow the terminology of [1].

A topological pospace X is called *$\mathscr{H}$-complete* if X is a closed subspace of every Hausdorff topological pospace in which it is contained.

In the report we discuss on the $\mathscr{H}$-completeness some classes topological pospaces with maximal compact anti-chains.

1. L. Nachbin, *Topology and Order*, D. van Nostrand Company, Inc., Princeton, New Jersey, 1965.

On pseudocompact primitive topological inverse semigroups

O. Gutik, K. Pavlyk

Ivan Franko National University of Lviv, Lviv, Ukraine
University of Tartu, Estonia
o_gutik@franko.lviv.ua

We follow the terminology of [1] and [2].

In the report we discuss the structure of pseudocompact primitive topological inverse semigroups. We show that a Tychonoff product of a family of pseudocompact primitive topological inverse semigroups is a pseudocompact topological space. Also we prove that the Stone-Čech compactification of a pseudocompact primitive topological inverse semigroup is a compact primitive one. This gives the positive answer to the Question 1, which we posed in [1].

1. T. Berezovski, O. Gutik, and K. Pavlyk, *Brandt extensions and primitive topological inverse semigroups*, Int. J. Math. Math. Sci. **2010** (2010) Article ID 671401, 13 pages, doi:10.1155/2010/671401.
2. J. H. Carruth, J. A. Hildebrant, and R. J. Koch, *The Theory of Topological Semigroups*, Vol. I, Marcel Dekker, Inc., New York and Basel, 1983; Vol. II, Marcel Dekker, Inc., New York and Basel, 1986.

On the semigroup of monotone co-finite partial bijections of $\mathbb{N}^2_{\leqslant}$

O. Gutik, I. Pozdnyakova

Ivan Franko National University of Lviv, Lviv, Ukraine
o_gutik@franko.lviv.ua

Let $\mathbb{N}$ be the set of positive integers with the usual linear order $\leqslant$. On the Cartesian product $\mathbb{N} \times \mathbb{N}$ we define the product partial order, i.e.,

$$(i, m) \leqslant (j, n) \qquad \text{if and only if} \qquad (i \leq j) \quad \text{and} \quad (m \leq n).$$

Later by $\mathbb{N}^2_{\leqslant}$ we denote the set $\mathbb{N} \times \mathbb{N}$ with so defined partial order.

By $\mathscr{PO}_{\infty}(\mathbb{N}^2_{\leqslant})$ we denote a subsemigroup of injective partial monotone self-maps of $\mathbb{N}^2_{\leqslant}$ with co-finite domains and images. In our report we discuss on the structure of the semigroup $\mathscr{PO}_{\infty}(\mathbb{N}^2_{\leqslant})$.

Subgroup structure of groups of infinite matrices

W. Hołubowski

Silesian University of Technology
w.holubowski@polsl.pl

In our talk we will give a survey of recent results on groups of infinite matrices. We will consider the lower central and derived series in triangular and unitriangular groups, parabolic subgroups in Vershik-Kerov group, free subgroups in groups of unitriangular infinite matrices.

Trace monoids and bisimulation

A. Husainov

Komsomolsk-on-Amur State Technical University
husainov51@yandex.ru

We propose to apply the methods of algebraic topology for classification and study the labeled asynchronous systems. We introduce homology groups for labeled asynchronous systems and show that Pom_L-bisimilar asynchronous systems have isomorphic homology groups.

Let E^* be a free monoid generated by a set E. For any symmetric irreflexive relation $I \subseteq E \times E$ on E, there exists a smallest congruence relation $(\equiv)$ on E^* containing all pairs (ab, ba) with $(a,b) \in I$. The *trace monoid* $M(E,I)$ is defined as the quotient monoid $E^*/(\equiv)$. It is well known [1] that *an asynchronous system* can be defined as a triple $(M(E,I), S, s_0)$ where S is a set of *states*, $s_0 \in S$ is an *initial state*, and $M(E,I)$ is a trace monoid with a right action on the set $S_* = S \sqcup \{*\}$ satisfying $* \cdot \mu = *$ for all $\mu \in M(E,I)$. A state $s \in S$ is *reachable* if there exists $\mu \in M(E,I)$ such that $s_0 \cdot \mu = s$.

For an arbitrary set L, a *strong label function* is any map $\lambda : E \to L$ such that $\lambda(a) \neq \lambda(b)$ for all $(a,b) \in I$. *A labeled asynchronous system* $(M(E,I), S, s_0, \lambda, L)$ is an asynchronous system $(M(E,I), S, s_0)$ with a strong label function $\lambda : E \to L$.

Let $(\leqslant)$ be a total order relation on L. If $\lambda : E \to L$ is a strong label function, then for any $A \subseteq E$ consisting of pairwise independent elements, the relation $(\leqslant) \cap (A \times A)$ is a total order relation on A. For every $n \geqslant 0$, consider a sequence of sets

$$\begin{aligned} Q_n(M(E,I), S, s_0, \lambda, L) = \{(\lambda(a_1), \cdots, \lambda(a_n)) | a_i \in E \text{ for all } 1 \leqslant i \leqslant n \ \& \\ (\exists s \in S(s_0)) s \cdot a_1 \cdots a_n \in S(s_0) \& \ a_1 < \cdots < a_n \ \& \ (a_i, a_j) \in I \text{ for all } 1 \leqslant i < j \leqslant n\}. \end{aligned}$$

Here $S(s_0)$ is the set of all reachable states. Denote by $\mathbb{Z}S$ a free Abelian group generated by S. We have a complex $\mathbb{Z}Q_n(M(E,I), S, s_0, \lambda, L)$ with differentials

$$d_n(\lambda(a_1), \cdots, \lambda(a_n)) = \sum_{i=1}^{n} (-1)^i (\lambda(a_1), \cdots, \lambda(a_{i-1}), \lambda(a_{i+1}), \cdots, \lambda(a_n)).$$

Its homology groups are called the *homology groups of the labeled asynchronous system* $H_n(M(E,I), S, s_0, \lambda, L)$. For the definition of the Pom_L*-bisimilar* labeled asynchronous systems, we refer the reader to [2].

Theorem 1. *Let $(M(E,I), S, s_0, \lambda, L)$ and $(M(E',I'), S', s_0', \lambda', L)$ are Pom_L-bisimilar labeled asynchronous systems, then for every $w = a_1 \cdots a_k \in E^*$, $k \geqslant 0$, satysfying $s_0 \cdot w \in S$, there exists $w' = a_1' \cdots a_k' \in E'^*$ such that $s_0' \cdot w' \in S'$ and*

$$(\forall n \geqslant 0) \ H_n(M(E,I), S, s_0 \cdot w, \lambda, L) \cong H_n(M(E',I'), S', s_0' \cdot w', \lambda', L).$$

1. A. Husainov, "On the homology of small categories and asynchronous transition systems", *Homology Homotopy Appl.*, 6, No. 1, 439–471 (2004).
2. M. Nielsen, G. Winskel, "Petri nets and bisimulation", *Theoretical Computer Sci.*, 153, No. 1-2, 211–244 (1996).

Asymptotic behavior of the degree of an algebra of SL_2-invariants

N. Ilash

Kamyanets-Podilsky College of Food Industry of National University of Food Technologies
ilashnadya@yandex.ua

Let $R = R_0 + R_1 + \cdots$ be a finitely generated graded complex algebra, $R_0 = \mathbb{C}$. Denote by $\mathcal{P}(R, z) = \sum_{j=0}^{\infty} \dim R_j z^j$, its Poincare series. The number $\deg(R) = \lim_{z\to 1}(1-z)^r \mathcal{P}(R, z)$ is called the degree of the algebra R. Here r is transcendence degree of the quotient field of R over $\mathbb{C}$. The first two terms of the Laurent series expansion of $\mathcal{P}(R, z)$ at the point $z = 1$ have the following form

$$\mathcal{P}(R, z) = \frac{\deg(R)}{(1-z)^r} + \frac{\tau(R)}{(1-z)^{r-1}} + \cdots .$$

Let $\mathcal{C}_d$ be the algebra of the covariants of the binary d-form, i.e. $\mathcal{C}_d \cong \mathbb{C}[V_1 \oplus V_d]^{SL_2}$. We calculate an integral representation and asymptotic behavior of the constants. For this purpose we use the explicit formula for the Poincare series $\mathcal{P}(\mathcal{C}_d, z)$ derived by L.Bedratyuk in [3].

Theorem 1.

$$\deg(\mathcal{C}_d) = \lim_{z\to 1}(1-z)^d \mathcal{P}(\mathcal{C}_d, z) = \frac{1}{d!} \sum_{0\le j<d/2} (-1)^j \binom{d}{j} \left(\frac{d}{2} - j\right)^{d-1},$$

$$\tau(\mathcal{C}_d) = \lim_{z\to 1} \Big(-(1-z)^d \mathcal{P}(\mathcal{C}_d, z)\Big)'_z = \frac{1}{2\, d!} \sum_{0\le j<d/2} (-1)^j \binom{d}{j} \left(\frac{d}{2} - j\right)^{d-1}.$$

We denote by $c_d := \deg(\mathcal{C}_d) \cdot d! = \sum_{0\le j<d/2} (-1)^j \binom{d}{j} \left(\frac{d}{2} - j\right)^{d-1}$. The following statements hold.

Theorem 2.

$$(i) \quad c_d = 2\pi^{-1}(d-1)! \int_0^{\infty} \frac{\sin^d x}{x^d} dx,$$

$$(ii) \quad \deg(\mathcal{C}_d) > 0.$$

Theorem 3.

$$\lim_{d\to\infty} d^{\frac{1}{2}} \int_0^{\infty} \frac{\sin^d x}{x^d} dx = \frac{(6\pi)^{\frac{1}{2}}}{2}$$

1. Springer, T.A., Invariant theory. Lecture Notes in Mathematics. 585., Springer-Verlag. (1977), 111 p.
2. Springer, T.A. On the invariant theory of SU_2, Indag. Math. 42, 339-345 (1980).
3. Bedratyuk, L. The Poincaré series for the algebra of covariants of a binary form. Int. J. Algebra 4, No. 25-28, 1201-1207 (2010).

On the adjoint groups of corner rings

Yu. Ishchuk

Ivan Franko National University of Lviv, Lviv, Ukraine
yu.ishchuk@gmail.com

Let R be an associative ring, not necessarily with an identity element. The set of all elements of R forms a semigroup with the identity element $0 \in R$ under the operation $a \circ b = a + b + ab$ for all a and b of R. The group of all invertible elements of this semigroup is called the *adjoint group* of R and is denoted by R°. Clearly, if R has the identity 1, then $1 + R^\circ$ coincides with the group of units $U(R)$ of the ring R and the map $a \to 1 + a$ with $a \in R$ is an isomorphism from R° onto $U(R)$.

If I is an ideal of R then $I^\circ = I \cap R^\circ$ is a normal subgroup of R°.

The elements of the Jacobson radical $J(R)$ of a ring R also forms a normal subgroup of the adjoint group R°. We will denote this group $J^\circ = (J(R), \circ)$ and refer to it as the Jacobson group of R.

If R is a ring with identity 1, then $J^\circ \cong 1 + J(R) \lhd U(R)$. (The normal subgroup $1 + J(R)$ of the group of units $U(R)$ is called the unitary group of ring R.)

Taking of the adjoint groups of rings commutes up to isomorphism with taking of homomorphic images of rings when the kernel of the ring homomorphism is contained in the Jacobson radical of a ring. As the corollary we obtain the following proposition.

Proposition 1. *Let A be a ring (not necessary with 1) and R a ring with 1. Then*

$$(A/J(A))^\circ \cong A^\circ/J^\circ(A) \quad \textit{and} \quad U(R/J(R)) \cong U(R)/(1 + J(R)).$$

The notations on the adjoint and associate groups of rings we refer to [1]. The foundations of a corner rings theory in noncommutative rings considered in [2, 3].

Let us recall that a ring $S \subseteq R$ is said to be a *corner ring* (or simply a corner) of R if there exists an additive subgroup $C \subseteq R$ such that

$$R = S \oplus C, \quad S \cdot C \subseteq C \text{ and } C \cdot S \subseteq C.$$

In this case, we write $S \prec R$ and we call any subgroup C satisfying conditions above a *complement* of the corner ring S in R.

We investigated the adjoint groups (group of units) of corners and conditions under which it has a complements in the adjoint group (group of units) of ring.

1. Ishchuk Yu., On Associate Group of Rings // London Math. Soc., Secture Note Series 304. "Groups St Andrews 2001 in Oxford", Vol. 1, Cambridge Univ. Press, P.284-293.
2. Lam T.Y., A First Course in Noncommutative Rings // Second Edition, Graduate Texts in Math., Vol. 131, Springer-Velgar, Berlin-Heidelberg-New York, 2001.
3. Lam T.Y., Corner Ring Theory: A Generalization of Peirce Decoomposition, I // Algebras, Rings And Their Representations: Proceedings of The International Conference on Algebras, Modules and Rings, Lisbon, Portugal, 14-18 July 2003, P.153–182.

Continuous logic and locally compact groups

A. Ivanov

University of Wrocław
ivanov@math.uni.wroc.pl

We study expressive power of continuous logic in classes of locally compact groups. We also describe locally compact groups which are separably categorical structures.

On groups of period 72

E. Jabara, D.V. Lytkina, V.D. Mazurov

Università di Ca' Foscari San Giobbe, Venezia
Siberian State University of Telecommunications and Information Sciencies, Novosibirsk
Sobolev Institute of Mathematics, Novosibirsk
jabara@unive.it, daria.lytkin@gmail.com, mazurov@math.nsc.ru

Suppose that G is a periodic group. The spectrum $\omega(G)$ is the set of element orders of G. If $\omega(G)$ is finite then $\mu(G)$ is the set of maximal with respect to division elements of $\omega(G)$.

Theorem 1. *Suppose that $\mu(G) = \{8, 9\}$. Then G is locally finite.*

Theorem 2. *Suppose that G is a locally finite $\{2, 3\}$-group without elements of order 6. Then one of the following statements holds:*

1) $G = O_3(G)T$ where $O_3(G)$ is Abelian and T is a locally cyclic or locally quaternion group acting freely on $O_3(G)$.

2) $G = O_2(G)R$ where $O_2(G)$ is nilpotent of class at most 2 and R is a locally cyclic 3-group acting freely on $O_2(G)$.

3) $G = O_2(G)D$ where D contains a subgroup R of index 2 and $O_2(G)R$ satisfies (b).

4) G is a 2-group or a 3-group.

In 1–3 G is soluble of length at most 4.

The work is supported by Russian Foundation of Basic Research (Grants 12–01–90006, 13-01-00505, 11–01–00456, 11–01–91158), the Federal Target Program (Contract No. 14.740.11.0346), and the Integration Project of the Siberian Division of the Russian Academy of Sciences for 2012–2014 (No. 14).

Representing real numbers by generalized Fibonacci sequences

D. M. Karvatsky

National Pedagogical Dragomanov University
dinaris-mail@mail.ru

Let us consider the positive series

$$\sum_{n=1}^{\infty} u_n,$$

whose terms satisfy the following conditions:

1. $u_{n+2} = pu_{n+1} + su_n,\ n \in N$, moreover $u_1,\ u_2,\ p,\ s \in R^+$;
2. $\left\{ \begin{array}{c} 0 < p < 1 \\ \frac{1}{2} - \frac{p}{2} \leq s < 1 - p \end{array} \right.$.

Let $A = \{0\,,\,1\}\,,\ L = A \times A \times A \times A \times ....$ Relation f between the sets L and R^1

$$L \supset (\alpha_n) \xrightarrow{f} x \in R^1,$$

which is determined as follows

$$x = \sum_{n=1}^{\infty} \alpha_n u_n,$$

is obviously functional.

Theorem 1. *For any* $x \in \left[0\,,\, \frac{u_1(1-p)+u_2}{1-p-s}\right]$, *there is a sequence of real numbers* (a_n), $a_n \in \{0,1\} \equiv A$, *such that*

$$x = \sum_{n=1}^{\infty} \alpha_n u_n.$$

We will also consider properties of cylindrical sets which generated by that represent, in detail will show the specificity of their overlap.

1. Василенко Н.М. Математичні структури в просторі послідовностей Фібоначчі / Н.М. Василенко, М. В. Працьовитий // Науковий часопис НПУ імені М. П. Драгоманова. Серія 1. Фіз.-мат. науки. – Київ.: НПУ імені М. П. Драгоманова, 2008. – 9. С. 129-150.
2. Працьовитий М.В. Фрактальний підхід у дослідженні сингулярних розподілів. — Київ: Видавництво НПУ імені М. П. Драгоманова, 1998. - 296 с.

Weakly hereditary and idempotent closure operators in module categories

A. Kashu

Institute of Mathematics and Computer Science, Chişinău
kashuai@math.md

A *closure operator* of the category R-Mod is a function C which associates to every pair of modules $N \subseteq M$ the submodule $C_M(N) \subseteq M$ such that: (c_1) $N \subseteq C_M(N)$; (c_2) if $N \subseteq P \subseteq M$, then $C_M(N) \subseteq C_M(P)$; (c_3) if $f : M \to M'$ is an R-morphism and $N \subseteq M$, then $f(C_M(N) \subseteq C_{M'}\big(f(N)\big)$ ([1], [2]). The closure operator C defines two functions $\mathcal{F}_1^C$ and $\mathcal{F}_2^C$ by the rules:

$$\mathcal{F}_1^C(M) = \{N \subseteq M \mid C_M(N) = M\}, \ \ \mathcal{F}_2^C(M) = \{N \subseteq M \mid C_M(N) = N\}.$$

The closure operator C is: a) *weakly hereditary* if $C_M(N) = C_{C_M(N)}(N)$; b) *idempotent* if $C_M(N) = C_M\big(C_M(N)\big)$ for every $N \subseteq M$.

The main results consists in the characterizations by the functions $\mathcal{F}_1^C$ and $\mathcal{F}_2^C$ of three types of closure operators of R-Mod: 1) weakly hereditary; 2) idempotent; 3) weakly hereditary and idempotent.

In particular, is true the

Theorem 1. *There exists a monotone* (*antimonotone*) *bijection between the weakly hereditary* (*idempotent*) *closure operators C of R-Mod and the abstract functions $\mathcal{F}$ of type $\mathcal{F}_1$* (*of type $\mathcal{F}_2$*).

Similarly the weakly hereditary and idempotent closure operators of R-Mod are described by means of *transitive* abstract functions $\mathcal{F}$ of type $\mathcal{F}_1$ or of $\mathcal{F}_2$. In the radical theory of R-Mod [2] the corresponding results are the characterizations of diverse kinds of preradicals by the classes of torsion or torsion-free modules.

1. D. Dikranjan, E. Giuli, *Factorizations, injectivity and compactness in categories of modules*, Commun. in Algebra, v. **19**, №1, 1991, pp. 45–83.
2. A.I. Kashu, *Radicals and torsion in modules*, Kishinev, Ştiinţa, 1983 (in Russian).

On generalized differential Hopfian modules

G. Kashuba

Vasyl Stefanyk Precarpathian National University
kgil@i.ua

All rings in question are considered associative with non-zero units, all modules are left and unitary. Let $\Delta = \{d_1, d_2, ..., d_n\}$ be a finite set of pairwise commuting differentials of a ring R. A left R-module M is also endowed with differentials $\{\delta_1, \delta_2, ...\delta_n\}$ compatible with the differentials from Δ.

A left differentiall module M is called generalized differential Hopfian module (abbreviated gdH) if each its surjective differential endomorphism has a small kernel (see [1] for the case of ordinary rings).

In the talk we present generalization of some results from [2] to the case of differentials modules.

Theorem 1. *A direct differential summand of a gdH module is a gdH module.*

Theorem 2. *for a Dedekind finite differential module M (see. [3]) the properties of differential Hopfians and of being gdH are equivalent.*

Theorem 3. *For a differential quasi-projective module M with a differential projective cover $\alpha : P \to M$ in the category $DMod - R$ the module M is gdH if and one if P is a gdH module.*

Theorem 4. *For a differential homomorphism $\varphi : R \to S$ of differentials rings and a left differential S-module M, the following statements are valid:*

(1) If the module M_φ thal is obtained from φ by taking the universal square is a gdH module, then M is a gdH module as well.

(2) If φ is surjective, then being a gdH module is equivalent for M and M_φ.

Theorem 5. *If R is a left differential duo ring, then each right differential ideal of R has gdH properts is a module.*

Theorem 6. *Each left differential duo ring is Dedekind finite.*

1. A.Haghany, M.R.Vedady, Modules whose injective endomorphisms are essential. J. Algebra 243(2001) 765-769
2. A.Ghorbani and A.Haghany, Generalized Hopfian modules, J. Algebra 255(2002) 324-341
3. Кашуба Г.І., Комарницький М. Я., Скінченні за Дедекіндом диференціальні кільця і модулі та їх первинний радикал. Вісник Львівського національного університету імені Івана Франка, серія мех.-мат., 2008, вип. 67, 114-121.

Maximal Galois subring of R

Nataliya Kaydan, Zoya Mekhtieva

Donbass State Pedagogical University
Slavyansk Energy and Build College
kaydannv@mail.ru, mehtieva-zv@mail.ru

In the 1979s Takao Sumiyama examined finite local rings.

Throughout R will represent a finite local ring with radical M. Let K be the residue field R/M, and R^* the unit group of R. Let $|K| = p^r$ (p a prime), $|R| = p^{nr}$, $|M| = p^{(n-1)r}$, and p^k ($k \leq n$) the characteristic of R. Let $Z_{p^k} = Z/p^k Z$ be the prime subring of R. The $r-$dimensional Galois extension $GR(p^{kr}, p^k)$ of Z_{p^k} is called a Galois ring [1]. By [2] Theorem 8(i), R contains a subring isomorphic to $GR(p^{rk}, p^k)$, which will be called a maximal Galois subring of R.

Theorem 1. *If an inner automorphism of R maps a maximal Galois subring of R into (and hence onto) itself, then it induces the identity map on the maximal Galois subring.*

1. B.R.McDonald. Finite Rings with identity. Pure Appl. Math. Ser.28. Marcel Dekker, New York, 1974.
2. R.Raghavendran. Finite associative rings, Compositio Math. 21 (1969), 195-229.
3. T. Sumiyama. Note on maximal Galois subrings of finite local rings. Math. J.Okayama Univ. 21 (1979), 31-32.

Algebras and superalgebras of Jordan brackets

I. Kaygorodov

Universidade de Sao Paulo, Brazil
kaygorodov.ivan@gmail.com

Poisson algebra is a vector space with two multiplications $\cdot$ and $\{,\}$. It is using in mathematical physics, differential geometry and others areas. For example, I. Shestakov and U. Umirbaev used the free Poisson algebra for proof of Nagata conjecture about wild automorphisms. Also, algebraic properties of Poisson algebras were studied in some papers of D. Farkass, L. Makar-Limanov, S. Mischenko, V. Petrogradsky and others. In particulary, D. Farkass proved that every PI Poisson algebra satisfy some polinomial identity with special type.

Algebras and superalgebras of Jordan brackets are generalization of Poisson algebras and superalgebras. Using Kantor process from every Poisson (super)algebra we can construct Jordan superalgebra. We define (super)algebra of Jordan bracket as (super)algebra where Kantor process obtain Jordan superalgebra.

D. King and K. McCrimmon proved that every unital algebra of Jordan brackets satisfies

$$(a \cdot b) \cdot c = a \cdot (b \cdot c), a \cdot b = b \cdot a,$$

$$\{a, b\} = -\{b, a\}, \{\{a, b\}, c\} + \{\{b, c\}, a\} + \{\{c, a\}, b\} = 0,$$

$$\{a \cdot b, c\} = \{a, c\} \cdot b + a \cdot \{b, c\} - D(c)ab, \text{ where } D(a) = \{a, 1\}.$$

If $D = 0$ we have Poisson algebra.

Early, (super)algebras of Jordan brackets was considered by C. Martinez, V. Kac, N. Cantarini, E. Zelmanov, I. Shestakov, I. Kaygorodov and V. Zhelyabin [1, 2, 3].

For polinomial algebras of Jordan bracket was proved an analogue of Farkass Theorem. It is following

Theorem 1. *Every PI algebra of Jordan bracket satisfy identity with type*

$$g = \sum_{i=0}^{[m/2]} \sum_{\sigma \in S_m} c_{\sigma,i} \Pi_{k=1}^{i} \langle x_{\sigma(2k-1)}, x_{\sigma(2k)} \rangle \Pi_{k=2i+1}^{m} D(x_{\sigma(k)}),$$

where $\langle x, y \rangle = \{x, y\} - (D(x)y - xD(y))$, $c_{\sigma,i}$ *from basic field.*

It is a joint work ith prof. I. Shestakov (USP, Brazil).

1. C. Martinez, I. Shestakov, E. Zelmanov, J. London Math. Soc. (2), 64, 2001, 2, 357–368.
2. N. Cantarini, V. Kac, J. Algebra, 313, 2007, 1, 100–124.
3. I. Kaygorodov, V. Zhelyabin, St.-Peterburg Math. J., 23, 2012, 4, 655–667.

Congruence of acts over groups

A. R. Khaliullina

National Research University of Electronic Technology MIET, Moscow
haliullinaar@gmail.com

A right act [1] over a semigroup S is defined as a set X with a mapping $X \times S \to X$, $(x, s) \mapsto xs$ satisfying the axiom $(xs)s' = x(ss')$ for $x \in X$. $s, s' \in S$. A left act Y over a semigroup S is defined analogously by a mapping $S \times Y \to Y$, $(s, y) \mapsto sy$ and the axiom $s(s'y) = (ss')y$ for $y \in Y$, $s, s' \in S$. We will consider only the right act, so we'll just call them act.

Congruence of act X over the semigroup S is equivalence relation ρ on X, that $(x, y) \in \rho \Rightarrow (xs, ys) \in \rho$ for all $x, y \in S$.

The main results are:

Theorem 1. *Let $X = G/H$, where H is a subgroup of group G. For any subgroup $H^{'} \supseteq H$ define an equivalence relation σ on G/H, assuming $(Hx, Hy) \in \sigma \Leftrightarrow H^{'}x = H^{'}y$. Then σ – a congruence of act G/H, in addition, any congruence of act G/H has the form of σ.*

Theorem 2. *Let $X_i = G/H_i$, $X_j = G/H_j$ are two components, $x \in X_i$ and $y \in X_j$. If $i \neq j$ and $(x, y) \in \sigma$, we will write $i \sim j$, where σ is a congruence on $\coprod_{i \in I} X_i$. σ induces on G/H_i congruence σ_i is a partition into right cosets of $H_i^{'}$. Then $(x, y) \in \sigma \Leftrightarrow H_i^{'} = a^{-1}H_j^{'}a$*

Theorem 3. *Any subdirectly indecomposable act over the group is as follows: a) G/H, b) $G/H \coprod \{0\}$, where H is a subgroup with the property: H has the lowest $H^{'} \supseteq H$.*

Theorem 4. *Given subdirectly irreducible unitary act over the group $X = G/H$. Consider the subgroup $H^{'} \supseteq H$, assuming that $(Hx, Hy) \in \sigma_i \Leftrightarrow H^{'}x = H^{'}y(*)$. This will be a congruence on X. Conversely, for any congruence on G/H, there exists a subgroup $H^{'}$, which will be carried out (*).*

1. Kilp M., Knauer U., Mikhalev A.V. Monoids, acts and categories. Berlin - New York: W. de Gruyter, 2000.

On arithmetic progressions on Legendre elliptic curves

Ya. Kholyavka, O. Kossak

Ivan Franko National University of Lviv, Lviv, Ukraine
ya_khol@franko.lviv.ua, evagata23@yahoo.com

An arithmetic progression is a sequence of numbers such that the difference between any two consecutive numbers is constant. When we talk about an arithmetic progression on a elliptic curve over a field $\mathbb{F}_q$ (char $\mathbb{F}_q \neq 2,\ 3$) $E : y^2 = x(x-1)(x-\lambda)$ $(\lambda \in \mathbb{F}_q,\ \ \lambda^2 \neq 0,\ 1)$, we mean an arithmetic progression in the x-coordinates $Q+P, Q+2P, \ldots, Q+nP$, $Q, P \in E$,where n is the smallest positive integer such that $nP = O$.

Theorem 1. *There exist at most $1 + q/3$ elements $(x_1 \ldots x_n)$ such that arbitrary $(x(P), x(2P), \ldots x_(nP))$ can be obtained as the product of the square of an element $\mathbb{F}_q$ by elements $x_1 \ldots x_n$ and their renumbering.*

1. Silverman J.H., The Arithmetic of Elliptic Curves, Graduate Texts in Mathematics, 106, Springer-Verlag, New York, 1986.
2. Тараканов В.Е., Несколько замечаний об арифметических свойствах рекуррентных последовательностей на эллиптических кривых над конечным полем // Матем. заметки – 2007. – Т.82,– Вып. 6. – С.926-933.

About n-ary analog of the theorem of Cheva

D.I. Kirilyuk

Francisk Skorina Gomel State University
kirilyuk.denis@gmail.com

Most evident of known ways of research of n-ary groups [1, 2] are certainly geometrical [3, 4]. So obtaining new results in this direction seems to us rather actual. In this work the n-ary analog of the theorem of Cheva is received.

Let's remind that a parallelogram of n-ary group G is a quadrangle $< a, b, c, d >$ for which the identity $d = (ab^{[-2]}\overset{2n-4}{b}\ c)$ is satisfied.

Directed segments $\overline{ab}$ and $\overline{cd}$ n-ary group G is called equal and write $\overline{ab} = \overline{cd}$, if $< a, c, d, b >$ is a parallelogram

Let $\overline{V}$ be the set of all directed segments n-ary group G. From Proposition 1 [3] follows that binary relation $=$ on the set of $\overline{V}$ is the equivalence relation and breaks a set $\overline{V}$ can be partitioned into disjoined classes. The class generated by the directed segment $\overline{ab}$ looks like

$$K(\overline{ab}) = \{\overline{uv}|\overline{ab} \in V, \overline{uv} = \overline{ab}\}.$$

As a vector $\overrightarrow{ab}$ n-ary group G understand a class $K(\overline{ab})$, i.e. $\overrightarrow{ab} = K(\overline{ab})$.

By symbol $\mathbb{Q}^{(r)}$ designate the following set

$$\mathbb{Q}^{(r)} = \{\frac{m}{r^t}|m \in \mathbb{Z}, t \in \mathbb{N}\}.$$

Other used designations and concepts can be found in [3].

Theorem 1. *Let G be a semiabelian n-ary group, a, b, c arbitrary and distinct points G. If a_1, b_1, c_1, x such that*

$$\begin{cases} \overrightarrow{ab_1} = t_1\overrightarrow{ac} \\ \overrightarrow{ac_1} = t_2\overrightarrow{ab} \\ \overrightarrow{ba_1} = t_3\overrightarrow{bc} \end{cases}, \begin{cases} \overrightarrow{ax} = l_1\overrightarrow{aa_1} \\ \overrightarrow{bx} = l_2\overrightarrow{bb_1} \\ \overrightarrow{cx} = l_3\overrightarrow{cc_1} \end{cases},$$

where $t_1, t_2, t_3, l_1, l_2, l_3 \in \mathbb{Q}^{(r)}$, then $k_1k_2k_3 = 1$, where $k_1, k_2, k_3 \in \mathbb{Q}^{(r)}$ and

$$\begin{cases} \overrightarrow{ba_1} = k_2\overrightarrow{a_1c} \\ \overrightarrow{c_1b} = k_3\overrightarrow{c_1a} \\ \overrightarrow{ab_1} = k_1\overrightarrow{b_1c} \end{cases}$$

1. A.M.Gal'mak (2003), N-ary groups(Russin). Part 1, Gomel.
2. W.A. Dudek (1980), Remarks on n-groups. Demonstr.Math. **13**, p. 165 - 181.
3. S.A. Rusakov (1998), Some applications of the theory of n-ary groups (Russian). Belaryskaya navuka
4. Yu. I. Kulazhenko (2011). Quasigroups and Related Systems. **19**, p. 265 - 278.

Exponent matrices and their applications

V. Kirichenko, M. Khibina

Kyiv National Taras Shevchenko University
Institute of Engineering Thermophysics of the NAS of Ukraine
vv.kirichenko@gmail.com

An integer matrix $\mathcal{E} = (\alpha_{ij}) \in M_n(\mathbb{Z})$ is called

(1) an exponent matrix if $\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}$ and $\alpha_{ii} = 0$ for all i, j, k;

(2) a reduced exponent matrix if $\alpha_{ij} + \alpha_{ji} > 0$ for all i, j, $i \neq j$.

We consider the quiver of a reduced exponent matrix. A reduced exponent matrix $\mathcal{E} = (\alpha_{ij}) \in M_n(\mathbb{Z})$ is Gorenstein if there exists a permutation σ of $\{1, 2, \ldots, n\}$ such that $\alpha_{ik} + \alpha_{k\sigma(i)} = \alpha_{i\sigma(i)}$ for $i, k = 1, \ldots, n$.

The quiver is called admissible if it is the quiver of reduced exponent matrix.

A ring A is called weakly prime if the product of any two nonzero two-sided ideals, which does not contain in the Jacobson radical, is nonzero.

We consider the relations of reduced exponent matrices with (1) tiled orders; (2) weakly prime rings; (3) elementary abelian 2-groups; (4) partially ordered sets; (5) Latin squares.

Cycles of linear and semilinear mappings

Tetiana Klymchuk

Taras Shevchenko Kyiv National University, Kyiv, Ukraine
`missklimchuck@yandex.ru`

This is the joint work of all the authors of [2, 3].

A mapping $\mathcal{A}$ from a complex vector space U to a complex vector space V is *semilinear* if

$$\mathcal{A}(u+u') = \mathcal{A}u + \mathcal{A}u', \qquad \mathcal{A}(\alpha u) = \bar{\alpha}\mathcal{A}u$$

for all $u, u' \in U$ and $\alpha \in \mathbb{C}$. We write $\mathcal{A} : U \to V$ if $\mathcal{A}$ is a linear mapping and $\mathcal{A} : U \dashrightarrow V$ if $\mathcal{A}$ is a semilinear mapping. A matrix of a semilinear operator $\mathcal{A} : U \dashrightarrow U$ is reduced by consimilarity transformations $A \mapsto \bar{S}^{-1}AS = B$ (S is nonsingular).

We obtained in [3] a canonical form of matrices of a cycle of linear and semilinear mappings

$$\mathcal{A}:\quad \begin{tikzcd} V_1 \arrow[r, dash, "\mathcal{A}_1"] \arrow[rrrr, dash, bend right, "\mathcal{A}_t"'] & V_2 \arrow[r, dash, "\mathcal{A}_2"] & \cdots \arrow[r, dash, "\mathcal{A}_{t-2}"] & V_{t-1} \arrow[r, dash, "\mathcal{A}_{t-1}"] & V_t \end{tikzcd}$$

(each line is $\longrightarrow$, or $\longleftarrow$, or $\dashrightarrow$, or $\dashleftarrow$).

Gelfand and Ponomarev [1] proved that the problem of classifying pairs of commuting linear operators contains the problem of classifying k-tuples of linear operators for any k. We extended in [2] this statement to semilinear mappings: *the problem of classifying pairs of commuting semilinear operators contains the problem of classifying $(p+q)$-tuples consisting of p linear operators and q semilinear operators, in which p and q are arbitrary nonnegative integers.*

1. I.M. Gelfand, V.A. Ponomarev. Remarks on the classification of a pair of commuting linear transformations in a finite dimensional vector space, Funct. Anal. Appl. . 3 (1969) 325–326..
2. D. Duarte de Oliveira, R.A. Horn, T. Klimchuk, V.V. Sergeichuk. Remarks on the classification of a pair of commuting semilinear operators, Linear Algebra Appl., 436 (2012) 3362–3372.
3. D. Duarte de Oliveira, V. Futorny, T. Klimchuk, D. Kovalenko, V.V. Sergeichuk. Cycles of linear and semilinear mappings, Linear Algebra Appl., 438 (2013), 3442–3453.

Asymptotic mean of digits of 4–adic representation of real number and it's properties

S.O. Klymchuk

Institute of Mathematics of National Academy of Sciences of Ukraine
svetaklymchuk@gmail.com

For any real number $x \in [0;1]$ exist sequence (α_n) that $\alpha_n \in \{0, 1, \dots, s-1\}$ and

$$x = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2} + \dots + \frac{\alpha_n}{s^n} + \dots \equiv \Delta^s_{\alpha_1\alpha_2\dots\alpha_n\dots}.$$

Let $N_i(x,k)$ be a number of digits $i \in \{0, \dots, s-1\}$ in $s-$adic representation $\Delta^s_{\alpha_1\alpha_2\dots\alpha_k\dots}$ of number $x \in [0;1]$ to $k-$th place.

Definition 1. *Frequency (asymptotic frequency) of digit i* in $s-$adic representation of real number $x \in [0;1]$ is called

$$\nu_i(x) = \lim_{k\to\infty} \frac{N_i(x,k)}{k},$$

if the limit exist.

Definition 2. If exist $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{i=1}^{n} \alpha_i(x) = r(x)$, where α_i – digits of $s-$adic representation of number $x \in [0;1]$, then its value (number $r(x)$) is called *asymptotic mean (or simply mean) of digits* of number x.

We study topological, metric and fractal properties of sets of real numbers represented in 4–adic scale with predefined asymptotic mean of digits.

Namely, we consider a set

$$S_\theta = \left\{ x : r(x) \equiv \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \alpha_i(x) = \theta \right\},$$

where θ is a predefined parameter from $[0;3]$, $x \in [0;1]$, $\alpha_i(x)$ is the $i-$th digit of number x.

It is clear that the set S_θ is union of three not intersected sets:
$\Theta_1 = \{x : \nu_i(x) \wedge \nu_j(x) \text{ — exists}, \ i \neq j, \ i,j \in \{0,1,2,3\}\}$
$\Theta_2 = \{x : \nu_1(x), \ \nu_2(x), \ \nu_3(x) \text{ — not exist}\}$
$\Theta_3 = \{x : \nu_i(x) \text{ exist}, \ i \in \{1,2,3\}\}$.

Theorem 1. *If asymptotic mean of digits $\theta = 0$ or $\theta = 3$, $\Theta_2 \equiv \Theta_3 \equiv \varnothing$ and Θ_1 is anomaly fractal, every where dense set.*

Theorem 2. *The set Θ_1 is continual, every where dense set in $[0;1]$. It is of full Lebesgue measure if $a = \dfrac{3}{2}$ and of zero Lebesgue measure if $a \neq \dfrac{3}{2}$.*

Theorem 3. *Sets Θ_2 and Θ_3 with given asymptotic mean of digits from $(0;3)$ is continual, every where dense sets from $[0;1]$ of zero Lebesgue measure. Hausdorff–Besicovitch dimension of which $\alpha_0(\Theta_2) > 0$, $\alpha_0(\Theta_3) > 0$*

Semigroups and graphs - on the genus of groups and semigroups

Ulrich Knauer

University of Oldenburg, Berlin, Germany
`emailadress`

Planar groups have been described already in the 1890 by Maschke. To define the genus of a group we use a suitable Cayley graph which then is embedded. That is we look for a system of generators of the group, such that the genus of the respective Cayley graph becomes minimal. There are many sometimes quite surprising results on this topic - for example there exists exactly one group with genus 2, for other genus there usually exist infinitely many groups. For semigroups which are not groups but unions of groups, i. e. completely regular semigroups, there are some new results. For some types of completely regular semigroups I present some of the problems which arise when determining their genus and give results on various levels. In analogy with graphs their exist many ideas and open questions for semigroups, for example crossing numbers, thickness, profile, non-orientable genus.

For information I include a list of references.

1. László Babai, Groups of graphs on given surfaces, Acta Math. Acad. Sci. Hungar. 24 (1973), 215-221.
2. László Babai, Some applications of graph contractions, J. Graph Theory 1 (1977), no. 2, 125-130, Special issue dedicated to Paul Turán.
3. Carl Droms, Infinite-ended groups with planar Cayley graphs, J. Group Theory 9 (2006), no. 4, 487-496.
4. A. Georgakopoulos, The planar cubic Cayley graphs, ArXiv e-prints (2011).
5. J. L. Gross and T. W. Tucker, Topological graph theory, Dover Publications Inc., Mineola, NY, 2001.
6. W. Imrich and S. Klavžar, Product graphs, Wiley-Interscience Series in Discrete Mathematics and Optimization, Wiley-Interscience, New York, 2000.
7. A. V. Kelarev, On undirected Cayley graphs, Australas. J. Combin. 25 (2002), 73-78.
8. A. V. Kelarev, On Cayley graphs of inverse semigroups, Semigroup Forum 72 (2006), no. 3, 411-418.
9. Andrei V. Kelarev and Cheryl E. Praeger, On transitive Cayley graphs of groups and semigroups, European J. Combin. 24 (2003), no. 1, 59-72.
10. M. Kilp, U. Knauer, and A. V. Mikhalev, Monoids, acts and categories, de Gruyter Expositions in Mathematics, vol. 29, Walter de Gruyter & Co., Berlin, 2000.
11. Kolja Knauer and Ulrich Knauer, Toroidal embeddings of right groups, Thai J. Math. 8 (2010), no. 3, 483-490.
12. U. Knauer, Y. Wang, and X. Zhang, Functorial properties of Cayley con- structions, Acta Comment. Univ. Tartu. Math. (2006), no. 10, 17-29.
13. Ulrich Knauer, Algebraic graph theory, de Gruyter Studies in Mathematics, vol. 41, Walter de Gruyter & Co., Berlin, 2011, Morphisms, monoids and matrices.
14. Heinrich Maschke, The Representation of Finite Groups, Especially of the Rotation Groups of the Regular Bodies of Three-and Four-Dimensional Space, by Cayley's Color Diagrams, Amer. J. Math. 18 (1896), no. 2, 156-194.
15. M. Petrich and N. R. Reilly, Completely regular semigroups, Canadian Mathematical Society Series of Monographs and Advanced Texts, 23, John Wiley & Sons Inc., New York, 1999.
16. V. K. Proulx, Classification of the toroidal groups, J. Graph Theory 2 (1978), no. 3, 269-273.
17. J. Scherphuis, Jaap's puzzle page, www.jaapsch.net/puzzles/cayley.htm.
18. Denis Vladimirovich Solomatin, Semigroups with outerplanar Cayley graphs, Sibirskie Èlektronnye Matematicheskie Izvestiya [Siberian Elec- tronic Mathematical Reports] 8 (2011), 191-212.
19. D.V. Solomatin, Direct products of cyclic semigroups admitting a planar Cayley graph., Sibirskie Ehlektronnye Matematicheskie Izvestiya [electronic only] 3 (2006), 238-252 (rus).
20. T. W. Tucker, The number of groups of a given genus, Trans. Amer. Math. Soc. 258 (1980), no. 1, 167-179.
21. T. W. Tucker, On Proulx's four exceptional toroidal groups, J. Graph Theory 8 (1984), no. 1, 29-33.
22. A. T. White, Graphs of groups on surfaces, North-Holland Mathematics Studies, vol. 188, North-Holland Publishing Co., Amsterdam, 2001, Inter- actions and models.
23. X. Zhang, Clifford semigroups with genus zero, Semigroups, acts and cate- gories with applications to graphs, Math. Stud. (Tartu), vol. 3, Est. Math. Soc., Tartu, 2008, pp. 151-160.

Recent advances in the field of model theory in topoi

M. Ya. Komarnytskyi, V. I. Stefaniak

Ivan Franko National University of Lviv, Lviv, Ukraine
Lviv Academy of Commerce, Lviv, Ukraine
`mykola_komarnytsky@yahoo.com, stef_vo@mail.ru`

Let $\mathscr{C}$ be a category with direct products and let $\mathcal{A}$ be a group object [1] of $\mathscr{C}$ endowed with three morphisms $\mathcal{A}\times\mathcal{A}\xrightarrow{+}\mathcal{A}$, $\mathcal{A}\xrightarrow{-}\mathcal{A}$, $\nVdash\xrightarrow{\mathbf{0}}\mathcal{A}$ ($\nVdash$ is the final object of $\mathscr{C}$). In a similar way, we can define a ring structure on $\mathcal{A}$ by considering additional morphisms $\mathcal{A}\times\mathcal{A}\xrightarrow{\cdot}\mathcal{A}$, $\mathcal{A}\xrightarrow{\mathbf{1}}\mathcal{A}$. Let $\mathcal{A}$ be a ring object of $\mathscr{C}$. We say that a subobject $\mathcal{I}\overset{\gamma}{\rightarrowtail}\mathcal{A}$ of $\mathcal{A}$ is a *left ideal* of $\mathcal{A}$ if there exist morphisms $\mathcal{I}\times\mathcal{I}\xrightarrow{\oplus}\mathcal{I}$, $\mathcal{I}\xrightarrow{\ominus}\mathcal{I}$, $\nVdash\xrightarrow{\odot}\mathcal{I}$, $\mathcal{A}\times\mathcal{I}\xrightarrow{\odot_l}\mathcal{I}$ such that the diagrams

$$\begin{tikzcd} \mathcal{A}\times\mathcal{A} \arrow[r, "+"] & \mathcal{A} \\ \mathcal{I}\times\mathcal{I} \arrow[u, "\gamma\times\gamma"] \arrow[r, "\oplus"] & \mathcal{I} \arrow[u, "\gamma"'] \end{tikzcd} \qquad \begin{tikzcd} \nVdash \arrow[rr, "\mathbf{0}"] \arrow[dr, "\odot"'] & & \mathcal{A} \\ & \mathcal{I} \arrow[ur, "\gamma"'] & \end{tikzcd}$$

$$\begin{tikzcd} \mathcal{I} \arrow[r, "\ominus"] \arrow[d, "\gamma"'] & \mathcal{I} \arrow[d, "\gamma"] \\ \mathcal{A} \arrow[r, "-"] & \mathcal{A} \end{tikzcd} \qquad \begin{tikzcd} \mathcal{A}\times\mathcal{I} \arrow[r, "\odot_l"] \arrow[d, "1_{\mathcal{A}}\times\gamma"'] & \mathcal{I} \arrow[d, "\gamma"] \\ \mathcal{A}\times\mathcal{A} \arrow[r, "\cdot"] & \mathcal{A} \end{tikzcd}$$

are commutative. We can consider also *right* and *two-sided ideals* of $\mathcal{A}$.

In the context of category theory there is a natural correspondence between the characterizations of group objects, ring objects and ideals by means of contravariant functors and Hom-sets.

It was first emphasized by Mulvey [3] that because the internal logic of a topos is intuitionistic the concept of a field can be defined by the axioms F1, F2, F3 which are equivalent in the classical logic but have different interpretations in other logics. In the talk we extend the ring-theoretic concept of a simple ideal in the intuitionistic logic of the topos of arrows (the Sierpiński topos) $\mathcal{S}et^{\rightarrow}$ [2].

Theorem 1. *Let $\mathcal{I}$ be a two-sided ideal of a commutative ring object $\mathcal{A}$ of $\mathcal{S}et^{\rightarrow}$. Hence we can construct by means of coequalizers the quotient ring $\mathcal{R}=\mathcal{A}/\mathcal{I}$ as a homomorphism of $\mathcal{S}et$-rings $R_1\xrightarrow{f}R_2$, and the following statements in $\mathcal{S}et^{\rightarrow}$ are true:*

1) $\mathcal{R}$ is a geometric field (F1) if and only if R_1 and R_2 are fields in $\mathcal{S}et$;

2) $\mathcal{R}$ is a field of fractions (F2) if and only if R_2 is a field, and R_1 is a commutative ring in which all the elements not belonging to $Ker(f)$ are invertible;

3) $\mathcal{R}$ is a residue field (F3) if and only if R_2 is a field, and R_1 is a commutative ring with $(\forall a\in R_1)\quad (a=0)\vee(a \text{ is invertible})\vee(a \text{ is not invertible, but } f(a) \text{ is invertible in } R_2)$;

4) $\mathcal{R}$ is an integral ring (I1) if and only if R_1 and R_2 are integral domains in $\mathcal{S}et$.

1. Bucur I. *Introduction to the Theory of Categories and Functors* / I. Bucur, A. Deleanu ; with the collabor. of P. J. Hilton. – New York : John Wiley & Sons Ltd, 1970. – 224 p.
2. Goldblatt R. *Topoi : The Categorial Analysis of Logic* / R. Goldblatt. – Amsterdam : North Holland Publishing Co., 1979. – 486 p. – ("Studies in Logic", 98).
3. Mulvey C. J. *Intuitionistic algebra and representations of rings* / C. J. Mulvey // Mem. Amer. Math. Soc. – 1974. – N 148. – P. 3-57.

On tame and wild cases for matrix representations of the semigroup

E. M. Kostyshyn

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
elina.kostyshyn@mail.ru

The problem of classifying the tame and wild finite groups G over a field k is completely solved in [1]. A similar problem for semigroups is very far from being solved. Moreover, the problem on representation type for a wide class of semigroups was considered only in certain cases (see [2], [3], [4]).

We consider matrix representations of the semigroup $T_2 \times T_2$ (T_2 denotes the semigroup of all transformations of a set of 2 elements) over a field k of characteristic 2, the restrictions of which to the subsemigroups T_2 satisfy some natural conditions. These studies were carried out together with V. M. Bondarenko.

1. V. M. Bondarenko, Ju. A. Drozd. The representation type of finite groups // Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI), 1977, 71, P. 24–41; English trans. in Journal of Soviet Math., 1982, 20, N6, P. 2515–2528.
2. C. Ringel. The representation type of the full transformation semigroup T4 // Semigroup Forum, 2000, N3, P. 429–434.
3. V. M. Bondarenko, O. M. Tertychna. On infiniteness of type of infinite semigroups generated by idempotents with partial null multiplication // Trans. Inst. of Math. of NAS of Ukraine, 2006, N3, P. 23-44 (in Russian).
4. V. M. Bondarenko, O. M. Tertychna. On tame semigroups generated by idempotents with partial null multiplication // Algebra Discrete Math., 2008, N4, P. 15–22.

Properties of sets of solutions of equations containing fractal functions

O. V. Kotova

Kherson State University
Olga-Kotova@ukr.net

Let $\Delta^s_{\alpha_1(x)\ldots\alpha_k(x)\ldots}$ be a formal s-adic representation for some number $x\in[0;1]$, $\alpha_i=\alpha_i(x)\in\{0,1,\ldots,s-1\}=A$, i.e., $x=\sum\limits_{m=1}^{\infty}s^{-m}\alpha_m$.

If limit $\lim\limits_{n\to\infty}\dfrac{N_i(x,n)}{n}$ exists, where $i\in A$, $N_i(x,n)=\#\{k:\alpha_k(x)=i,k\le n\}$, then its value $\nu_i^s(x)$ is called a *frequency of digit "i"* in s-adic representation of number x.

Theorem 1. *Equation $\nu_i^s(x)=x$, $i\in A$, does not have rational solutions in s-adic numeral sustem except $x=0$.*

Theorem 2. *Number* $x=\dfrac{\gamma_1}{s}+\dfrac{\gamma_2}{s^2}+\ldots+\dfrac{\gamma_k}{s^k}+\sum\limits_{j=1}^{\infty}\sum\limits_{l=1}^{e_j}\dfrac{i\cdot\beta_{lj}+i_{lj}\cdot(1-\beta_{lj})}{s^{s_j+l}}$, *where*

$$\begin{aligned}
\gamma_1,i,i_{lj} &\in A,k\in N,i_{lj}\ne i,x_1=\frac{\gamma_1}{s}+\ldots+\frac{\gamma_k}{s^k},\\
\beta_{ln} &= [(s_n+l)\,x_n]-[(s_n+l-1)\,x_n]\,,\\
x_n &= \frac{\gamma_1}{s}+\frac{\gamma_2}{s^2}+\ldots+\frac{\gamma_k}{s^k}+\sum_{j=1}^{n-1}\sum_{l=1}^{e_j}\frac{i\cdot\beta_{lj}+i_{lj}\cdot(1-\beta_{lj})}{s^{s_j+l}},\\
s_1 &= k,s_n=s^{s_{n-1}},e_n=s_{n+1}-s_n=s^{s_n}-s_n,
\end{aligned}$$

is a solution of equation $\nu_i^s(x)=x$.

Theorem 3. *For any infinite sequence $\{\varepsilon_n\}$ of zeros and ones there exist $\gamma_{2(k-1)s+1},\gamma_{2(k-1)s+2},\ldots,\gamma_{2ks-1},k=1,2,\ldots$ such that number*

$$x=\Delta^s_{\gamma_1\ldots\gamma_{2s-1}\varepsilon_1\gamma_{2s+1}\ldots\gamma_{4s-1}\varepsilon_2\ldots\gamma_{2(k-1)s+1}\ldots\gamma_{2ks-1}\varepsilon_k\ldots},$$

is a solution of equation $\nu_1^s(x)=x$. Hausdorff-Besicovitch dimension of the set K_s of solutions of equation $\nu_i^s(x)=x$ obtained by this algorithm is equal to $\dfrac{1}{2s}\log_s 2$.

Theorem 4. *Hausdorff-Besicovitch dimension of the set E_s of all solutions of the set $\nu_i^s(x)=x$ is not less than* $\dfrac{1}{2s}\log_s 2$.

1. *Котова О.В.* Континуальність множини розв'язків одного класу рівнянь, які містять функцію частоти трійкових цифр числа // Укр. мат. журн. — 2008. — т.60, № 10. — С. 1414–1421.
2. *Працьовитий М.В.* Фрактальний підхід у дослідженнях сингулярних розподілів. — Київ: Вид-во НПУ імені М.П. Драгоманова, 1998. — 296 с.

One class of Σ_t-closed formations of finite groups

V.A. Kovalyova, A.N. Skiba

Francisk Skorina Gomel State University
vika.kovalyova@rambler.ru, alexander.skiba@gmail.com

Throughout this paper, all groups are finite. We use $\mathcal{N}$ and $\mathcal{N}^r$ to denote the class of all nilpotent groups and the class of soluble groups of nilpotent length at most r ($r \geq 1$). If p is a prime, then we use $\mathcal{G}_p$ to denote the class of all p-groups.

By definition, every formation is 0-*multiply saturated* and for $n \geq 1$ a formation $\mathcal{F}$ is called n-*multiply saturated* if $\mathcal{F} = LF(f)$, where every non-empty value of the function f is an $(n-1)$-multiply saturated formation (see [1] and [2]).

Let $\mathcal{F}$ be a class of groups and t a natural number with $t \geq 2$. Recall that $\mathcal{F}$ is called Σ_t-*closed* if $\mathcal{F}$ contains all such groups G that G has subgroups $H_1, \ldots, H_t$ whose indices are pairwise coprime and $H_i \in \mathcal{F}$, for $i = 1, \ldots, t$.

Theorem 1. *Let $\mathcal{M}$ be an r-multiply saturated formation and $\mathcal{N} \subseteq \mathcal{M} \subseteq \mathcal{N}^{r+1}$ for some $r \geq 0$. Then, for any prime p, both formations $\mathcal{M}$ and $\mathcal{G}_p\mathcal{M}$ are Σ_{r+3}-closed.*

Corollary 1. *(See [3, Satz 1.3]) Every saturated formation contained in $\mathcal{N}^2$ is Σ_4-closed.*

Corollary 2. *The class of all soluble groups of nilpotent length at most r ($r \geq 2$) is Σ_{r+2}-closed.*

1. A.N. Skiba. A Characterization of Finite Soluble Groups of Given Nilpotent Length. *Problems in Algebra* **3** (1987), 21–31.
2. L.A. Shemetkov and A.N. Skiba. *Formations of Algebraic Systems* (Nauka, 1989).
3. Otto-Uwe Kramer. Endliche Gruppen mit paarweise teilerfremden Indizes. *Math. Z.* **139**(1) (1974), 63–68.

Orthogonal operators on associative algebras

S.V. Kovriga, V.S. Luchko, A.P. Petravchuk

Kyiv national Taras Shevchenko University
apetrav@gmail.com

In the paper [1], abelian complex structures on real Lie algebras L were studied (in connection with some problems in geometry of Kaehler manifolds); these structures are linear operators $J : L \to L$ such that $J^2 = -E$ and $[J(x), J(y)] = [x, y]$ for all $x, y \in L$. Since the last condition is similar to orthogonality, some papers appeared recently where Lie algebras possessing orthogonal operators and groups of such bijective operators were investigate (see, for example [2], [3]). Analogous question for associative algebras is also of interest (note that an "orthogonal" operator on an associative algebra induces an operator with the same property on the adjoint Lie algebra).

Let $\mathbb{K}$ be an algebraically closed field of characteristic zero and A be an associative algebra over $\mathbb{K}$ (not necessarily with 1). A linear map $T : A \to A$ will be called orthogonal if $T(x)T(y) = xy$ for all $x, y \in A$. Of course, the identity operator E and its opposite $-E$ are orthogonal, but such trivial operators are not interesting. We study associative algebras possessing bijective nontrivial orthogonal operators and the group $O(A)$ of all such operators on A. If the algebra A has the unit element, then the group $O(A)$ is isomorphic to a subgroup of invertible elements of the center $Z(A)$. The most interesting is the case of nilpotent algebras, when the group of all bijective orthogonal operators can be relatively large (in case of the algebra A with zero multiplication the group $O(A)$ coincides obviously with the general linear group $GL(A)$).

As in case of Lie algebras the group $O(A)$ has the trivial intersection with the automorphism group $Aut(A)$ in the general linear group $GL(A)$, and the subgroup $Aut(A)$ is invariant under action of orthogonal operators on A. We have found orthogonal groups of some classes of nilpotent Lie algebras. For example, if $A = K\langle a, a^2, \ldots, a^n \mid a^{n+1} = 0\rangle$, then $O(A)$ consists of upper triangular matrices (in this basis) with some additional restrictions on elements outside the main diagonal.

1. M.Barberis, I.G.Dotti, Abelian complex structures on solvable Lie algebras, Journal of Lie Theory, 14 (2004), 25–34.
2. S.V. Bilun, D.V. Maksimenko, and A.P, Petravchuk, On the group of Lie-orthogonal operators on a Lie algebra, Methods of Functional Analysis and Topology, (2011) v.17, no.3, P.199-203
3. D.R. Popovych, Lie-orthogonal operators, Linear Algebra Appl., 438, no.5 (2013), 2090-2106.

On minimal nontrivial quasigroup identities

Halyna Krainichuk

University "Ukraina", Vinnytsia Institute of Economics and Social Sciences
kraynichuk@ukr.net

Two quasigroups $(Q;\cdot)$ and $(Q;\circ)$ are said to be *orthogonal* if the system $\{x\cdot y=a,\ \ x\circ y=b$ has a unique solution for all $a,b\in Q$. A τ-parastrophe $(Q;\overset{\tau}{\cdot})$ of a quasigroup $(Q;\cdot)$ is defined by

$$x_{1\tau}\overset{\tau}{\cdot}x_{2\tau}=x_{3\tau}\ \Leftrightarrow\ x_1\cdot x_2=x_3,$$

where $\tau\in S_3:=\{\iota,\ell,r,s,s\ell,sr\}$, $\ell:=(13)$, $r:=(23)$, $s:=(12)$. A quasigroup is said to be *self-orthogonal* if some of its parastrophes are orthogonal. A quasigroup identity is called *self-orthogonal* if its every model is self-orthogonal. V.D. Belousov [1] proved that there exist seven minimal quasigroup identities up to parastrophic equivalency [2] and all of them are self-orthogonal:

Schröder laws: $xy\cdot y=x\cdot xy,\qquad xy\cdot yx=y;$

Stein laws: $x\cdot xy=yx,\qquad xy\cdot x=y\cdot xy,\qquad yx\cdot xy=y;$

Belousov laws: $x(x\cdot xy)=y,\qquad y(x\cdot xy)=x.$

Theorem. *The variety defined by a Schröder law is parastrophically closed, i.e., if a quasigroup satisfies one of them, then all of its parastrophes satisfy the same identity.*

The class of all τ-parastrophes of quasigroups from the class $\mathfrak{A}$ is called *τ-parastrophe* of $\mathfrak{A}$ and is denoted by ${}^{\tau}\mathfrak{A}$.

So, the above theorem means that all parastrophes of a Schröder's variety coincide. It is not true for the Stein's and Belousov's varieties. The relationships are given in the following table.

	the first Stein law	*the second Stein law*	*the third Stein law*	*the first Belousov law*	*the second Belousov law*
$\mathfrak{A}$	$x\cdot xy=yx$	$xy\cdot x=y\cdot xy$	$yx\cdot xy=y$	$x(x\cdot xy)=y$	$y(x\cdot xy)=x$
${}^{\ell}\mathfrak{A}$	$(y\overset{\ell}{\cdot}xy)\cdot z=xy$	$y\cdot(x\cdot yx)=x$	$(xy\cdot x)\cdot xy=y$	$y\cdot(xy\overset{\ell}{\cdot}x)=xy$	$xy\cdot(xy\overset{\ell}{\cdot}x)=y$
${}^{r}\mathfrak{A}$	$xy\cdot(xy\cdot y)=x$	$(xy\cdot x)\cdot y=x$	$xy\cdot(y\cdot xy)=x$	$x(x\cdot xy)=y$	$x\cdot(x\cdot yx)=y$
${}^{s}\mathfrak{A}$	$yx\cdot x=xy$	$xy\cdot x=y\cdot xy$	$yx\cdot xy=y$	$(yx\cdot x)\cdot x=y$	$(yx\cdot x)\cdot y=x$
${}^{s\ell}\mathfrak{A}$	$(x\cdot xy)\cdot xy=y$	$y\cdot(x\cdot yx)=x$	$(xy\cdot x)\cdot xy=y$	$(yx\cdot x)\cdot x=y$	$(xy\cdot x)\cdot x=y$
${}^{sr}\mathfrak{A}$	$(x\cdot yx)\cdot yx=y$	$(xy\cdot x)\cdot y=x$	$xy\cdot(y\cdot xy)=x$	$y\cdot(xy\overset{\ell}{\cdot}x)=xy$	$(x\overset{r}{\cdot}yx)\cdot yx=y$

1. Belousov V. D. Parastrophic-orthogonal quasigroups. Quasigroups and Related Systems 13 (2005), P.25-72
2. Sokhatsky F. M., On classification of functional equations on quasigroups, Ukrainian mathem. journ., (2004), **56**, No 4, 1259-1266p.

Exceptional collections on the derived categories of singularities

Oleksandr Kravets

Kyiv National Taras Shevchenko University
Moscow Higher School of Economics
xander.krav@gmail.com

Derived categories of singularities measure how far is an algebraic variety from being smooth. They appear on the algebraic side of the Homological Mirror Symmetry for the Landau-Ginzburg models. The talk will be dedicated to the case when the superpotential is given by the so called invertible polynomial. In this case I will try to give the description of mentioned categories in terms of exceptional collections of good kind.

Definition 1. [1] We define a triangulated category $D_{sg}(X)$ of the scheme X as the quotient of the triangulated category $D^b(\mathrm{coh}(X))$ by the full triangulated subcategory $\mathrm{Perf}(X)$ and call it a triangulated category of singularities of X.

Definition 2. A collection of objects $(E_i)_{1\leqslant i\leqslant n}$ in a triangulated category T is called full exceptional collection if $\mathrm{Ext}^\bullet(E_i,E_i)=k$ (the base field) for each i, $\mathrm{Ext}^\bullet(E_i,E_j)=0$ for each $i>j$ and all given objects generate the whole category by taking cones.

Hypothesis 1. For the polynomial $f\in k[x_1,\dots,x_n]$ of special kind (*quasi-homogenous with the isolated singularity in zero) the triangulated category of the corresponding hypersurface singularity* $D_{sg}(Spec(k[x_1,\dots,x_n]/(f)))$ *admits a strong full exceptional collection which can be divided into* $n+1$ *blocks s.t. in each of them objects don't have Ext-s between each other.*

I will show how these exceptional collections look like explicitly in the cases of low dimensions.

1. Dmitri Orlov, *Triangulated categories of singularities and D-branes in Landau-Ginzburg models*, arXiv:math/0302304 [math.AG].

On asymptotic Assouad-Nagata dimension and G-symmetric powers

J. Kucab, M. M. Zarichnyi

University of Rzeszów, Ivan Franko National University of Lviv
jacek.kucab@wp.pl, mzar@litech.lviv.ua

The asymptotic Assouad-Nagata dimension of a metric space X does not exceed n, $\mathrm{AN} - \operatorname{asdim} X \leqslant n$, if there exist $c > 0$ and $r_0 > 0$ such that for every $r \geqslant r_0$, there is a cover $\mathcal{U}$ of X such that $\operatorname{mesh}\mathcal{U} \leqslant cr$, $L(\mathcal{U}) > r$, and $\mathcal{U}$ has multiplicity $\leqslant n + 1$ (see [1]; here $L(\mathcal{U})$ is the Lebesgue number of $\mathcal{U}$).

The aim of this talk is to show the estimates given in [2] for the asymptotic dimension of the G-symmetric powers (and the hypersymmetric powers; even more general functorial constructions are considered) are valid also in the case of the asymptotic Assouad-Nagata dimension and the so-called semigroup-controlled asymptotic dimension [3].

1. A.N. Dranishnikov, J. Smith, On asymptotic Assouad-Nagata dimension, Topology and its Applications 154 (2007) 934–952.
2. T.Radul, O.Shukel'. Functors of finite degree and asymptotic dimension. Mathem. Studii. - 2009. - V. 31, no 2. -P. 204–206.
3. J. Higes, Semigroup-controlled asymptotic dimension, arXiv:math/0608736v1.

On Artinian hereditary rings

I.Kulakovskaya

Petro Mohyla Black Sea State University
kulaknic@ukr.net

Let A be an Artinian ring. Then right regular A-module A_A has the following decomposition into a direct sum of indecomposable pairwise non-isomorphic projective ideals:

$$A_A = P_1^{n_1} \oplus \ldots \oplus P_s^{n_s}$$

If X is A-module the denote with X^n the direct sum of n copies of X and $X^0 = 0$. Each module P_i has exactly one maximal submodule $P_i R$.

A module us called uniserial if the lattice of its submodules is a chain, i.e. the set of all its submodules is linearly ordered by inclusion.

The direct sum of uniserial modules is called serial module.

A ring is called right (left) serial if it is a right (left) serial module over itself. A ring which is both a right and left serial is called serial ring.

We consider quivers of Artinian rings. Remind that a quiver is called acyclic if it has no oriented cycles.

Theorem 1. *A quiver of hereditary Artinian ring is acyclic.*

Theorem 2. *Artinian hereditary serial ring is Morita equivalent to the direct product of the rings $T_{n_i}(\alpha_i)$ of the upper triangular matrices over the division ring D_i.*

On the structure of groups whose non-abelian subgroups are subnormal

L. A. Kurdachenko, M. M. Semko, Sevgi Atlihan

National University of The State Tax Service of Ukraine
n_semko@mail.ru

A group G is said to be metahamiltonian, if every non-abelian subgroup of G is normal. The properties of matahamiltonian groups have been studied by many authors. Full description of these groups was obtained in a series of papers of M.F. Kuzennyj and M.M. Semko [1]. We start to study here groups, whose non-abelian subgroups are subnormal. The next theorems give a description of finite groups with these property.

Theorem 1. *Let G be a non-nilpotent finite group whose non-abelian subgroups are subnormal. Suppose that p is a prime such that Sylow p-subgroup P of G is non-abelian. Then the following assertions hold:*

(i) $G = P\lambda Q$ where Q is an abelian Hall p' -subgroup of G;

(ii) $C = C_P(Q)$ is a G-invariant abelian subgroup of P such that P/C is a G-chief factor;

(iii) $G/C_G(P/C)$ is a cyclic p'-group;

(iv) P/C is a $< g >$-chief factor for every element $g \notin CG(P/C)$;

(v) $P = CD$, where $D = [P, Q]$ is a special p-subgroup.

*Recall that a finite p-group is called **special**, if $[\mathbf{P}, \mathbf{P}] = \xi(\mathbf{P}) = \mathbf{Fratt}(\mathbf{P})$ is an elementary abelian p - subgroup.*

Theorem 2. *Let G be a non-nilpotent finite group whose non-abelian subgroups are subnormal. Suppose that Sylow s - subgroups of G is abelian for all primes s. Then there exists a prime p such that G has a normal Sylow p-subgroup P and the following assertions hold:*

(i) $G = P\lambda Q$ where Q is an abelian Hall p' - subgroup of G;

(ii) $P = C \times D$, where $C = C_P(Q)$ and $D = [P, Q]$ is a minimal G-invariant subgroup of P;

(iii) $G/C_G(D)$ is a cyclic p' -group;

(iv) D is a minimal $< g >$ - invariant subgroup of P for every element $g \notin C_G(D)$.

1. Kuzennyj M.F., Semko M.M. Meta-Hamiltonian groups and their generalizations.- Kyiv:Institute of NAS of Ukraine,1996.- 232 pp.

On some properties of the upper and lower central series

Leonid A. Kurdachenko, Igor Ya. Subbotin

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
Los Angeles National University, Los Angeles, USA
`lkurdachenko@i.ua, isubboti@nu.edu`

It is well known that if G is a nilpotent group, then the length of its upper central series coincides with the length of the lower central series. In other words, if $G = \zeta_k(G)$, then $\gamma_{k+1}(G) =< 1 >$. R. Baer [1] has obtained the following generalization of this result:

Suppose that the factor-group $G/\zeta_k(G)$ is finite. Then $\gamma_{k+1}(G)$ is likewise finite.

On the other hand, Baer's theorem is a generalization of another classical result obtained by I. Schur. I. Schur considered relations between the central factor-group $G/\zeta(G)$ of a group G and its derived subgroup $[G, G]$ [2]. From his results it follows that if $G/\zeta(G)$ is finite, then $[G, G]$ is also finite. This fact brought to life the following question: How the order t of the factor-group $G/\zeta(G)$ is associated with the order of $[G, G]$? Related to this problem results were covered in several articles. The last result here has been obtained by J. Wiegold. In the paper [3] he proved that if $t = |G/\zeta(G)|$, then $|[G, G]| \leq \mathfrak{w}(t) = t^m$ where $m = (1/2)(log_p t - 1)$ and p is the least prime dividing t.

We were able to obtaine the following version of Baer's theorem.

Theorem 1. *Let G be a group and suppose that $|G/\zeta_k(G)| = t$. Then there a function β_1 such that $|\gamma_{k+1}(G)| \leq \beta_1(t, k)$.*

In this theorem we defined the function $\beta_1(t, k)$ recursively:

$\beta_1(t, 1) = \mathfrak{w}(t), \beta_1(t, 2) = \mathfrak{w}(\mathfrak{w}(t)) + t\mathfrak{w}(t), ..., \beta_1(t, k) = \mathfrak{w}(\beta_1(t, k-1)) + t\beta_1(t, k-1)$.

In the above situation, $\gamma_{k+1}(G)$ includes the nilpotent residual L of G. Therefore, the question about the order of L naturally arises here.

Theorem 2. *Let G be a group and L be a nilpotent residual of G. Suppose that $|G/\zeta_k(G)| = t$. Then the following conditions hold:*

(i) L is finite and there exists a function β_2 such that $|L| \leq \beta_2(t)$;

(ii) G/L is nilpotent and there exists a function β_3 such that $\mathbf{ncl}(G/L) \leq \beta_3(k, t)$.

It is interesting to observe that the order of nilpotent residual depends only of the order of the factor-group by the upper hypercenter.

1. Baer R. Endlichkeitskriterien fur Kommutatorgruppen. Math. Annalen 124(1952), 161- 177.
2. Schur I. Uber die Darstellungen der endlichen Gruppen durch gebrochene lineare substitutionen. J. reine angew. Math. - 127(1904), 20 - 50.
3. Wiegold J. Groups with boundedly finite classes of conjugate elements. Proc. Roy. Soc. - A 238(1956), 389 - 401.

On the structure of some modules over generalized soluble groups

L.A. Kurdachenko, I.Ya. Subbotin, V.A. Chupordya

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
Los Angeles National University, Los Angeles, USA
Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
`lkurdachenko@i.ua, isubboti@nu.edu, vchupordya@mail.ru`

The modules over group rings RG are classic objects of research with well established links to various areas of algebra. The case when G is a finite group has been studying in sufficient details for a long time. For the case when G is an infinite group, the situation is different. A new approach based on studying of properties of modules of cocentralizers of some important families of subgroups has formed recently. Let R be a ring, G a group and A an RG-module. For a subgroup H of G the R-factor-module $A/C_A(H)$ is called the *cocentralizer of H in A*. The largest family of subgroups is a family of all proper subgroups. If R is an integral domain, then $\mathbf{Tor}_R(A) = \{a \in A \mid \mathbf{Ann}_R(a) \neq \langle 0 \rangle\}$ is a submodule of A. We say that an R-module A is an *artinian-by-(finite rank)*, if $\mathbf{Tor}_R(A)$ is artinian and $A/\mathbf{Tor}_R(A)$ has finite R-rank.

A group G is called *generalized radical*, if G has an ascending series whose factors are locally nilpotent or locally finite.

Theorem 1. *Let G be a locally generalized radical group and A a $\mathbb{Z}G$-module. If the factor-module $A/C_A(H)$ is artinian-by-(finite rank) (as a $\mathbb{Z}$-module) for every proper subgroup H of G, then either $A/C_A(G)$ is artinian-by-(finite rank) or $G/C_G(A)$ is a cyclic or quasicyclic p-group for some prime p.*

Corollary 1. *Let G be a locally generalized radical group and A a $\mathbb{Z}G$-module. If the factor-module $A/C_A(H)$ is minimax for every proper subgroup H of G, then either $A/C_A(G)$ is minimax or $G/C_G(A)$ is a cyclic or quasicyclic p-group for some prime p.*

Corollary 2. *Let G be a locally generalized radical group and A a $\mathbb{Z}G$-module. If a factor-module $A/C_A(H)$ is finitely generated for every proper subgroup H of G, then either $A/C_A(G)$ is finitely generated or $G/C_G(A)$ is a cyclic or quasicyclic p-group for some prime p.*

Corollary 3. *Let G be a locally generalized radical group and A a $\mathbb{Z}G$-module. If the factor - module $A/C_A(H)$ is artinian for every proper subgroup H of G, then either $A/C_A(G)$ is artinian or $G/C_G(A)$ is a cyclic or quasicyclic p-group for some prime p.*

We remark that it is possible to show that for every quasicyclic group one can find a $\mathbb{Z}G$-module A such that $C_G(A) = \langle 1 \rangle$ but the factor-module $A/C_A(H)$ is artinian-by-(finite rank) for every proper subgroup H of G.

On modules with few minimax cocentralizers

L.A. Kurdachenko, I.Ya. Subbotin, V.A. Chupordya

Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
Los Angeles National University, Los Angeles, USA
Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
lkurdachenko@i.ua, isubboti@nu.edu, vchupordya@mail.ru

Let R be a ring, G a group and A an RG-module. For a subgroup H of G the R-factor-module $A/C_A(H)$ is called the *cocentralizer of H in A*. An R-module A is said to be *minimax* if A has a finite series of submodules, whose factors are either noetherian or artinian. We say that A is *minimax-antifinitary RG-module* if the factor-module $A/C_A(H)$ is minimax as an R-module for each not finitely generated proper subgroup H and the R-module $A/C_A(G)$ is not minimax.

It was began to consider the first natural case when $R = \mathbb{Z}$ is the ring of all integers. The main result is devoted to a minimax-antifinitary $\mathbb{Z}G$-modules where G belongs to the following very large class of groups.

A group G is called *generalized radical* if G has an ascending series whose factors are locally nilpotent or locally finite. Hence a generalized radical group G either has an ascendant locally nilpotent subgroup or an ascendant locally finite subgroup.

Theorem 1. *Let G be a locally generalized radical group, A a minimax-antifinitary $\mathbb{Z}G$-module, and $D = \mathbf{Coc}_{\mathbb{Z}-mmx}(G) = \{x \in G \mid A/C_A(x) \text{ is minimax }\}$. Suppose that G is not finitely generated, $G \neq D$ and $C_G(A) = \langle 1 \rangle$. Then G is a group of one of the following types:*

1. *G is a quasicyclic q-group for some prime q.*
2. *$G = Q \times \langle g \rangle$ where Q is a quasicyclic q-subgroup, g is a p-element and $g^p \in D$, p, q are prime (not necessary different).*
3. *G includes a normal divisible Chernikov q-subgroup Q, such that $G = Q\langle g \rangle$ where g is a p-element, p, q are prime (not necessary different). Moreover, G satisfies the following conditions:*
   - *(a) $g^p \in D$;*
   - *(b) Q is G-quasifinite;*
   - *(c) If $q = p$, then Q has special rank $p^{m-1}(p-1)$ where $p^m = |\langle g \rangle / C_{\langle g \rangle}(Q)|$;*
   - *(d) if $q \neq p$, then Q has special rank $\mathbf{o}(q, p^m)$ where again $p^m = |\langle g \rangle / C_{\langle g \rangle}(Q)|$ and $\mathbf{o}(q, p^m)$ is the order of q modulo p^m.*

Furthermore, for the types 2, 3 $A(\omega\mathbb{Z}D)$ is a Chernikov subgroup and $\Pi(A(\omega\mathbb{Z}D)) \subseteq \Pi(D)$.

Here $\omega\mathbb{Z}G$ be the *augmentation ideal* of the group ring $\mathbb{Z}G$, the two-sided ideal of $\mathbb{Z}G$ generated by all elements $g - 1$, $g \in G$.

On normalizers in fuzzy groups

L.A. Kurdachenko, K.O. Grin, N.A. Turbay

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
lkurdachenko@i.ua, k.grin@mail.ru, nadezhda.turbay@gmail.com

Let G be a group with a multiplicative binary operation denoted by juxtaposition and identity e. Fuzzy subset $\gamma : G \to [0,1]$ is said to be a *fuzzy group on G* (see, for example, [1, S 1.2]), if it satisfies the following conditions:

(FSG 1) $\gamma(xy) \geq \gamma(x) \wedge \gamma(y)$ for all $x, y \in G$,

(FSG 2) $\gamma(x^{-1}) \geq \gamma(x)$ for every $x \in G$.

If X is a set, for every subset Y of X and every $a \in [0,1]$ we define a fuzzy subset $\chi(Y,a)$ as follows:

$$\chi(Y,a) = \begin{cases} a, x \in Y, \\ 0, x \notin Y. \end{cases}$$

Clearly $\chi(H,a)$ is a fuzzy group on G for every subgroup H of G. If $Y = \{y\}$, then instead of $\chi(\{y\},a)$ we will write shorter $\chi(y,a)$. A fuzzy subset $\chi(y,a)$ is called a *fuzzy point* (or *fuzzy singleton*).

Recall that if γ, κ are the fuzzy groups on G and $\kappa \preccurlyeq \gamma$, then it is said that κ is a *normal fuzzy subgroup of* γ, if $\kappa(yxy^{-1}) \geq \kappa(x) \wedge \gamma(y)$ for every elements $x, y \in G$ [1, 1.4]. We denote this fact by $\kappa \trianglelefteq \gamma$.

Let γ, κ be the fuzzy groups on G and $\kappa \preccurlyeq \gamma$. We define *a normalizer $N_\gamma(\kappa)$ of κ in γ* as an union of all fuzzy points $\chi(x,a) \subseteq \gamma$, satisfying the following condition $\chi(x^{-1},a) \circledcirc \kappa \circledcirc \chi(x,a) = \kappa$.

Theorem 1. *Let G be a group, γ, κ be the fuzzy groups on G and $\kappa \preccurlyeq \gamma$. Then a normalizer $N_\gamma(\kappa)$ is a fuzzy subgroup of γ.*

Let γ, κ be the fuzzy groups on G and $\kappa \preccurlyeq \gamma$. We say that γ *satisfies a normalizer condition* if $N_\gamma(\kappa) \neq \kappa$ for every subgroup κ of γ.

Theorem 2. *Let G be a group, γ be a fuzzy group on G. If γ satisfies a normalizer condition, then* ***Supp***(γ) *satisfies normalizer condition.*

Let G be a group, γ be a fuzzy group on G. A fuzzy subgroup κ of γ is called an *ascendant subgroup* of γ, if there exists an ascending series $\kappa = \kappa_0 \trianglelefteq \kappa_1 \trianglelefteq \ldots \trianglelefteq \kappa_\alpha \trianglelefteq \kappa_{\alpha+1} \trianglelefteq \ldots \trianglelefteq \kappa_\beta = \gamma$,where κ_α is a normal fuzzy subgroup of $\kappa_{\alpha+1}$ for all $\alpha < \beta$.

Theorem 3. *Let G be a group, γ be a fuzzy group on G. Then γ satisfies a normalizer condition if and only if every fuzzy subgroup of γ is ascendant.*

1. J.N. Mordeson, K.R. Bhutani, A. Rosenfeld, *Fuzzy Group Theory*, Springer: Berlin, 2005.

Transversals of general form in groups

E. Kuznetsov

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova
kuznet1964@mail.ru

The classical notion of a transversal in a group by its subgroup ([1]) is generalized by removing of any conditions on a choice of the representatives in left (right) cosets.

Definition 1. Let G be a group and H be its subgroup. Let $\{H_i\}_{i\in E}$ be the set of all left (right) cosets in G to H. A set $T = \{t_i\}_{i\in E}$ of representativities of the left (right) cosets (by one from each coset H_i) is called a **left (right) transversal of general form** in G to H.

The variuos properties of these transversals of general form as well as properties of their transversal operations are studied.

Theorem 1. *For an arbitrary left transversal of general form T in G to H the following statements are fulfilled:*

1. For any $h \in H$ the set

$$T_h = Th = \{t_i h\}_{i\in E}$$

be a left transversal of general form in G to H too;

2. For any $\pi \in G$ the set

$$_{\pi}T = \pi T = \{\pi t_i\}_{i\in E}$$

be a left transversal of general form in G to H too.

Theorem 2. *For an arbitrary left transversal of general form T in G to H the following propositions are equivalent:*

1. A set T is a left loop transversal of general form in G to H;

2. A set T is a left reduced transversal of general form in G to H and for any $\pi \in G$ set $T\pi = \{t_i\pi\}_{i\in E}$ is a left transversal of general form in G to H;

3. For any $\pi \in G$ set $T^{\pi} = \pi T \pi^{-1} = \left\{\pi t_i \pi^{-1}\right\}_{i\in E}$ is a left reduced transversal of general form in G to H;

4. For any $\pi \in G$ set T is a left reduced transversal of general form in G to $H^{\pi} = \pi H \pi^{-1}$.

1. Kuznetsov E. *Transversals in groups. 1. Elementary properties.* Quasigroups and related systems, 1994, **1**, No. 1, 22-42.

Explicit formulas for determinantal representations of the Drazin inverse solution of the quaternion matrix equation $\mathbf{AXB} = \mathbf{D}$

I. Kyrchei

Pidstrygach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine
kyrchei@lms.lviv.ua

By $\mathbb{H}^{m\times n}$ denote the set of all $m\times n$ matrices over the quaternion skew field $\mathbb{H}$. Consider a matrix equation

$$\mathbf{AXB} = \mathbf{D}, \tag{1}$$

where $\{\mathbf{A},\mathbf{B},\mathbf{D}\} \subset \mathbb{H}^{n\times n}$ are given, and $\mathbf{A},\mathbf{B}$ are Hermitian, $\mathbf{X}\in\mathbb{H}^{n\times n}$ is unknown. By using determinantal representations of the Drazin inverse over the quaternion skew field obtained by the author, we get explicit formulas for determinantal representations of the Drazin inverse solution of (1) in the following theorem within the framework of theory of the column and row determinants (see, e.g. [1]).

Theorem 1. *If $Ind\mathbf{A} = k_1$ and $Ind\mathbf{B} = k_2$, and $\operatorname{rank}\mathbf{A}^{k_1+1} = \operatorname{rank}\mathbf{A}^{k_1} = r_1 \leqslant n$ for $\mathbf{A}\in\mathbb{H}^{n\times n}$, $\operatorname{rank}\mathbf{B}^{k_2+1} = \operatorname{rank}\mathbf{B}^{k_2} = r_2 \leqslant n$ for $\mathbf{B}\in\mathbb{H}^{n\times n}$, then for the Drazin inverse solution $\mathbf{X} = \mathbf{A}^D\mathbf{D}\mathbf{B}^D =: (x_{ij}) \in \mathbb{H}^{n\times n}$ of (1) we have*

$$x_{ij} = \frac{\sum\limits_{\beta\in J_{r_1,\,n}\{i\}} \operatorname{cdet}_i\left(\left(\mathbf{A}^{k_1+1}\right)_{.\,i}\left(\mathbf{d}^{\mathbf{B}}_{.j}\right)\right)^{\beta}_{\beta}}{\sum\limits_{\beta\in J_{r_1,n}}\left|\left(\mathbf{A}^{k_1+1}\right)^{\beta}_{\beta}\right|\sum\limits_{\alpha\in I_{r_2,n}}\left|\left(\mathbf{B}^{k_2+1}\right)^{\alpha}_{\alpha}\right|} = \frac{\sum\limits_{\alpha\in I_{r_2,n}\{j\}} \operatorname{rdet}_j\left(\left(\mathbf{B}^{k_2+1}\right)_{j\,.}\left(\mathbf{d}^{\mathbf{A}}_{i\,.}\right)\right)^{\alpha}_{\alpha}}{\sum\limits_{\beta\in J_{r_1,n}}\left|\left(\mathbf{A}^{k_1+1}\right)^{\beta}_{\beta}\right|\sum\limits_{\alpha\in I_{r_2,n}}\left|\left(\mathbf{B}^{k_2+1}\right)^{\alpha}_{\alpha}\right|},$$

where

$$\mathbf{d}^{\mathbf{B}}_{.j} = \left(\sum_{\alpha\in I_{r_2,n}\{j\}} \operatorname{rdet}_j\left(\left(\mathbf{B}^{k_1+1}\right)_{j.}\left(\tilde{\mathbf{d}}_{k.}\right)\right)^{\alpha}_{\alpha}\right)\in\mathbb{H}^{n\times 1},$$
$$\mathbf{d}^{\mathbf{A}}_{i.} = \left(\sum_{\beta\in J_{r_1,n}\{i\}} \operatorname{cdet}_i\left(\left(\mathbf{A}^{k_1+1}\right)_{.i}\left(\tilde{\mathbf{d}}_{.l}\right)\right)^{\beta}_{\beta}\right)\in\mathbb{H}^{1\times n}$$

are respectively the column vector and the row vector for $k,l = 1,...,n$. $\tilde{\mathbf{d}}_{i.}$ and $\tilde{\mathbf{d}}_{.j}$ are the ith row and the jth column of $\widetilde{\mathbf{D}} := \mathbf{A}^{k_1}\mathbf{D}\mathbf{B}^{k_2}$ for all $i,j = 1,...,n$.

We use the notations from [2].

1. Ivan I. Kyrchei. The theory of the column and row determinants in a quaternion linear algebra. Advances in Mathematics Research 15 (2012), pp. 301-359. Nova Science Publishers. ISBN 978-1-62417-511-4
2. Ivan Kyrchei, Explicit formulas for determinantal representations of the Drazin inverse solutions of some matrix and differential matrix equations, Applied Mathematics and Computation, 219 (2013), pp. 7632-7644.

Equivalence of pairs of matrices with relatively prime determinants over quadratic principal ideal rings

N. Ladzoryshyn, V. Petrychkovych

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the NAS of Ukraine

`natalja.ladzoryshyn@gmail.com, vas_petrych@yahoo.com`

In many problems there is need for studied for certain types of equivalences of pairs of matrices over different domains and constructing their canonical forms [1] - [4]. We study the equivalence of pairs of matrices over quadratic rings $\mathbb{K} = \mathbb{Z}[\sqrt{k}]$, where $k \in \mathbb{Z}$ and k is a square-free integer different from 1. It is known that for some values of k such rings $\mathbb{K}$ are Euclidean and there exist some other values of k, that they are non-Euclidean principal ideal rings. There are examples of quadratic rings are not principal ideal rings.

Theorem 1. *Let* $\mathbb{K} = \mathbb{Z}[\sqrt{k}]$ *be quadratic principal ideal ring and* A, B *be* $n \times n$*-matrices over* $\mathbb{K}$ *and* $(\,detA,\ detB) = 1$*. Then there exist invertible matrices* U *over* $\mathbb{Z}$ *and* V_A*,* V_B *over* $\mathbb{K} = \mathbb{Z}[\sqrt{k}]$ *such that*

$$UAV_A = \left\| \begin{array}{cccc} \mu_1^A & 0 & \dots & 0 \\ a_{21}\mu_1^A & \mu_2^A & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1}\mu_1^A & a_{n2}\mu_2^A & \dots & \mu_n^A \end{array} \right\|, \ UBV_B = \left\| \begin{array}{cccc} \mu_1^B & 0 & \dots & 0 \\ b_{21}\mu_1^B & \mu_2^B & \dots & 0 \\ \dots & \dots & \dots & \dots \\ b_{n1}\mu_1^B & b_{n2}\mu_2^B & \dots & \mu_n^B \end{array} \right\|,$$

where $\mu_i^A \mid \mu_{i+1}^A$, $\mu_i^B \mid \mu_{i+1}^B$, $i = 1, \dots, n-1$, *i.e.* μ_i^A, μ_i^B *are invariant factors of matrices* A *and* B*, respectively.*

1. Dlab V., Ringel C.M. *Canonical forms of pairs of complex matrices* // Linear Algebra Appl.– 1991.– **147**.– P. 387–410.
2. Gaiduk T.N., Sergeichuk V.V. *Generic canonical form of pairs of matrices with zeros* // Linear Algebra Appl.– 2004.– **380**.– P. 241–251.
3. Petrichkovich V.M. *Semiscalar equivalence and Smith normal form of polynomial matrices* // J. Sov. Math.– 1993.– **66**, No. 1.– P. 2030–2033.
4. Petrychkovych V. *Generalized equivalence of pairs of matrices* // Linear Multilinear Algebra.– 2000. – **48**, No 2.–P. 179–188.

Joint universality of the Riemann zeta-function and Hurwitz zeta-functions

A. Laurinčikas

Vilnius University
antanas.laurincikas@mif.vu.lt

Let $\zeta(s)$ and $\zeta(s,\alpha)$, $s=\sigma+it$, $0<\alpha\leq 1$, as usual, denote the Riemann and Hurwitz zeta-functions, respectively. It is well known that the functions $\zeta(s)$ and $\zeta(s,\alpha)$ with rational and transcendental parameter α are universal in the sense that their shifts $\zeta(s+i\tau)$ and $\zeta(s+i\tau,\alpha)$, $\tau\in\mathbb{R}$, approximate with a given accuracy any analytic function. Also, the functions $\zeta(s)$ and $\zeta(s,\alpha)$ with transcendental α are jointly universal, i. e., any pair of analytic functions simultaneously are approximated by $\zeta(s+i\tau)$ and $\zeta(s+i\tau,\alpha)$.

Let $D=\left\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\right\}$, and $H(D)$ denote the space of analytic functions on D equipped with the topology of uniform convergence on compacta. Our report is devoted to the universality of functions $F(\zeta(s),\zeta(s,\alpha))$ with transcendental α for some classes of operators $F:\ H^2(D)\to H(D)$. For example, from general theorems the universality of the functions

$$c_1\zeta(s)+c_2\zeta(s,\alpha),\quad c_1\zeta'(s)+c_2\zeta'(s,\alpha),\quad c_1,c_2\in\mathbb{C}\setminus\{0\},$$

$$e^{\zeta(s)+\zeta(s,\alpha)},\quad \sin(\zeta(s)+\zeta(s,\alpha))$$

follows.

The statements of results with their proofs can be found in [1].

1. A. Laurinčikas. On joint universality of the Riemann zeta-function and Hurwitz zeta-functions. *J. Number Theory* **132** (2012), 2842–2853.

Functions involving exponential divisors

A. V. Lelechenko

Odessa National University
1@dxdy.ru

Consider a set of arithmetic functions $\mathcal{A}$, a set of multiplicative prime-independent functions $\mathcal{M}_{PI}$ and operator $E\colon \mathcal{A} \to \mathcal{M}_{PI}$, which is defined as follows

$$(Ef)(p^a) = f(a).$$

The behaviour of Ef for various special cases of f has been widely studied by different authors, starting with the pioneering paper of Subbarao [1] on $E\tau$ and $E\mu$.

Our aim is to investigate asymptotic properties of multiple applications of E.

Theorem 1. *Let $\gamma(0) = 2$ and $\gamma(m) = 2^{\gamma(m-1)}$; let τ be the divisor function:*

$$\tau(n) = \sum_{d|n} 1.$$

Then for a fixed integer $m > 1$ we have

$$\sum_{n \leqslant x} E^m \tau(n) = A_m x + B_m x^{1/\gamma(m)} + R_m(x),$$

where A_m and B_m are computable constants and

$$R_m(x) \ll x^{1/(\gamma(m)+1)}, \qquad R_m(x) \gg x^{1/2(\gamma(m)+1)},$$

Under Riemann hypothesis

$$R_m(x) \ll x^{\beta_m + \varepsilon},$$

where

$$\beta_m = \frac{\gamma(m) + \gamma(m-1) - 1}{\gamma(m)^2 + 2\gamma(m) + \gamma(m)\gamma(m-1) + 2\gamma(m-1) - 2\gamma(m) - 2}$$

We also prove analogous theorems on asymptotic properties of $E^m \tau_k$, where τ_k is k-dimensional divisor function, and for several other families of E-functions. For example, Tóth's estimates [2] are significantly improved.

1. Subbarao M. V. On some arithmetic convolutions // The theory of arithmetical functions, Lecture Notes in Mathematics. Springer Verlag, 1972. Vol. 251. P. 247–271.
2. Tóth L. An order result for the exponential divisor function // Publ. Math. Debrecen. 2007. Vol. 71, no. 1-2. P. 165–171.

On the diameters of the commuting graphs of permutational wreath products

Yu. Leshchenko

Cherkasy National University, Cherkasy, Ukraine
ylesch@ua.fm

Let G be a non-abelian group and $Z(G)$ is the center of G. Consider a simple undirected graph $\Gamma(G) = (V, E)$, here $V = G \setminus Z(G)$ is the set of vertices and E is the set of edges. Two distinct vertices x, y are connected with an edge (i.e. $\{x, y\} \in E$) if and only if $xy = yx$. Then $\Gamma(G)$ is called the *commuting graph* of the group G. Also, given a graph Γ let $\mathrm{d}(\Gamma)$ denotes the diameter of Γ.

In [1] A. Iranmanesh and A. Jafarzadeh conjectured that there is a natural number d such that $\mathrm{d}(\Gamma(G)) \leq d$ for any non-abelian finite group G with $\Gamma(G)$ connected. In 2012 the conjecture was disproved by M. Giudici and C. Parker [2] who presented a counterexample: an infinite family of finite 2-groups whose commuting graphs are connected and have arbitrarily large diameters.

We study the commuting graphs of permutational wreath products $G \wr H$, where G is a transitive permutation group acting on X (the "active" group of the wreath product) and (H, Y) is an abelian permutation group acting on Y. We obtain the following

Theorem 1. *Suppose $(G, X), (H, Y)$ are permutation groups acting on X and Y respectively, (G, X) is transitive on X, (H, Y) is abelian and Γ denotes the commuting graph of the wreath product $G \wr H$. Then*

1. *If $|X|$ is a prime number then Γ is disconnected.*
2. *If $|X|$ is not a prime number then Γ is connected and $\mathrm{d}(\Gamma) \leq 5$. Furthermore, if G is imprimitive then $\mathrm{d}(\Gamma) \leq 4$; and if G does not contain cycles of maximal length then $\mathrm{d}(\Gamma) \leq 3$.*

Theorem 2. *Let $(G, X), (H, Y)$ are permutation groups with cycles of maximal length, $|X|$ is not a prime number, (G, X) is imprimitive and (H, Y) is abelian. Then $\Gamma(G \wr H)$ is connected with diameter 4.*

These results allows to obtain the diameters of the commuting graphs for some classical finite p-groups (for example, Sylow p-subgroups of symmetric and general linear group over finite fields).

1. Iranmanesh A., Jafarzadeh A. On the commuting graph associated with the symmetric and alternating groups, J. Algebra Appl. **7** (2008), 129-146.
2. Giudici M., Parker C. There is no upper bound for the diameter of the commuting graph of a finite group, arXiv:1210.0348v1 (2012).

Estimates for systems of differential operators in Sobolev spaces

D. Limanskii

Donetsk National University, Donetsk, Ukraine
4125aa@gmail.com

We consider the problem of description the linear space $L(P)$ of minimal differential operators $Q(D)$ with constant coefficients subordinated in the $C(\mathbb{R}^n)$ norm to the tensor product

$$P(D) := P_1(D_1, \dots, D_{p_1}, 0, \dots, 0) \cdot P_2(0, \dots, 0, D_{p_1+1}, \dots, D_n).$$

Subordination means that the operators $Q(D)$ and $P(D)$ satisfy the following a priori estimate:

$$\|Q(D)f\|_{C(\mathbb{R}^n)} \leqslant C_1 \|P(D)f\|_{C(\mathbb{R}^n)} + C_2 \|f\|_{C(\mathbb{R}^n)}, \qquad \forall f \in C_0^\infty(\mathbb{R}^n).$$

We present the next main result.

Theorem 1. *[2] Suppose that $P(D) = P(D_1, D_2) := p_1(D_1)p_2(D_2)$, where $p_1(\xi_1)$ is a polynomial of degree $l \geqslant 1$ such that all its zeros are real and simple, and $p_2(\xi_2)$ is an arbitrary polynomial of degree $m \geqslant 1$. Then the inclusion $Q \in L(P)$ is equivalent to the equality*

$$Q(D) = P(\xi)\frac{q(\xi_2)}{p_{22}(\xi_2)} + r(\xi_1),$$

where $p_{22}(\xi_2)$ is the non-degenerate divisor of $p_2(\xi_2)$ of maximal degree; $q(\xi_2)$ is an arbitrary polynomial of degree $\leqslant s := \deg p_{22}$; $r(\xi_1)$ is an arbitrary polynomial of degree $\leqslant l-1$ if $s = m$, and an arbitrary constant $r(\xi_1) \equiv r$ if $s < m$. Moreover, $\dim L(P) = m + l + 1$ *in the first case and* $\dim L(P) = s + 2$ *in the second one.*

1. D. V. Limanskii, M. M. Malamud, Elliptic and Weakly Coercive Systems of Operators in Sobolev Spaces, *Mat. Sbornik*, **199**, No. 11, 75-112 (2008).
2. D. V. Limanskii, Subordination Conditions for a Tensor Product of Two Ordinary Differential Operators *Dopovidi NAN Ukrainy*, No. 4, 25-29 (2012).

Derivations with regular values in rings

M.P. Lukashenko

PreCarpathian National University of Vasyl Stefanyk
bilochka.90@mail.ru

Let R be an associative ring with an identity. J. Bergen, I.N. Herstein and C. Lanski [1] have investigated rings R in which there is a derivation d such that, for every $x \in R$, $d(x) = 0$ or $d(x)$ is invertible. We study rings with a wider condition. Namely we say that R *satisfies the condition* $(*)$ if R has a derivation d such that, for every $x \in R$, $d(x) = 0$ or $d(x)$ is regular. Recall that $x \in R$ is regular if it is neither left, nor right zero divisor. If R satisfies the condition $(*)$, then:

- (i) if $u \in R$ is a nilpotent element, then $u^2 = 0$,
- (ii) if $\mathrm{char}R = 0$ (respectively $\mathrm{char}R > 2$), then the nil-radical $N(R) = 0$ is trivial (i.e., the ring R is semiprime).

Every semiprime ring with the condition $(*)$ is prime. A derivation $d : R \to R$ is called *cofinite* if its image $\mathrm{Im}d$ is a subgroup of finite index in the additive group R^+ of R [2]. If a ring R has a cofinite derivation d with $(*)$, then R is a domain.

In a commutative ring R with $(*)$ every zero divisor is nilpotent.

We prove the following

Theorem 1. *Let R be a commutative ring with identity. Then R satisfies the condition $(*)$ if and only if one of the following holds:*

- (1) *R is a domain,*
- (2) *the quotient ring $Q(R) = D[X]/(X^2)$ is of characteristic 2, $d(D) = 0$ and $d(X) = 1 + aX$ for some $a \in D$.*

1. J. Bergen, I.N. Herstein and C. Lanski, Derivations with invertible values, Canad. J. Math., 35(1983), 300–310.
2. O.D. Artemovych, Cofinite derivations in rings, Annales Math. Inform., 40(2012), 3–11.

About generalized norms in periodic locally nilpotent groups

T.D. Lukashova, F.M. Lyman

Sumy Makarenko State Pedagogical University, Ukraine
mathematicsspu@mail.ru

In this paper authors investigate one of the possible generalizations of the classical notion of groups' norm – the norm of decomposable subgroups. Let's remind that a subgroup of G, which can be represented as a direct product of two nontrivial subgroups is called decomposable subgroup [1].

The norm N_G^d of decomposable subgroups of G is the intersection of the normalizers of all decomposable subgroups of the group (in condition that the system of such subgroups is non-empty). The intersection of the normalizers of all Abelian non-cyclic subgroups of group G is called the norm of Abelian non-cyclic subgroups of G and denoted by N_G^A [2]. The connections between those norms in periodic locally nilpotent groups are described by the following statements.

Theorem 1. *If G is a locally finite p-group, then its norm N_G^A of Abelian non-cyclic subgroups coincides with the norm N_G^d of decomposable subgroups.*

Theorem 2. *Any periodic locally nilpotent group G has non-Dedekind norm N_G^d of decomposable subgroups if and only if group G is a locally finite p-group with non-Dedekind norm N_G^A of Abelian non-cyclic subgroups.*

Corollary 1. *Arbitrary infinite periodic locally nilpotent group G with non-Dedekind norm N_G^d is a finite extension of a quasicyclic p-subgroup.*

Corollary 2. *In any periodic locally nilpotent group G all Abelian non-cyclic and decomposable subgroups are normal if norm N_G^d of G is infinite and non-Dedekind.*

1. Lyman F.N. (1970). Ukr. Math. Journal. 22(6), p. 725-733.
2. Lukashova T.D. (2004). Bulletin of Kyiv University. 3, p. 35-39.

About non-periodic groups with non-Dedekind norms of decomposable subgroups

F.M. Lyman, T.D. Lukashova

Sumy Makarenko State Pedagogical University, Ukraine
mathematicsspu@mail.ru

Let Σ be the system of all subgroups of a group with some theoretical-group property ($\Sigma \neq \emptyset$). Let's remind that the intersection of normalizers of all subgroups of group G, included in Σ, is called the Σ-norm of group G. In particular, if Σ consists of all subgroups of G, the corresponding Σ-norm is called the norm of the group [1].

In this paper the connections between the properties of the group G and its norm N_G^d of decomposable subgroups for class non-periodic groups are studied. Let's remember that a subgroup H of G is called decomposable if it can be represented as a direct product of two proper subgroups [2]. Respectively, the norm of decomposable subgroups of G is the intersection of normalizers of all decomposable subgroups of this group (provided that the system of such subgroups is non-empty).

Theorem 1. *Let G is a non-periodic group with non-Dedekind norm N_G^d of decomposable subgroups. Then the following statements hold:*

1) the group G doesn't contain the decomposable subgroups if and only if the norm N_G^d of this group doesn't contain such subgroups;

2) the group G contains a free Abelian subgroup of rank $r \geq 2$ if and only if the norm N_G^d contains the free Abelian subgroup of such rank;

3) the group G contains non-primary Abelian subgroups if and only if the subgroups with such property are contained in the norm N_G^d of group G;

4) every decomposable Abelian subgroup of group G is mixed if and only if every decomposable Abelian subgroup of its norm N_G^d is mixed.

Corollary 1. *Let G be the non-periodic group with non-Dedekind norm N_G^d of decomposable subgroups. If all the Abelian decomposable subgroups of norm N_G^d are mixed, then $N_G^d \supseteq N_G^A$, where N_G^A is norm of Abelian non-cyclic subgroups of G. Moreover, the case $N_G^d \neq N_G^A$ is true.*

1. Baer R. (1935). Comp. Math. 1. S. 254 – 283.
2. Lyman F.N. (1970). Ukr. Math. Journal. 22(6), p. 725-733.

Curved operads and curved cooperads

V. Lyubashenko

Institute of Mathematics, Kyiv, Ukraine
lub@imath.kiev.ua

The dual notion to a differential graded algebra is not a differential graded coalgebra, as one might expect, but an augmented curved coalgebra [1]. We show that the dual notion to a differential graded operad is an augmented curved cooperad. The duality exhibits itself in two functors adjoint to each other:

$$\text{bar-construction} : \{\text{differential graded algebras}\} \to \{\text{augmented curved coalgebras}\},$$
$$\text{cobar-construction} : \{\text{augmented curved coalgebras}\} \to \{\text{differential graded algebras}\}.$$

The same for operads and cooperads.

Broader picture incorporates duality between curved operads (which are a generalization of differential graded operads) and curved cooperads.

1. Leonid Positselski, *Two kinds of derived categories, Koszul duality, and comodule-contramodule correspondence*, Memoirs Amer. Math. Soc. **212** (2011), no. 996, vi+133.

Joint value distribution of the Riemann zeta-function and Lerch zeta-functions

R. Macaitienė

Siauliai University, Šiauliai State College
renata.macaitiene@mi.su.lt

Let $s = \sigma + it$ be a complex variable, $\zeta(s)$ denote the Riemann zeta-function, $\lambda \in \mathbb{R}$, $0 < \alpha \leq 1$, and let $L(\lambda, \alpha, s)$ denote the Lerch zeta-function, for $\sigma > 1$, by

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{\mathrm{e}^{2\pi i\lambda m}}{(m+\alpha)^s},$$

and by analytic continuation elsewhere. It is well known that the functions $\zeta(s)$ and $L(\lambda, \alpha, s)$ for some parameters λ and α are universal in the sense that their shifts $\zeta(s+i\tau)$ and $L(\lambda, \alpha, s+i\tau)$, $\tau \in \mathbb{R}$, can approximate in a certain region any analytic function. Moreover, the Lerch zeta-functions are jointly universal: a collection of their shifts approximate any collection of analytic functions.

In the report, we consider the joint universality of the function $\zeta(s)$ and Lerch zeta-functions. Let $D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$.

Theorem 1. *Suppose that the numbers $\alpha_1, ..., \alpha_r$ are algebraically independent over the field of rational numbers. For $j = 1, \ldots, r$, let $\lambda_j \in (0, 1]$, $K_j \subset D$ be a compact set with connected complement and $f_j(s)$ be a continuous function on K_j which is analytic in the interior of K_j. Moreover, let $K \subset D$ be a compact set with connected complement and $f(s)$ be a continuous non-vanishing function on K which is analytic in the interior of K. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \operatorname{meas}\Big\{\tau \in [0;T] : \sup_{1\leq j\leq r} \sup_{s\in K_j} \big|L(\lambda_j, \alpha_j, s+i\tau) - f_j(s)\big| < \varepsilon, \sup_{s\in K} \big|\zeta(s+i\tau) - f(s)\big| < \varepsilon\Big\} > 0.$$

Proof of the teorem is given in [1].

1. A. Laurinčikas, R. Macaitienė. Joint universality of the Riemann zeta-function and Lerch zeta-functions, *Nonlinear Analysis: Modelling and Control*, 2013 (to appear).

On centralizers of elements in some Lie algebras of derivations

Ie.O. Makedonskyi, A.P. Petravchuk, V.V. Stepukh

Kyiv National Taras Shevchenko University, Kyiv, Ukraine
apetrav@gmail.com

Let K be an algebraically closed field of characteristic zero and $R = K(x_1, \ldots, x_n)$ be the field of rational functions in n variables. Recall that a linear map $D : R \to R$ is called a K-derivation if $D(\varphi\psi) = D(\varphi)\psi + \varphi D(\psi)$ for all $\varphi, \psi \in R$. Every K-derivation D can be written uniquely in the form $D = a_1\frac{\partial}{\partial x_1} + \cdots + a_n\frac{\partial}{\partial x_n}$, where $\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}$ are usual partial differentiations and $a_1, \ldots, a_n \in R$. All derivations form a Lie algebra relatively to the operation $[D_1, D_2] = D_1D_2 - D_2D_1$. This Lie algebra will be denoted by $\widetilde{W}_n(K)$, it is closely connected with the group of birational automorphisms of the projective space $\mathbb{P}^n$ (note that Lie algebras of derivations of the polynomial ring $K[x_1, \ldots, x_n]$ was studied by many authors (see, for example, [1], [2], [3]). Now we give a description of centralizers $C_{\widetilde{W}_2(K)}(D)$ of elements D in the Lie algebra $\widetilde{W}_2(K)$. The structure of this centralizer depends on the kernel $kerD$ of the derivation D which is a subfield of the field $R = K(x, y)$ (the field of constants of D).

Every rational function $\varphi \in R = K(x, y)$ determines a derivation D_φ by the rule: $D_\varphi(h) = detJ(\varphi, h)$ where $J(\varphi, h)$ is the Jacobi matrix of the functions φ and h. A rational function φ is called closed if the subfield $K(\varphi)$ of $K(x, y)$ is algebraically closed in $K(x, y)$, for such functions it holds $kerD_\varphi = K(\varphi)$. Then we have for centralizers of elements $D \in \widetilde{W}_2(K)$ the next statement:

Theorem 1. *Let $D \in \widetilde{W}_2(K)$ and $C = C_{\widetilde{W}_2(K)}(D)$. Then it holds:*

(1) If $kerD = K$ then either $C = KD$, or $C = K\langle D, D_1\rangle$ where $D_1 \in \widetilde{W}_2(K)$ such that $[D, D_1] = 0$ and D, D_1 are linearly independent over R.

(2) If $kerD \neq K$ then $D = hD_\varphi$, where $h \in R$ and φ is a closed rational function and $C = K(\varphi)D$, or $C = K(\varphi)D + K(\varphi)D_1$ for some D_1 such that $[D_1, D] = 0$, D, D_1 are linearly independent over R, and $D_1(\varphi) = 1$.

We also characterized centralizers of elements from the Lie algebra $\widetilde{W}_n(K)$ in case when the the field of constants $KerD$ has transcendence degree 0 or 1 over K. For example, if $\mathrm{tr.deg}_K KerD = 0$, then the centralizer $C_{\widetilde{W}_n(K)}(D)$ is a vector space over $KerD$ of dimension $\leq n$.

1. I.V. Arzhantsev, E.A. Makedonskii and A.P. Petravchuk, Finite-dimensional subalgebras in polynomial Lie algebras of rank one// Ukrainian Math. Journal, **63** (2011), no. 5, 708–712.
2. A. Nowicki, Rings and fields of constants for derivations in characteristic zero// J. Pure Appl. Algebra – 1994. - v.96, no. 1 - P.47–55.
3. A.P.Petravchuk, O.G.Iena, On centralizers of elements in the Lie algebra of the special Cremona group SA2 K) // J. Lie Theory. – 2006. – v. 16, no. 3. – P. 561–567.

Infinite Linear Recurrent Equations

A. Maliarchuk

Precarpathian Vasyl Stefanyk National University
lesua197112@mail.ru

The article deals with application of the means of triangular matrix parafunctions [1] to solution of infinite linear recurrent equations. In particular, the following theorems are proved

Theorem 1. *Let us have a vector*

$$a = (a_1, a_2, a_3, \ldots).$$

For the sequence $\{u_n\}_{n=0}^{\infty}$, the following equalities hold:
1.

$$u_0 = 1,\ u_n = a_1 u_{n-1} + a_2 u_{n-2} + a_3 u_{n-3} + \ldots,\ n = 1, 2, \ldots$$

2.

$$u_0 = 1,\ u_1 = [a_1],\ u_2 = \left[\begin{array}{cc} a_1 & \\ \frac{a_2}{a_1} & a_1 \end{array} \right], \ldots,$$

3.

$$\frac{1}{1 - a_1 z - a_2 z^2 - a_3 z^3 - \ldots} = 1 + \sum_{i=1}^{\infty} u_i z^i.$$

Theorem 2. *The members of the normal sequence*

$$u_0 = 1, u_1, u_2, \ldots$$

satisfy the linear recurrent equation

$$u_n = a_1 u_{n-1} + a_2 u_{n-2} + a_3 u_{n-3} + \ldots,$$

where

$$a_i = (-1)^{i-1} \left\langle \begin{array}{cccc} u_1 & & & \\ \frac{u_2}{u_1} & u_1 & & \\ \cdots & \cdots & \cdots & \\ \frac{u_i}{u_{i-1}} & \frac{u_{i-1}}{u_{i-1}} & \cdots & u_1 \end{array} \right\rangle,\ i = 1, 2, \ldots.$$

1. *Zatorsky R.A.* Theory of paradeterminants and its applications // Algebra and Discrete Mathematics №1, 2007, pp. 109-138.

Generalization of Bruhat and LU-decomposition

G.I. Malaschonok

Tambov State University
malaschonok@gmail.com

A matrix decomposition of a form $A = VwU$ is called the Bruhat decomposition of the matrix A if V and U are nonsingular upper triangular matrices and w is a matrix of permutation. Bruhat decomposition plays an important role in the theory of algebraic groups. The generalized Bruhat decomposition was introduced and developed by D.Grigoriev[1].

In [2] there was constructed a pivot-free matrix decomposition method in a common case of singular matrices over a field of arbitrary characteristic. This algorithm has the same complexity as matrix multiplication and does not require pivoting.

Exact triangular decomposition is a generalization of Bruhat and LU-decomposition.

Definition 1. A decomposition of the matrix A of rank r over a commutative domain R in the product of five matrices $A = PLDUQ$ is called *exact triangular decomposition* if P and Q are permutation matrces, L and PLP^T are nonsingular lower triangular matrices, U and Q^TUQ are nonsingular upper triangular matrices over R, $D = \mathbf{diag}(d_1^{-1}, d_2^{-1}, .., d_r^{-1}, 0, .., 0)$ is a diagonal matrix of rank r, $d_i \in R\backslash\{0\}$, $i = 1, \dots$.

Theorem 1. *Any matrix over a commutative domain has an exact triangular decomposition.*

Exact triangular decomposition relates the LU decomposition and the Bruhat decomposition.

If D matrix is combined with L or U, we get the expression $A = PLUQ$. This is the LU-decomposition with permutations of rows and columns. If the factors are grouped in the following way:

$$A = (PLP^T)(PDQ)(Q^TUQ),$$

then we obtain **LDU**-decomposition. If S is a permutation matrix in which the unit elements are placed on the secondary diagonal, then $(S\mathbf{L}S)(S^T\mathbf{D})\mathbf{U}$ is the Bruhat decomposition of the matrix (SA).

1. Grigoriev D. Analogy of Bruhat decomposition for the closure of a cone of Chevalley group of a classical serie. Soviet Math. Dokl., vol.23, N 2, 393–397 (1981)
2. Malaschonok G.I.: Fast Generalized Bruhat Decomposition. In: Ganzha, V.M., Mayr, E.W., Vorozhtsov, E.V. (eds.) 12th International Workshop on Computer Algebra in Scientific Computing, pp.194-202. LNCS 6244. Springer, Berlin Heidelberg, (2010)

An algorithm for analytical solving of partial differential equations systems by algebraization of a problem

N.A. Malaschonok

Tambov State University
nmalaschonok@yandex.ru

An algebraization of a problem, related to various areas of pure and applied mathematics is of great importance now because of possibilities to construct suitable fast algorithms for symbolic solving of a problem.

There is produced an algorithm for symbolic solving systems partial differential equations by means of multivariate Laplace-Carson transform.

The Laplace and Laplace–Carson transform enables to construct algorithms for solving linear partial differential equations with constant coefficients and systems of equations of various order, size and types. The application of Laplace–Carson transform permits to obtain compatibility conditions in symbolic way for many types of PDE equations and systems of PDE equations.

There is considered a system of n equations with m as the greatest order of partial derivatives and right hand parts of a special type. Initial conditions are inposed.

As a result of Laplace-Carson transform of the system according to initial conditions we obtain an algebraic system of equations which can be solved with effective methods for each kind of a system. Efficient methods of solving such systems are developed (for example [1]).

A method to obtain compatibility conditions is discussed. This problem is also resolved in algebraic way due to the correspondent algebraic system, obtained in the proceeder, suggested in this work. It arises just at the stage of consideration of the solution of the algebraic system and produces the analysis of its components.

With respect to compatible conditions we use the inverse Laplace–Carson transform and obtain the correct solution of PDE system.

1. Malaschonok G.I. Effective Matrix Methods in Commutative Domains. Formal Power Series and Algebraic Combinatorics. pp. 506-517. Springer, Berlin. (2000)

Some relationship between left and torsion-theoretic spectrums of rings and modules

M. Maloid-Glebova

Ivan Franko National University of Lviv, Lviv, Ukraine
martamaloid@gmail.com

Let R be associative ring with $1 \neq 0$. Left ideal $\mathfrak{p}$ of ring R is called prime ideal, if $xRy \subseteq \mathfrak{p}$ implies that either $x \in \mathfrak{p}$ or $y \in \mathfrak{p}$ for any $x, y \in R$. Set of all prime ideals is denoted by $Spec(R)$ and is called (prime) spectrum of ring R.

Left ideal $\mathfrak{p}$ of the ring R is called strongly-prime ideal, if it has the property, that for any $x \in R - \mathfrak{p}$ there exist such finite subset V of ring R, that $(\mathfrak{p} : Vx) = \{r \in R : rVx \subseteq \mathfrak{p}\} \subseteq \mathfrak{p}$. Set $spec(R)$ of all strongly-prime left ideals of the ring R is called left spectrum of ring R. Nonzero left module M over ring R is called strongly-prime, if for any nonzero $x, y \in M$ there exist finite subset $\{a_1, a_2, \dots, a_n\} \subseteq R$, that $Ann_R\{a_1x, a_2x, \dots, a_nx\} \subseteq Ann_R\{y\}$, $(ra_1x = ra_2x = \dots = ra_nx = 0)$, $r \in R$ implies $ry = 0$. Submodule P of some module M is called strongly-prime, if quotient module M/P is strongly-prime R-module. The set of all strongly-prime submodules of module M is called left prime spectrum of M and is denoted by $spec(M)$.

Relation of preorder on $spec(R)$ do not have trivial generalization for modules, but it is possible for cyclic modules. For instance, let M by an cyclic module $Rm = R/Ann(m)$ and L, K are some submodules. We can represent L and K as $\mathfrak{A}/Ann(m)$ and $\mathfrak{B}/Ann(m)$ respectively, were $\mathfrak{A}$ and $\mathfrak{B}$ are some left ideals of ring R. Then we define $K \leq L$ iff $\mathfrak{A} \leq \mathfrak{B}$.

Lemma 1. *Submodule L of module $Rm = R/Ann(m)$ is prime iff left ideal $\mathfrak{A}$ is left prime ideal of R.*

Rosenberg points of $spec(Rm)$ are submodules of Rm, that have the form $\mathfrak{A}/Ann(m)$, were $\mathfrak{A}$ is the Rosenberg point of $spec(R)$. All properties are carried out. So spectrum of cyclic module coincides with set of all it's Rosenberg point. Then cyclic spectrum of arbitrary module M is defined as an union of all spectrums of its cyclic submodules. Cyclic spectrum of module M is denoted by $Cspec(M)$.

Lemma 2. *Let L and K are left cyclic R-modules. Then $L \leq K$ iff there exists cyclic left R-module X, monomorphism $X \hookleftarrow L^n$ and epimorphism $X \twoheadrightarrow K$. In other words, there exist diagram $(L)^n \hookleftarrow X \twoheadrightarrow K$.*

This lemma generalizes one statement from [2].

For torsion-theoretic spectrum of module M, the concept of $supp(M) = \{\sigma \mid \sigma(M) \neq 0\}$ is given, where $\sigma(M)$ is the set of prime torsion-theories of module M. Torsion-theoretic spectrum of module M is defined as $M - sp = R - sp \cap supp(M)$, where $R - sp$ is torsion-theoretic spectrum of ring R. (see [1])

Lemma 3. *If σ is torsion theory and $\mathfrak{P}$ is left Rosenberg point of module M, then $M/\mathfrak{P}$ is either σ-torsion module or σ-torsionfree module.*

Proposition 1. *If M is an fully bounded left noetherian module and $\mathfrak{P} \in Cspec(M)$, then torsion theory $\tau_{\mathfrak{P}} = \chi(M/\mathfrak{P})$, cogenerated by module $M/\mathfrak{P}$, is prime.*

1. Golan J. S. Topologies on the Torsion-Theoretic Spectrum of a Noncommutatie Ring // Pacific Journal of Mathematics, 1974 Vol. 51, No. 2, p 439-450;
2. Jara P., Verhaeghe P., Verschoren A. On the left spectrum of a ring //Communs. Algebra, - 1994, - 22(8), p. 2983-3002.
3. Rosenberg A.L. The left spectrum, the Levitski radical, and noncommutative schemes. Proc. Nat. Acad. Sci. USA, 1990, 87, 8583-8586

The p-adic class number formula revisited

F. Marko

The Pennsylvania State University Hazleton, USA
fxm13@psu.edu

The purpose of this talk is to interpret results of Jakubec, Jakubec-Lassak, Marko and Jakubec-Marko on congruences of Ankeny-Artin-Chowla type for cyclic totally real fields as an elementary version of the p-adic class number formula modulo powers of p. Explicit formulas for quadratics and cubic fields will be given.

Tate homology and inverse limits

A. Martsinkovsky, J. Russell

Northeastern University
alexmart@neu.edu

Three different constructions of Tate cohomology over arbitrary rings were given by Pierre Vogel [2], R.-O. Buchweitz [1] (both in the mid 1980s), and G. Mislin [4] (1994). The comparison maps between these three theories are all isomorphisms.

For Tate homology over arbitrary rings, there has been only one construction, also due to P. Vogel [2]. It is remarkably similar to its cohomological counterpart. We will propose two more constructions, one is similar to the construction of Mislin, and the other can be viewed as a homological analog of the construction of Buchweitz. For the former, we simply "reverse" arrows in Mislin's construction and interchange left and right satellites. This procedure is colloquially referred to as the mirror Mislin construction. For the other, we need a new concept, which we call an asymptotic stabilization of the tensor product. The result is a connected sequence of functors, which is isomorphic to that arising from the mirror Mislin construction.

We also construct a comparison map from Vogel homology to the asymptotic stabilization of the tensor product. This map is an epimorphism in each degree and, under suitable Mittag-Leffler - type conditions, it becomes an isomorphism. Conjecturally, the kernel of the comparison map should be expressed in terms of the first derived functor of the inverse limit.

1. R.-O. Buchweitz. Maximal Cohen-Macaulay modules and Tate-cohomology over Gorenstein rings. *http://hdl.handle.net/1807/16682*
2. F. Goichot. Homologie de Tate-Vogel équivariante. *J. Pure Appl. Algebra*, 82(1):39–64, 1992.
3. A. Martsinkovsky and J. Russell, Asymptotic stabilization of the tensor product, in preparation.
4. G. Mislin. Tate cohomology for arbitrary groups via satellites. *Topology Appl.*, 56(3):293–300, 1994.

On modular lattices of π-normal Fitting classes

A.V. Martsynkevich, N.T. Vorob'ev

Vitebsk State University of P.M. Masherov
hanna-t@mail.ru, vorobyovnt@tut.by

All groups considered are finite and soluble. The notation used in this paper are standard [1].

Let $\mathfrak{F}$ and $\mathfrak{H}$ be Fitting classes. The smallest of Fitting classes containing their union $\mathfrak{F} \cup \mathfrak{H}$ is denoted as $\mathfrak{F} \vee \mathfrak{H}$. Such class is called the lattice join of Fitting classes $\mathfrak{F}$ and $\mathfrak{H}$ [2].

Definition 1. Let π be a non-empty set of primes. A Fitting class $\mathfrak{F}$ is called π-normal or normal in a class of all π-groups, if $\mathfrak{F} \subseteq \mathfrak{S}_\pi$ and for each group G its $\mathfrak{F}$-radical is $\mathfrak{F}$-maximal among the subgroups which lie in G .

If $\pi = P$, where P is a set of all primes, the Fitting class $\mathfrak{F}$ is normal [3].

The Lausch result [4] is known in the theory of normal Fitting classes. It was proved [4] that the lattice of all soluble normal Fitting classes is isomorphic to the lattice of subgroups of some infinite abelian group, which is known as Lausch group (X.4.2 [1]) in the theory of classes of groups. Therefore the lattice of all soluble normal Fitting classes is modular.

In particular we have found an alternative proof of this statement (without using the Lausch group) in the case of π-normal Fitting classes. The following theorem is proved.

Theorem 1. *Let π be a non-empty set of primes. The lattice of all π-normal Fitting classes is modular, full and atomic.*

1. K. Doerk, T.Hawkes (1992). Finite soluble groups. Berlin-New York: Walter de Gruyter.
2. E. Cusack (1979). The join of two Fitting classes. Math. Z. Vol.167.
3. D. Blessenohl, W.Gaschütz (1970). Uber normale Schunk und Fittingklassen. Math. Z. Vol.118.
4. H. Lausch (1973). On normal Fitting classes. Math. Z. Vol.130.

Characterizations of rings by means of some quotient lattice of the lattice of two-sided ideals

Yu.P. Maturin

Drohobych Ivan Franko State Pedagogical University, Drohobych, Ukraine
yuriy_maturin@hotmail.com

Let R be an associative ring with 1. The set of all two-sided ideals of R is denoted by $TI(R)$. It is well known that $(TI(R), \supseteq)$ is a modular lattice.

Define the relation η on the set $TI(R)$ via

$$(I, J) \in \eta \Leftrightarrow \forall a_1, a_2, ... \in I \forall b_1, b_2, ... \in J \exists n \in \mathbb{N} :$$

$$(a_1 + b_1)(a_2 + b_2)...(a_n + b_n) \in I \bigcap J.$$

Theorem 1. *The relation η is a congruence on the lattice $(TI(R), \supseteq)$.*

Theorem 2. *The quotient lattice $TI(R)/\eta$ is distributive.*

Theorem 3. *The quotient lattice $TI(R)/\eta$ is Boolean if and only if $J(R)$ is left T-nilpotent and $R/J(R) \cong R_1 \times ... \times R_n$ for some simple rings $R_1, ..., R_n$.*

Corollary 1. *A commutative ring R is perfect if and only if the quotient lattice $TI(R)/\eta$ is Boolean.*

1. Frank W. Anderson, Kent R. Fuller (1992). Rings and Categories of Modules (2nd ed.). Springer. ISBN 3-540-97845-3.
2. George Gratzer (1978). General Lattice Theory. Academic Press. ISBN 0-12295750-4
3. A.I. Kashu (1983). Radicals and Torsions in Modules. Stiinca. (in Russian)

Divisibility of recursive sequence elements

A.Matyukhina, L.Oridoroga

Donetsk National University, Donetsk, Ukraine
a-l-i-n-a.matyukhina@yandex.ru

For recursive sequences of second order we studied the divisibility of elements defined by arithmetic expressions from the root of a number of D discriminate of the characteristic equation, on the prime number p. Depending on whether D is quadratic residue in the field of residues modulo p, we derive two theorems that allow knowing the recursive sequence, its first element and its reciprocal equation, you can specify the elements of this sequence, which are divided without remainder by giving us a prime number p.

We explored a variety of tasks on the divisibility of numbers given by the arithmetic expressions from the root of an integer D, discriminate of the characteristic equation of recursive sequences of the second degree. Depending on whether D is a quadratic residue a prime modulo p, apply one of two methods. If D is a quadratic residue modulo p, this expression is itself an element of the field of residues modulo p. Otherwise, it is part of the extension of the field of second order, which was built by using the vD. In the first case using Fermat's little theorem for study of the divisibility, the second is its analogue for finite fields.

The results are applied to study the divisibility of Fibonacci sequence elements, and then arbitrary recursive sequences elements, by the prime number p. In this case the answer is very different, depending on whether the discriminate of the characteristic equation of a sequence is a quadratic residue of modulo p. Also, these methods are applied to the solution and the generalization of Olympiad problems on divisibility. Collated task # 4 Class XI, XXXIX-th Ukrainian Mathematical Olympiad and the problem # 2 XXV-th International Mathematical Olympiad

Theorem 1. *"Divisibility of the recursive sequence elements of the second degree: If the discriminant of the characteristic equation of the second degree is a quadratic residue (quadratic nonresidue) a prime modulo p, then $u_{n(p-1)}$ ($u_{n(p+1)}$) a member of any recursive sequence $\{u_i\}$ is divided without remainder, so $u_{n(p-1)} \equiv 0 (mod p)$, $n \in Z$, p is a prime ($u_{n(p+1)} \equiv 0 (mod p)$).*

This work was awarded the Gold Medal from Yale Science & Engineering Association for Most Outstanding Eleventh Grade Exhibit in Mathematics at the Intel Isef 2011 Science Fair

1. Воробьев Н.Н. Числа Фибоначчи. – 1978. – С. 142.
2. Laslo Geroecs. Some properties of divisibility of higher-order linear recursive sequences, Fibonacci Quarterly, 20, – 1982. – P. 354-359.
3. Joseph H. Silverman. Divisibility sequences and powers of algebraic integers.Documenta Math. Extra Volume Coates – 2006. – P. 711-727.

On lattice subformations of finite groups

A.P. Mekhovich, N.N. Vorob'ev

P.M. Masherov Vitebsk State University
amekhovich@yandex.ru, vornic2001@yahoo.com

All groups considered are finite. All unexplained notations and terminologies are standard (see [1, 2]).

Recall that a group class closed under taking homomorphic images and finite subdirect products is called a *formation*.

In each group G we select a system of subgroups $\tau(G)$. It is said that τ is a subgroup functor [1] if the following conditions hold: 1) $G \in \tau(G)$ for every group G; 2) for every epimorphism $\varphi : A \mapsto B$ and all groups $H \in \tau(A)$ and $T \in \tau(B)$ we have $H^{\varphi} \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$.

A formation $\mathfrak{F}$ is called *τ-closed* [1] if $\tau(G) \subseteq \mathfrak{F}$ for every group $G \in \mathfrak{F}$.

A function of the form $f : \mathbb{P} \to \{\text{formation of groups}\}$ is called a local satellites [2]. For every satellite f, we consider the class $LF(f) = (G \mid G/F_p(G) \in f(p)$ for all $p \in \pi(G))$ where $\pi(G)$ is the set of all prime divisors of the order of a group G. If $\mathfrak{F}$ is a formation such that $\mathfrak{F} = LF(f)$ for a local satellite f, then $\mathfrak{F}$ is said to be saturated and f is said to be a local satellite of $\mathfrak{F}$ [2].

Every formation is 0-multiply saturated by definition. For $n > 0$, a formation $\mathfrak{F}$ is called n-multiply saturated if $\mathfrak{F} = LF(f)$ and all non-empty values of f are $(n-1)$-multiply saturated formations (see [1]). If a formation $\mathfrak{F}$ is n-multiply saturated for all natural n, then $\mathfrak{F}$ is called totally saturated.

The symbol $l_\infty^\tau \mathrm{form} G$ denotes the intersection of all τ-closed totally saturated formations containing a group G. Every formation in this form is called a one-generated τ-closed totally saturated formation.

Let $\mathfrak{F}$ be a τ-closed totally saturated formation. The symbol $L_\infty^\tau(\mathfrak{F})$ denotes the lattice of all τ-closed totally saturated subformations of $\mathfrak{F}$.

We prove the following

Theorem 1. *Let $\mathfrak{F} = l_\infty^\tau \mathrm{form} G$ be a one-generated τ-closed totally saturated formation. Then the lattice $L_\infty^\tau(\mathfrak{F})$ contains a finite number of atoms only.*

Corollary 1. *Let $\mathfrak{F} = l_\infty \mathrm{form} G$ be a one-generated totally saturated formation. Then the lattice $L_\infty(\mathfrak{F})$ contains a finite number of atoms only.*

1. Alexander N. Skiba (1997). Algebra of formations. Minsk: Belaruskaya navuka. (in Russian). ISBN 985-08-0078-x.
2. Klaus Doerk, Trevor Hawkes (1992). Finite soluble groups. De gruyter expositions in mathematics 4. Berlin-New York: Walter de Gruyter. ISBN 3-11-012892-6.
3. Leonid A. Shemetkov, Alexander N. Skiba (2000). Multiply ω-local formations and Fitting classes of finite groups. *Siberian Advances in Mathematics* 10(2):112–141.

On p-supersolvability of finite factorisable group

V. S. Monakhov, I. K. Chirik

Gomel Francisk Skorina State University, Gomel,
Gomel Engineering Institute of MES of the Republic of Belarus, Gomel
Victor.Monakhov@gmail.com, chyrykira@mail.ru

Let p be a prime. A finite group is called p-supersolvable if orders of the chief factors either equal to p or not divisible by p. It is easy to verify that a finite solvable group with cyclic Sylow p-subgroup is p-supersolvable. Berkovich [1] established p-supersolvability of group $G = AB$ of odd order if Sylow p-subgroups of A and B are cyclic. Hence supersolvability of group $G = AB$ of odd order if all Sylow subgroups of A and B are cyclic. For groups of even order similar results are incorrect. The simplest example is the symmetric group S_4, which coincides with the product of its subgroups S_3 and $< (1234) >$, but S_4 is not 2-supersolvable. For any prime $p > 2$ Mazurov [2, p. 75] found examples of non-p-supersolvable solvable group of even order, which is the product of two subgroups with cyclic Sylow p-subgroups.

Theorem 1. *Let $G = AB$ be a finite group, and let Sylow p-subgroups of A and B are cyclic. If G is 2-closed, then G is p-supersolvable. In particular, if $G = AB$ is a 2-closed and all Sylow subgroups of A and B are cyclic, then G is supersolvable.*

Example 1. The semidirect product $[E_{7^2}]S_3$, in which symmetric group S_3 acts irreducibly on the elementary Abelian subgroup E_{7^2} of order 49, is a minimal non-supersolvable group. It is 2- and 3-supersolvable, but it is not 7-supersolvable. This group has the following factorisable:

$$[E_{7^2}]S_3 = ([U]Z_2)([V]Z_3), \ E_{7^2} = U \times V, \ U \simeq V \simeq Z_7.$$

All Sylow subgroups of factors $[U]Z_2$ and $[V]Z_3$ are cyclic, but group is not 7-supersolvable.

Therefore, in Theorem 1 the condition of 2-closed group can not be replaced by condition of 2-nilpotency and more so can not be replaced by $l_2(G) \leqslant 1$.

1. Berkovich J. G. Math. Sb. 1967. V. 74 (116), № 1. P. 75-92 (In Russian).
2. XII All-Union. algebr. coll. Theses of reports. Exercise book 1. Sverdlovsk, 1973 (In Russian).

On finite groups with subnormal non-cyclic subgroups

V.S. Monakhov, A.A. Trofimuk

Gomel Francisk Skorina State University
Brest State University named after A.S. Pushkin
Victor.Monakhov@gmail.com, Alexander.Trofimuk@gmail.com

All groups considered bellow will be finite.

We say that G has a Sylow tower if there exists a normal series in which factors are isomorphic to the Sylow subgroups of G.

Let G be a group of order $p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$, where $p_1 > p_2 > \ldots > p_k$. We say that G has an ordered Sylow tower of supersolvable type if there exists a normal series

$$1 = G_0 \subseteq G_1 \subseteq G_2 \subseteq \ldots \subseteq G_{k-1} \subseteq G_k = G$$

such that G_i/G_{i-1} is isomorphic to a Sylow p_i-subgroup of G for each $i = 1, 2, \ldots, k$.

Recall that supersolvable group is a group which has a normal series with cyclic factors. If G is supersolvable then G has an ordered Sylow tower of supersolvable type, see [1, VI.9.1]. The alternating group A_4 of degree 4 has a Sylow tower of non-supersolvable type.

By the Zassenhaus Theorem [1, IV.2.11], a group G with cyclic Sylow subgroups has a normal cyclic Hall subgroup such that the corresponding quotient group is also cyclic. Hence G is supersolvable.

In 1980 Srinivasan, see [2, Theorem 1], proved that if all maximal subgroups of the Sylow subgroups of G are normal in G then G is supersolvable.

If the condition of normality is weakened to subnormality then the group can be non-supersolvable. An example is the alternating group A_4 of degree 4. But the group with subnormal maximal subgroups of the Sylow subgroups has an ordered Sylow tower, see [2, Theorem 3].

Developing this result we prove the following theorem.

Theorem 1. *Let G be a group such that every non-cyclic maximal subgroups in its Sylow subgroups are subnormal in G. Then:*

1) *if G is non-solvable then $G/S(G) \simeq PSL(2, p)$, p is prime, $p \equiv \pm 3 \pmod 8$;*

2) *if G is solvable then G has a Sylow tower.*

Here $S(G)$ is a largest normal solvable subgroup of G.

1. Huppert B. Endliche Gruppen I, Berlin-Heidelberg-New York, Springer, 1967.
2. Srinivasan S. Two sufficient conditions for supersolvability of finite groups, Israel J. Math. 35 (1980), 210–214.

Recursive criterion of conjugation of finite-state binary tree's automorphisms

D. Morozov

National University of "Kyiv-Mohyla Academy", Kyiv, Ukraine
denis.morozov178@gmail.com

The conjugation problem in the group of finite-state automorphisms of rooted binary tree is investigated.

Definition 1. Define $FAutT_2$ as group of finite-state automorphisms of rooted binary tree.

Definition 2. Denote **the marked tree** for automorphism $f \in AutZ_2$ like this.

- The root of the tree note by automorphism f.
- If the vertex of the n-th level of the marked-type tree marked automorphism $a = (b, c) \circ \sigma$, then only one edge connects the n+1-th level with this vertex. Other vertex of this edge marked with automorphism $\pi_L(a) \circ \pi_R(a)$.
- If the vertex of the n-th level of the marked-type tree marked automorphism $a = (b, c)$, then two edges connect the n+1-th level with this vertex. Other vertex of one edge marked with automorphism $\pi_L(a)$ and another edge marked with $\pi_R(a)$.

Automorphism that marked the vertex $t\ inD_f$ of marked-type tree denote as $D_f(t)$. The set of vertices of n-th level of tree D denote as $L_n(D)$.

Lemma 1. *Let*

$$a = (a_1, a_2) \circ \sigma, b = (b_1, b_2) \circ \sigma$$

$$a' = a_1 \circ a_2, b' = b_1 \circ b_2$$

If a' and b' conjugated in $FAutT_2$ then a and b conjugated in $FAutT_2$.

Theorem 1. *Automorphisms a and b conjugated in $FAutT_2$ if, and only if*

$$\forall t \in L_n(D_a), \exists x \in FAutT_2,\ D_a(t)^x = D_b(t * \alpha)$$

Corollary 1. *Automorphisms a and b conjugated in $FAutT_2$ if, and only if*

$$\exists n \in \mathbb{N}, \forall t \in L_n(D_a), \exists x \in FAutT_2\ D_a(t)^x = D_b(t * \alpha)$$

This techniques applied for finite-state conjugation problem solving of differentiable finite-state izometries of the ring of integer 2-adic numbers.

1. Denis Morozov. Differentiable finite-state izometries and izometric polynomials of the ring of integer 2-adic numbers. 8th International Algebraic Conference, July 5-12 (2011), Lugansk, Ukraine.

On R-subnormal subgroups

V. I. Murashka

Skoryna Gomel State University
mvimath@yandex.ru

All groups considered are finite. One of important concepts of group theory is the concept of subnormal subgroup. Recall that a subgroup H of a group G is called subnormal if there is a chain of subgroups $H = H_0 \leq H_1 \leq \cdots \leq H_n = G$ such that H_i is a normal subgroup of H_{i+1} for all $i = 1, \ldots, n-1$. In 1939 Wielandt [1] found main properties of subnormal subgroups. In particular he showed that the set of all subnormal subgroups is sublattice of the subgroup lattice.

There are some generalizations of subnormality. For example such generalizations are the concepts of $\mathfrak{F}$-subnormal and K-$\mathfrak{F}$-subnormal subgroups. In this paper we present the following generalization of subnormality:

Definition 1. [2] Let R be a subgroup of a group G. We shall call a subgroup H of G the R-subnormal subgroup if H is subnormal in $\langle H, R\rangle$.

In this work we study properties and applications of R-subnormal subgroups.

Proposition 1. *Let H, K and R be subgroups of a group G. If H and K are R-subnormal then $H \cap K$ is R-subnormal.*

It is clear that every subnormal subgroup is R-subnormal. But in the general case R-subnormal subgroup is not subnormal.

Example 1. Let $G \simeq S_4$ be the symmetric group of degree 4. Then the Fitting subgroup $F(G) \simeq \langle (1,2)(3,4), (1,3)(2,4)\rangle$. Let $H_1 \simeq \langle (1,2), F(G)\rangle$ and $H_2 \simeq \langle (1,3), F(G)\rangle$. Then $|H_1| = |H_2| = 8$ and $H_1 \neq H_2$. Since H_1 and H_2 are nilpotent, we see that $\langle (1,2)\rangle$ and $\langle (1,3)\rangle$ are $F(G)$-subnormal. Note that H_1 is $F(G)$-subnormal but not subnormal subgroup of G.

In addition $\langle\langle (1,2)\rangle, \langle (1,3)\rangle\rangle \simeq S_3$ is not subnormal in $\langle\langle\langle (1,2)\rangle, \langle (1,3)\rangle\rangle, F(G)\rangle = G$. It means that $\langle\langle (1,2)\rangle, \langle (1,3)\rangle\rangle$ is not $F(G)$-subnormal. Hence, in the general case the join of two R-subnormal subgroups need not to be R-subnormal.

Recall that $F^*(G)$ is a quasinilpotent residual for a group G. As an application of our concept we obtain the following theorem.

Theorem 1. *A group G is nilpotent if and only if every cyclic primary subgroup of G are $F^*(G)$-subnormal.*

1. *H. Wielandt,* Eine Verallgemeinerung der invariant Untergruppen / Math Z. 45, 209-244 (1939)
2. *V.I. Murashka and A.F. Vasil'ev,* On the products of partially subnormal subgroups of finite groups / Vestnik VGU 70(4), 24-27 (2012). (In Russian)

Continuity of symmetric products of capacities

O. Mykytsey

Vasyl Stefanyk Precarpathian National University
oksana39@i.ua

Let MX be the space of all normalized capacities on a compactum X. It is known that MX is a compactum with the weak* topology. The construction of MX extended to the capacity functor M in the category of compacta.

For capacities $c_1 \in MX$, $c_2 \in MY$, the *tensor product* [2] $c_1 \otimes c_2$ is the unique capacity $c \in M(X \times Y)$ such that, for each closed $F \subset X \times Y$, the inequality $c(F) \geqslant \alpha \in I = [0;1]$ is valid if and only if there exists a closed subset $H \subset X$ with $c_1(H) \geqslant \alpha$, and for every $x \in H$ we have $c_2(pr_2(H \cap (\{x\} \times Y))) \geqslant \alpha$.

The operation $\otimes$ is associative and continuous because the functor M is the functorial part of the capacity monad [1], but it fails to be commutative.

Assume that continuous operations $\odot : I \times I \to I$ and $\oplus : I \times I \to I$ are given, $\odot$ is associative, commutative, monotone in both variables and with 1 being its twoside unit, and $\oplus$ is defined as $\alpha \oplus \beta = 1 - (1-\alpha) \odot (1-\beta)$ for all $\alpha, \beta \in I$. Then we can assume that $\odot$ is a fuzzy conjunction and $\oplus$ the corresponding fuzzy disjunction.

We propose to introduce the symmetric product operation $\boxtimes : MX \times MY \to M(X \times Y)$ by the formula: $(c_1 \boxtimes c_2)(F) = \sup\{c_1(A) \odot c_2(B) \mid A \subset X, B \subset Y, A \times B \subset F\}$ for any closed $F \subset X \times Y$ and capacities $c_1 \in MX$, $c_2 \in MY$. Clearly the result is a capacity.

Theorem 1. *Symmetric multiplication $\boxtimes : MX \times MY \to M(X \times Y)$ is associative, commutative, and continuous.*

It is obvious that this operation is associative and commutative (although it is not determined by a monad). Therefore, our task is to prove its continuity.

Dual operation to the symmetric multiplication is the symmetric addition: $c_1 \boxplus c_2 = \widetilde{\tilde{c}_1 \boxtimes \tilde{c}_2}$ or $(c_1 \boxplus c_2)(F) = \inf\{c_1(A) \oplus c_2(B) \mid \text{for all} A \subset X, B \subset Y \text{ such that} F \subset (A \times Y) \cup (X \times B)\}$ for any closed $F \subset X \times Y$ and $c_1 \in MX, c_2 \in MY$.

Corollary 1. *Symmetric addition $\boxplus : MX \times MY \to M(X \times Y)$ is associative, commutative, and continuous.*

1. Заричный М.М. *Функтор емкостей в категории компактов* / М.М. Заричный, О.Р. Никифорчин // Мат. Сборник. — 2008. — № **199**(2). — 3–26.
2. Teleiko A. Categorical Topology of Compact Hausdorff Spaces / A. Teleiko, M. Zarichnyi. — VNTL Publishers, Lviv, 1999. — (Math. Studies Monograph Series, vol. 5).

On zerous of polynomial of the elliptic functions

Olha Mylyo

Ivan Franko National University of Lviv, Lviv, Ukraine
olga.mylyo@gmail.com

Let $sn\ z$ be the Jacobi elliptic function that is defined in terms of the elliptic modulus k and satisfies the equation

$$\left(\frac{dy}{dz}\right)^2 = (1-y^2)(1-k^2y^2).$$

Let us consider arbitrary nonzero polynomial $P \in \mathbf{C}[X,Y]$ of degree at most L in X and of degree at most M in Y, where $M \geq 1$.

Theorem 1. *The order of arbitrary zero of function $F(z) = P(sn^2 z, z)$ is not greater than*

$$16LM + 4M + 1.$$

Similar estimates can be used, for example, while proving of algebraic independence of numbers connected with the elliptic functions.

From a technical point of view it is more convenient to use functions $sn^2\ z$, $cn^2\ z$ instead of $sn\ z$, $cn\ z$.

1. Fel'dman N.I., Nesterenko Yu.V. Transcendental Numbers. - Springer-Verlag, Berlin, 1998. - 346 p.
2. Reyssat E. Approximation algebrique de nombres lies aux fonctions elliptique et exp // Bull. Soc. Math. France. - 1980. - 108. - P. 47-79

On formation of finite ca-$\mathfrak{F}$-groups

E. Myslovets

Francisk Scorina Gomel State University
myslovets@gmail.com

All the groups considered are finite.

Recall that a group G is c-supersoluble if it has a chief series all the factors of which are simple groups and a group G is ca-supersoluble if it is a c-supersoluble and it has a chief series all abelian factors of which are central in G ([1]).

Definition 1. [2] Let $\mathfrak{F}$ be a class of groups. A chief factor H/K of a group G is called $\mathfrak{F}$-central provided $H/K \leftthreetimes G/C_G(H/K) \in \mathfrak{F}$.

In the article [3] A.N. Skiba and W. Guo have introduced a concept of quazi-$\mathfrak{F}$-groups by using this definition. We introduce next definition by analogy with [3].

Definition 2. Let $\mathfrak{F}$ be a class of groups and G is a group. We say G is a ca-$\mathfrak{F}$-group if every abelian chief factor of G is $\mathfrak{F}$-central and every non-abelian chief factor of G is a simple group.

We use $\mathfrak{F}_{ca}$ to denote the class of all ca-$\mathfrak{F}$ groups. For any class $\mathfrak{F}$ of groups, the class $\mathfrak{F}_{ca}$ is a nonempty formation.

Theorem 1. *Let $\mathfrak{F}$ be a saturated formation. Then $\mathfrak{F}_{ca}$ is a composition formation.*

Class of groups $\mathfrak{F}$ is called semiradical [4], if $\mathfrak{F}$ is S_n-closed and it contains any group $G = HK$, where H и K are normal $\mathfrak{F}$-subgroups in G such that G/H and G/K doesn't have any common (up to isomorphism) abelian composition factors.

Theorem 2. *Let $\mathfrak{F}$ be a saturated semiradical formation. Then $\mathfrak{F}_{ca}$ is a semiradical formation.*

Corollary 1. *Let $\mathfrak{F}$ be a saturated semiradical formation. If group $G = HK$, where H and K are normal ca-$\mathfrak{F}$-subgroups of G and $(|G : H|, |G : K|) = 1$, then G is a ca-$\mathfrak{F}$-group.*

1. V.A. Vedernikov (1988). On some classes of finite groups. Dokl. Akad. Nauk BSSB, 32(10). P. 872-875.
2. L.N. Shemetkov, A.N. Skiba (1989). Formations of algebraic systems. Nauka, Moscow.
3. W. Guo, A.N. Skiba (2009). On finite quazi-$\mathfrak{F}$-groups. Communication in Algebra. Vol. 37. P. 470–481.
4. T.I. Vasil'eva (1999) Semiradical formations of finite groups. Proc. Gomel State Univ. Problems in Algebra. V. 1, № 1 (15). P. 71–77.

About semigroups of linear relations

M.I. Naumik

P.M. Masherov Vitebsk State University
naumik@tut.by

Let V be an n-dimensional vector space over a field F, $LR(V)$ be the multiplicative semigroup of all linear relations [1], $L(V)$ be subsemigroup of the semigroup $LR(V)$ of all linear relations of a rank ≤ 1 [2].

In this paper, we give characteristic of the semigroup $LR(V)$ of all linear relations over the field F with the help of its subsemigroup $L(V)$.

Theorem 1. *Let $LR(V)$ be the multiplicative semigroup of all linear relations over the field F. If and only if the semigroup is isomorphic to the semigroup $LR(V)$, when if contains the tightly embedded ideal, what is isomorphic to $L(V)$.*

This theorem generalizes Theorem 1 from [3].

1. Sneperman L.B. The Shur theorem for periodic semigroups of linear relations // Semigroup Forum. – 1982 – V. 25. – P. 203–211.
2. Naumik, M.I. Semigroups of Linear Relations // Reports of AS of Belarus, 2004. – V. 48, № 3. – P. 34–37.
3. Gluskin, L.M. About the Matrik Semigroips // News of the Accademy of Sciences of the USSR. The mathematical series. – V. 22, 1958. – P. 439–448.

On application of self-induced metric on groupoids

M. Nazarov

National Research University of Electronic Technology
Nazarov-Maximilian@yandex.ru

The application of finite groupoids to modeling the interaction of living cells in biology is considered. Assume that we know a multitude D of cell types, which can exist within a given organism. In this case we could consider this multitude D of cell types a finite groupoid with the following operation defined thereon:

- $a \cdot b = c$, if and only if the cell of type a under the influence of signals from cells of type b will tend to change into type c.

For practical application of a groupoid as a parameter of some mathematical model the first thing to do is to find small variations of the groupoid so that the latter could be changed iteratively within the optimization task. The concept of a small variation of groupoids can be introduced via metrics defined on the groupoid elements.

Metrics that are unambiguously determined based on the operation on groupoid will be referred to as self-induced metrics. For the construction of this metrics, special numerical parameters of finite groupoids are defined in this study.

Definition 1. By the **idempotency index** of groupoid $(D, \cdot)$ we understand the function $K : D \to \mathbb{R}$, which is defined by the system of linear equations of the form

$$2\,K(a) = 2 + \begin{cases} 0, & \text{if} \quad a^2 = a \\ K(a^2), & \text{if} \quad a^2 \neq a \end{cases}$$

Definition 2. By the **right quasiderivative** of finite groupoid $(D, \cdot)$ we will mean the function $\delta_R : D \times D \to \mathbb{R}$, which is defined by the system (assume $\delta_R(a, a) = 0$)

$$\pi \cdot |D| \cdot \, \delta_R(a, b) = \pi \cdot |D| \, + \sum_c K(c)\, \delta_R(a \cdot c,\, b \cdot c) \cdot \begin{cases} \sqrt{2}, & \text{if} \quad a \cdot c = b \\ 1, & \text{if else} \end{cases}$$

The left quasiderivative $\delta_L : D \times D \to \mathbb{R}$ will be defined similar to the right one δ_R up to the substitution of $c \cdot a$ for $a \cdot c$ and $c \cdot b$ for $b \cdot c$.

Definition 3. By the **left self-induced metric** ρ_L of groupoid we will understand function expressed as (assume $\rho_L(a, a) = 0$)

$$\rho_L(a, b) = \sum_c |K(a)\delta_L(a, c) - K(b)\delta_L(b, c)| + |K(b)\delta_L(c, b) - K(a)\delta_L(c, a)| +$$
$$+ \sum_c |K(a)\delta_L(a \cdot c, c \cdot a) - K(b)\delta_L(b \cdot c, c \cdot b)| \qquad (\forall a \neq b) \tag{1}$$

The right self-induced metric will be defined fully similar to the left one up to substitution of δ_R for δ_L.

Theorem 1. *The following conditions will hold for any finite groupoid D.*

1. *Systems of linear equations for K and quasiderivatives δ_R (δ_L) are solvable.*
2. *Self-induced metric ρ_L and ρ_R will induce metric spaces on D.*

On generalized automata

B. Novikov, G. Zholtkevych

V.N. Karazin Kharkiv National University, Kharkiv, Ukraine
boris.v.novikov@univer.kharkov.ua, g.zholtkevych@gmail.com

Using the concept of partial action of the semigroup [2, 4], we have defined the notion of preautomaton in [1]. Here we consider a more general class of machines — protoautomata.

Definition 1. Given a set X and a free monoid Σ^* over the alphabet Σ, a **protoautomaton** is a partial mapping $X \times \Sigma^* \dashrightarrow X : (x, a) \mapsto xa$, such that

a) $x\varepsilon = x$;

b) if $xu \neq \varnothing$ and $(xu)v \neq \varnothing$, then $x(uv) \neq \varnothing$ and $x(uv) = (xu)v$.

We denote the category of the protoautomata (preautomata, automata) by $\mathcal{P}t\mathcal{A}ut(\Sigma)$ (resp. $\mathcal{PA}ut(\Sigma)$, $\mathcal{A}ut(\Sigma)$).

It follows from the theory of partial action of semigroups [2] that the protoautomaton, which is not a preautomaton, has no globalization. In this situation, the concept of a reflector [3] is useful.

Theorem 1. *$\mathcal{A}ut(\Sigma)$ and $\mathcal{PA}ut(\Sigma)$ are reflective subcategories of $\mathcal{P}t\mathcal{A}ut(\Sigma)$.*

Some category $\mathcal{R}el(\Sigma)$ is built for which the following assertion is fulfilled:

Theorem 2. *For every object of $\mathcal{R}el(\Sigma)$ there is a free protoautomaton relatively a forgetting functor.*

1. M. Dokuchaev, B. Novikov, G. Zholtkevych. *Partial actions and automata.* Algebra and Discr. Math., **11**(2011), N 2, 51–63.
2. C. Hollings. *Partial actions of monoids.* Semigroup Forum, **75** (2007), 293–316.
3. S. MacLane. *Categories for the Working Mathematician.* Springer, Berlin, 1971.
4. M. Megrelishvili, L. Schröder. *Globalization of confluent partial actions on topological and metric spaces.* Topology and its Appl., **145**(2004), 119–145.

On natural liftings of functors in the category of compacta to categories of topological algebra and analysis

O. Nykyforchyn, D. Repovš

Vasyl' Stefanyk Precarpathian National University
University in Ljubljana
`oleh.nyk@gmail.com, dusan.repovs@guest.arnes.si`

We consider natural liftings [1] of a normal functor F in the category of compacta $\mathcal{C}omp$ to the following categories: $\mathcal{CSG}r$ of compact semigroups, $\mathcal{CA}b\mathcal{SG}r$ of compact Abelian semigroups, $\mathcal{CM}on$ and $\mathcal{CA}b\mathcal{M}on$ of compact monoids and compact Abelian monoids, respectively.

We show that each natural lifting of a normal functor F in $\mathcal{C}omp$ to one of the mentioned categories is determined with a unique natural trsnsformation $t : F(-) \times F(-) \to F(- \times -)$, i.e., a collection of continuous mappings $t(X,Y) : FX \times FY \to F(X,Y)$ for all compacta X, Y, such that for all continuous mappings of compacta $f : X \to X'$, $g : Y \to Y'$ and all elements $a \in FX$, $b \in FY$ the equality $t(X',Y')(Ff(a), Fg(b)) = F(f \times g)(t(X,Y)(a,b))$ is valid.

In what follows the mapping $p : X \times Y \to Y \times X$, $\bar{p} : FX \times FY \to FY \times FX$ for all $x \in X$, $y \in Y$, $a \in FX$, $b \in FY$ are defined as $p(x,y) = (y,x)$, $\bar{p}(a,b) = (b,a)$.

Consider possible properties of a natural transformation t:

$$t(X \times Y, Z)(t(X,Y)(a,b),c) = t(X, Y \times Z)(a, t(Y,Z)(b,c)) \tag{$*$}$$

for all $a \in FX$, $b \in FY$, $c \in FZ$ ("associativity");

$$F\operatorname{pr}_1 \circ t(X,\{y\})(a, \eta\{y\}(y)) = a, F\operatorname{pr}_2 \circ t(\{x\},Y)(\eta\{x\}(x), b) = b \tag{$**$}$$

for all $a \in FX$, $b \in FY$, $x \in X$, $y \in Y$ ("two-sided unit");

$$Fp \circ t(X,Y)(a,b) = t(Y,X)(b,a) \tag{$***$}$$

for all $a \in FX$, $b \in FY$ ("commutativity").

Theorem 1. *Let $\mathcal{C}$ be one of the categories $\mathcal{CSG}r$, $\mathcal{CM}on$, $\mathcal{CA}b\mathcal{SG}r$, and $\mathcal{CA}b\mathcal{M}on$. For each natural lifting $\bar{F} : \mathcal{C} \to \mathcal{C}$ of the functor $F : \mathcal{C}omp \to \mathcal{C}omp$ that preserves monomorphisms, preimages, intersections, the empty set and singletons, there exists a unique natural transformation $t(-,-) : F(-) \times F(-) \to F(- \times -)$ such that the operation $\bar{\odot}$ on $\bar{F}X$ for all compactt semigroup $(X, \odot)$ is defines by the formula $a \bar{\odot} b = F\odot \circ t(X,X)(a,b)$. This natural transformation satisfies: $(*)$ for $\mathcal{C} = \mathcal{CSG}r$, $(*)$, $(**)$ for $\mathcal{C} = \mathcal{CM}on$, $(*)$, $(***)$ for $\mathcal{C} = \mathcal{CA}b\mathcal{SG}r$, and $(*)$, $(**)$, $(***)$ for $\mathcal{C} = \mathcal{CA}b\mathcal{M}on$. Conversely, each natural transformation $t(-,-) : F(-) \times F(-) \to F(- \times -)$ satisfying the respective conditions determines a natural lifting of F to $\mathcal{C}$ by the above formula.*

There are plenty of natural transformations that satisfy the mentioned conditions and therefore define natural liftings. On the other hand, for the category $\mathcal{C}onv$ of convex compacta and affine continuous mappings we prove the analogue of the result of M.Zarichyi [2] on the category of compact topological groups.

Theorem 2. *For a normal functor $F : \mathcal{C}omp \to \mathcal{C}omp$ there exists a natural lifting to $\mathcal{C}onv$ if and only if F is a power functor.*

1. *A. Teleiko, M. Zarichnyi,* Categorical Topology of Compact Hausdorff Spaces, Math. Studies Monograph Series, V. 5, VNTL Publishers, Lviv, 1999.
2. *М.М. Заричный,* Мультипликативный нормальный функтор — степенной // Мат. Заметки, **41**:1 (1987), 93–110.

Free products and infinite unitriangular matrices

A. Oliynyk

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
olijnyk@univ.kiev.ua

Let p be a prime. Consider a unitary associative ring R of characteristic p. Denote by $UT_\infty(R)$ the group of upper unitriangular infinite matrices over R ([1]). For any $n \geqslant 1$ denote the identity matrix and the null matrix of size n by E_n and O_n correspondingly. A unitriangular matrix is called narrow ([2]) if it has the form

$$\begin{pmatrix} E_n & A & O_n & O_n & \dots \\ O_n & E_n & A & O_n & \dots \\ O_n & O_n & E_n & A & \dots \\ . & . & . & . & \dots \end{pmatrix}$$

for some $n \geqslant 1$ and $A \in M_n(R)$. Denote by $FBUT_\infty(R)$ the subgroup of $UT_\infty(R)$ generated by all narrow matrices.

We give a constructive proof of the following result.

Theorem 1. *Let $m, k_1, \dots, k_m$ be positive integers, $m \geqslant 2$. The group $FBUT_\infty(\mathbb{F})$ contains the free product of cyclic groups of orders $p^{k_1}, \dots, p^{k_m}$.*

1. Olijnyk A., Sushchansky V. Representations of free products by infinite unitriangular matrices over finite fields// International Journal of Algebra and Computation. 2004. — Vol. 14, no. 5-6. — Pp. 741–749.
2. Oliynyk A. Free products of cyclic 2-groups in groups of infinite unitriangular matrices (in Ukrainian)// Mathematical Bulletin of the Shevchenko Scientific Society. 2012. — Vol. 9, Pp. 231–237.

The normal structure of the diagonal limit of hyperoctahedral groups with doubling embeddings

B. Oliynyk

National Taras Shevchenko University of Kyiv, Kyiv, Ukraine
bogdana.oliynyk@gmail.com

Let n be a positive integer. Consider the n-dimension Hamming space H_n, i.e. the space of all n-tuples $(a_1, \ldots, a_n)$, $a_i \in \{0, 1\}$, $1 \leq i \leq n$, with the distance d_{H_n} defined by the rule:

$$d_{H_n}(\bar{x}, \bar{y}) = |\{k : x_k \neq y_k, 1 \leq k \leq n\}|,$$

where $\bar{x} = (x_1, \ldots, x_n), \bar{y} = (y_1, \ldots, y_n) \in H_n$. The isometry group $IsomH_n$ is isomorphic to the wreath product $W_n = S_n \wr Z_2$, where Z_2 is the cyclic group of order 2 and S_n is the symmetric group of degree n (e.g. [1]), i.e. the group $IsomH_n$ decomposes into the semidirect product of its subgroup S_n and its normal subgroup $K_n = \underbrace{Z_2 \times \ldots \times Z_2}_{n}$.

A mapping $f_n : \frac{1}{n}H_n \to \frac{1}{2n}H_{2n}$ is said to be *doubling* if it is determined as

$$f_n(x_1, \ldots, x_n) = (x_1, x_1, \ldots, x_n, x_n). \tag{1}$$

The direct spectrum $\Phi = \langle \frac{1}{2^n}H_{2^n}, f_{2^n} \rangle$ of scaled Hamming spaces $\frac{1}{2^n}H_{2^n}$ is said to be the 2^∞-*periodic Hamming space* H_{2^∞} (see [1], [2]).

The doubling defined by (1) induces a diagonal embedding φ_n of the isometry group $IsomH_{2^n}$ into the group $IsomH_{2^{n+1}}$. Then we obtain an embedding of $S_{2^n} \wr Z_2$ into $S_{2^{n+1}} \wr Z_2$ where K_{2^n} embeds into $K_{2^{n+1}}$ as we defined in (1) and S_{2^n} embeds into $S_{2^{n+1}}$ diagonally in sense of [3]. Denote by W_{2^∞} the diagonal limit of W_n. The group W_{2^∞} is an everywhere dense subgroup of the isometry group $IsomH_{2^\infty}$ of the 2^∞-periodic Hamming space (see [2]).

Let K_{2^∞} be the group defined on the set of all 2^∞-periodic sequences with coordinate-wise addition. Denote by C the subgroup of K_{2^∞} containing only sequences $(0, 0, \ldots)$ and $(1, 1, \ldots)$.

Theorem 1. *The normal structure of the group W_{2^∞} has the form*

$$E \lhd C \lhd K_{2^\infty} \lhd W'_{2^\infty} \lhd W_{2^\infty},$$

where W'_{2^∞} is the commutator subgroup of the group W_{2^∞}.

1. *Cameron P. J., Tarzi S.* Limits of cubes // Topology and its Applications. — 2008. — V. 155. — P. 1454–1461.
2. *Oliynyk B.V. , Sushchanskii V.I.* The isometry groups of Hamming spaces of periodic sequences // Siberian Mathematical Journal. — 2013. — Vol. 54, № 1. — P. 124-136.
3. *Kroshko N.V., Sushchansky V.I.* Direct limits of symmetric and alternating groups with strictly diagonal embeddings // Arch. Math. — 1998. — V. 71, № 3. — P. 173–182.

On the lattice of quasi-filters of left congruence on a Clifford semigroups

R.M. Oliynyk

Lviv National Agrarian University, Lviv, Ukraine
forvard-or@ukr.net

The concept of torsion theory for $S-$acts was introduced by J. K. Luedeman [2] in 1983. R. Zhang, W. Gao, F. Xu [5] have introduced the concept of quasi-filter of right congruence on a semigroup S. Their results have a new influence on the study of the torsion theories of $S-$acts [3], [4].

Throughout this paper S is always a multiplicative semigroup with 0 and 1. The terminologies and definitions not given in this paper can be found in [1]. Denoted by $Con(S)$ the set of all left congruence on S.

Definition 1. A quasi-filter (see [5]) of S is defined to be subset $\mathcal{E}$ of $Con(S)$ satisfying the following conditions:

1. If $\rho \in \mathcal{E}$ and $\rho \subseteq \tau \in Con(S)$, then $\tau \in \mathcal{E}$.
2. $\rho \in \mathcal{E}$ implies $(\rho : s) \in \mathcal{E}$ for every $s \in S$.
3. If $\rho \in \mathcal{E}$ and $\tau \in Con(S)$ such that $(\tau : s), (\tau : t)$ are in $\mathcal{E}$ for every $(s, t) \in \rho$, then $\tau \in \mathcal{E}$.

Denoted by $S - q - fil$ the set of all quasi-filters of left congruence on S.

A meet of the quasi-filters $\mathcal{E}_1$ and $\mathcal{E}_2$ is the quasi-filter $\mathcal{E}_1 \wedge \mathcal{E}_2 = \{\rho | \rho \in \mathcal{E}_1 \text{ i } \rho \in \mathcal{E}_2\}$.

A join of quasi-filters $\mathcal{E}_1$ and $\mathcal{E}_2$ is the least quasi-filter $\mathcal{E}_1 \vee \mathcal{E}_2$ which contain both $\mathcal{E}_1$ and $\mathcal{E}_2$.

The unique minimal element in $S - q - fil$ is $\omega = S \times S$ and the unique maximal element is $\mathcal{E}_{\triangle_S}$, which contains $\triangle_S$, when $\triangle_S = \{(s, s) | s \in S\}$. Also we call a quasi-filter $\mathcal{E}$ trivial if either it contains $\triangle_S$ or only contains ω.

The lattice structure on quasi-filters of left congruence on a Clifford semigroup are described.

1. Kilp M., Knauer U., Mikhalev A. V. Monoids, Acts and Categories, Walter de Gruyter, Berlin, 2000. - 529 p.
2. Luedeman J. K. Torsion theories and semigroup of quotients, Lecture Notes in Mathematics 998, Springer-Verlag, Berlin, New York, (1983), 350-373.
3. Qiu D. Hereditary Torsion Theory of Pseudo-Regular-Systems, Semigroup Forum Vol. 66 (2003) 131-139
4. Wiegandt R. Radical and torsion theory for acts, Semigroup Forum 72 (2006), 312-328.
5. Zang R. Z., Gao W. M., Xu F. Y. Torsion theories and quasi-filters of right congruences, Algebra Colloq. 1(3) (1994), 273-280.

Prime abelian groups in a localic topos and test of injectivity

I.A. Palas

Ivan Franko National University of Lviv, Lviv, Ukraine
irapalas@yandex.ru

In this talk we announce our results about injectivity abelian groups in a topos of sheaves on a locale, describe prime abelian groups in some concrete localic topoi and find their injective hulls. The main result of our talk shows that injectivity of prime abelian groups in such topoi is test property of injectivity in general.

1. K. R. Bhutani, *Injectivity and injective hulls of abelian groups in a localic topos*, Bull. Austral. Math. Soc. Vol.37 (1988), 43—59.
2. K. R. Bhutani, *Abelian groups in a topos of sheaves on a locale*, ProQuest Dissertations and Theses, Canada (1983), p.1—149.
3. P. Vamos, *Ideals and modules testing injectivity*, Communications in Algebra (1983), 2495—2505.

Asymptotic structures of cardinals

O.V. Petrenko, I.V. Protasov, S. Slobodianiuk

Kyiv National University, Kyiv, Ukraine
opetrenko72@gmail.com, i.v.protasov@gmail.com, slobodianiuk@yandex.ru

A ballean is a set X endowed with some family $\mathcal{F}$ of its subsets, which are called balls, in such a way that $(X, \mathcal{F})$ can be considered as an asymptotic counterpart of a uniform topological space. Given a cardinal κ, we define $\mathcal{F}$ using a natural order structure on κ. We characterize balleans up to coarse equivalence, give the criterions of metrizability and cellularity, calculate the basic cardinal invariant of those balleans. Next, we discuss properties of the combinatorial derivation on κ. Finally, for a cardinal κ, we define T_κ-points, analogues of T-point ultrafilters on ω and prove that an ultrafilter on ω is a T-point if and only if it is a $T_{\aleph_0}$-point.

1. O. Petrenko, I.V. Protasov, *Thin ultrafilters*, Notre Dame J. Formal Logic, **53** (2012), 79–88.
2. I.V. Protasov, *Combinatorial derivation*, to appear in Appl. Gen. Topology, preprint available at arXiv:1210.0696.
3. I.V. Protasov, *Asymptotically scattered spaces*, preprint available at arXiv:1212.0364
4. I. Protasov, M. Zarichnyi, *General Asymptotology*, Math. Stud. Monogr. Ser., Vol. 12, VNTL Publishers, Lviv, 2007

The chain length in POset and its connection with solution of linear equations over $\mathbb{Z}$

M. Plakhotnyk

Kyiv National Taras Shevchenko University, Kyiv, Ukraine
makar_plakhotnyk@ukr.net

In our talk we will consider minimal exponent matrices of dimension 5 with zero the first line.

Exponent matrices itself were appeared at V.V. Kirishenko's works for example [1] in connection of studying of Gorenstein Orders.

Exponent matrix $A = (\alpha_{pq})$ is a square integer matrix with zero diagonal such that for all indices $i,\ j,\ k \in [1,\ n]$ inequalities $\alpha_{ij} + \alpha_{jk} \geqslant \alpha_{ik}$ hold.

Exponent matrices whose the first line is zero are widely studied. All elements of these matrices are non-negative. We also stay our attention on such matrices.

Minimal vectors as elements of the basis of the Partial Ordered Set of non-negative solutions of system of liner equations or inequalities were introduced in [2].

Minimal elements of Partial Ordered Set of non-negative minimal exponent matrices were described in [3]. These matrices are called superminimal.

In our talk we consider the following construction.

First consider the set of superminimal matrices of dimension 5. Their height as elements of POset of all non-negative exponent matrices will be 1.

Then consider those minimal exponent matrices which difference with at least one superminimal matrix is non-negative (0, 1) matrix. From these set find minimal exponent matrices with height 2 in POset of non-negative exponent matrices. Then we may continue in the same way with higher heights.

All found matrices are are incomparable as non-negative vectors – solutions of system of linear equations in the sense of [2] where correspond system of linear equations is taken from the definition of exponent matrix. That is why their number is finite and according to well known R.P. Dilworth theorem it is not bigger then the number of minimal non-negative exponent matrices of the same order.

We have calculated the number of exponent matrices which are found in such a way (it is equal to 2305). Nevertheless intermediate calculation gives the conclusion that the number of minimal exponent matrices of dimension 5 whose the first line is zero is much more (not less then 1.5 times) bigger then them.

1. *Zavadskii A. G., Kirichenko V. V.*, Torsion free modules on the primery rings, Modules in the representation theory and algebraic geometry, Zap. Nauchn. Sem. LOMI, 57, "Nauka", Leningrad. Otdel., Leningrad, 1976, 100–116
2. *Plakhotnyk, V.V.* On minimal elements in the set of integer nonnegative solutions of a system of linear equations. Visn., Ser. Fiz.-Mat. Nauky, Kyïv. Univ. Im. Tarasa Shevchenka 2000, No.3, 74-77 (2000).
3. *Kirichenko, V.V.; Plakhotnyk, M.V.*, Superminimal exponent matrices, Visn., Ser. Fiz.-Mat. Nauky, Kyïv. Univ. Im. Tarasa Shevchenka 2011, No. 2, 20-22 (2011).

Sharpening of explicit lower bounds on elements order for finite field extensions $F_q[x]/\Phi_r(x)$

R. Popovych

Lviv Polytechnic National University, Lviv, Ukraine
`rombp07@gmail.com`

The problem of constructing efficiently a primitive element for a given finite field is notoriously difficult in the computational theory of finite fields. That is why one considers less restrictive question: to find an element with high multiplicative order [1, 2, 3, 4, 5]. It is sufficient in this case to obtain a lower bound on the order. High order elements are needed in several applications. Such applications include cryptography, coding theory, pseudo random number generation and combinatorics.

Let q be a power of prime number p, r be an odd prime number coprime with q, q be a primitive root modulo r, a be any non-zero element in the finite field F_q. Set $F_q(\theta) = F_{q^{r-1}} = F_q[x]/\Phi_r(x)$, where $\Phi_r(x) = x^{r-1} + x^{r-2} + ... + x + 1$ is the r-th cyclotomic polynomial and $\theta = x\ (mod\ \Phi_r(x))$. We use below the following denotations:
$\beta = \theta + \theta^{-1}$, $\gamma = (\theta^{-1} + a)(\theta + a)^{-1}$ and $z = \begin{cases} \beta^2\gamma\ if\ \nu_2(q^{(r-1)/2} - 1) = 2 \\ \beta\gamma^2\ if\ \nu_2(q^{(r-1)/2} + 1) = 2 \end{cases}$.

It is shown in [3] that the order of β is at least $2^{\sqrt{r-1}}/4$. Better explicit lower bounds on orders of β and similar elements in terms of p and r are given in [4]. However, such bounds are not general, because they are obtained only for the cases $r \geqslant p^2$ and $r < p$. Important in applications (particularly in cryptography) case $p \leqslant r < p^2$ remains not described. That is why we give in this paper explicit lower bounds for any p and r.

Theorem 1. *Let $p \geqslant 5$, e be any integer, f be any integer coprime with r. Then*
(a) $\theta^e(\theta^f + a)$ has the multiplicative order at least $5^{\sqrt{(r-2)/2}}$,
(b) $\theta^e(\theta^f + a)$ for $a^2 \neq \pm 1$ has the multiplicative order at least $5^{\sqrt{r-3}}/2$,
(c) z for $a^2 \neq 1$ has the multiplicative order at least $5^{(\sqrt{2}/2+1)\sqrt{r-3}}/2$.

1. Q. Cheng (2005). On the construction of finite field elements of large order, Finite Fields Appl., 11 (3), P.358-366.
2. S. Gao (1999). Elements of provable high orders in finite fields, Proc. Amer. Math. Soc., 107 (6), P.1615-1623.
3. J. von zur Gathen, I.E. Shparlinski (1998). Orders of Gauss periods in finite fields, Appl. Algebra Engrg. Comm. Comput., 9 (1), P.15-24.
4. R.Popovych (2012). Elements of high order in finite fields of the form $F_q[x]/\Phi_r(x)$, Finite Fields Appl., 18 (4), P.700-710.
5. R.Popovych (2013). Elements of high order in finite fields of the form $F_q[x]/(x^m - a)$, Finite Fields Appl., 19 (1), P.86-92.

Using a variety of images of real numbers for modeling and analysis of fractal properties of nowhere differentiable functions

M. Pratsiovytyi, N. Vasylenko

Dragomanov National Pedagogical University, Kyiv, Ukraine
prats4@yandex.ru, nata_va@inbox.ru

Let $1 < s$ be a fixed odd positive integer, $A_s = \{0, 1, 2, \dots, s-1\}$ – alphabet. Recall that [1] Q-representation of real number x is called representation it in the form

$$x = \varphi_{\alpha_1} + \sum_{i=2}^{\infty} \left(\varphi_{\alpha_i} \cdot \prod_{j=1}^{i-1} q_{\alpha_j} \right) \equiv \Delta^{Q}_{\alpha_1\, \alpha_2\, \dots \alpha_k \dots}, \ \ \alpha_k \in A_s,$$

where $Q = \{q_0, q_1, \dots, q_{s-1}\}$, $q_i > 0$, $\sum\limits_{i=0}^{s-1} q_i = 1$, $\varphi_0 = 0$, $\varphi_k = \sum\limits_{i=1}^{k-1} q_i$.

Let p is the given number, $p < s$. Let us define a function with argument representation in the form

$$x = \varphi_{\alpha_1} + \sum_{i=2}^{\infty} \left(\varphi_{\alpha_i} \cdot \prod_{j=1}^{i-1} q_{\alpha_j} \right) \equiv \Delta^{Q_s}_{\alpha_1\, \alpha_2\, \dots \alpha_k \dots}, \ \ \alpha_k \in A_s, \ Q_s = \{q_0, q_1, \dots, q_{s-1}\},$$

and value of function has the following Q_2-representation

$$f(x) = \psi_{\beta_1} + \sum_{i=2}^{\infty} \left(\psi_{\beta_i} \cdot \prod_{j=1}^{i-1} g_{\beta_j} \right) \equiv \Delta^{Q_2}_{\beta_1 \beta_2 \dots \beta_k \dots}, \ \ \beta_k \in \{0, 1\}, \ Q_2 = \{g_0, g_1\},$$

$$\beta_1 = \begin{cases} 0, \ \text{якщо} \ \alpha_1 = p, \\ 1, \ \text{якщо} \ \alpha_1 \neq p, \end{cases} \quad \beta_k = \begin{cases} \beta_{k-1}, \ \text{якщо} \ \alpha_k = \alpha_{k-1}, \\ 1 - \beta_{k-1}, \ \text{якщо} \ \alpha_k \neq \alpha_{k-1}. \end{cases}$$

Function f is a correctly defined in Q_s-rational point (ie the point $\Delta^{Q_s}_{\alpha_1 \alpha_2 \dots \alpha_{k-1} \alpha_k (0)} = \Delta^{Q_s}_{\alpha_1 \alpha_2 \dots \alpha_{k-1} [\alpha_k - 1](s-1)}$), continuous, non-monotonic.

In the report are given results of differential, self-affine and integral properties of a continuous nowhere differentiable function belonging to this family.

1. *Pratsiovytyi M. V.* Fractal approach to investigations of singular probability distributions. – Kyiv: Dragomanov National Pedagogical Univ., 1998. – 296 p.
2. *Pratsiovytyi M. V.* Fractal properties of one continuous nowhere differentiable function. Naukovi zapiski NPU (Ukrainian) 3 (2002) 351-362

Normal form with respect to similarity of involutory matrices over a principal ideal domain

V.M. Prokip

IAPMM, Lviv, Ukraine
v.prokip@gmail.com

Let $M_{n,m}(\mathrm{R})$ be a set of $n \times m$ matrices over a principal ideal domain R, $\operatorname{char} \mathrm{R} \neq 2$, with identity $e \neq 0$ (see [1]); I_n is the $n \times n$ identity matrix and $0_{n,m}$ is the $n \times m$ zero matrix.

Proposition 1. *Let $A \in M_{n,n}(\mathrm{R})$ be an involutory matrix, i.e. $A^2 = I_n$, with characteristic polynomial* $\det(I_n\lambda - A) = (\lambda - e)^k(\lambda + e)^{n-k}$, *where $1 \leqslant k < n$. If*

$$A + 2I_n = 0_{n,n} \pmod{2e}$$

then for the involutory matrix A there exists a matrix $T \in GL(n, \mathrm{R})$ such that

$$TAT^{-1} = \begin{bmatrix} I_k & 0_{n,n-k} \\ 0_{n-n,k} & -I_{n-k} \end{bmatrix}.$$

Proposition 2. *Let $A \in M_{n,n}(\mathrm{R})$ be an involutory matrix with characteristic polynomial* $\det(I_n\lambda - A) = (\lambda - e)^k(\lambda + e)^{n-k}$, *where $1 \leqslant k < n$. If*

$$A + 2I_n \neq 0_{n,n} \pmod{2e}$$

then for the involutory matrix A there exists a matrix $T \in GL(n, \mathrm{R})$ such that

$$TAT^{-1} = J_m(A) = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{I_k} & 0_{n,n-k} \\ \hline I_m & 0_{m,k-m} & \\ 0_{n-k-m,m} & 0_{n-k-m,k-m} & -I_{n-k} \end{array}\right].$$

The matrix $J_m(A)$ is unique for involutory matrix A with given characteristic polynomial $\det(I_n\lambda - A) = (\lambda - e)^k(\lambda + e)^{n-k}$.

1. Newman M. *Integral Matrices.* – Academic Press: New York, 1972.

Ultracompanion's characterization of subsets of a group

I.V. Protasov, S. Slobodianiuk

Kyiv Taras Shevchenko University, Kyiv, Ukraine
slobodianiuks@gmail.com

For any infinite group G any its subset A and any free ultrafilter $p \in \beta G$ we define a p-companion:

$$\Delta_p(A) = \{gp : g \in G, g^{-1}A \in p\}$$

We will consider the properties for a set to be thin, sparse, small, prethick, thick, large in the language of ultracompanions applying to this set.

Theorem 1. *[1] For any infinite group G and any $A \subseteq G$ the following holds:*
1) A is thin if and only if for any $p \in \beta G \setminus G$ $|\Delta_p(A)| \leqslant 1$.
2) A is sparse if and only if for any $p \in \beta G \setminus G$ $\Delta_p(A)$ is finite.
3) A is thick if and only if there exists $p \in \beta G$ such that $\Delta_p(A) = Gp$.

Due to [2] it is easy to show that for any prethick set $A \subseteq G$ (that is an intersection of large and thick subset) there exists $p \in \beta G \setminus G$ such that ultracompanion $\Delta_p(A)$ is equal to Lp, for some large L and olso has no isolated points in βG.

1. I. Protasov *Ultracompanions of subsets of a group*, preprint.
2. N. Hindman, D. Strauss, *Algebra in the Stone-Čech Comactification*, 2nd edition, de Grueter, 2012.

Algebras of Language Transformations in Labelled Graphs

O. Prianychnykova

Donetsk National Technical University, Donetsk, Ukraine
pryanichnikovae@gmail.com

In recent years, labeled oriented graphs have found much interest in theoretical computer science, as well as in several applied areas, such as software engineering and robotics. They have served as useful tools in software verification and validation to represent behavior of a system and have been successfully applied to model checking and source-to-source program transformation. Other important application areas have been object-oriented modeling, description and verification of protocols. In robotics labeled graphs have been used to describe a topological environment for navigation problems.

In this paper we introduce and study algebras of language transformations in labelled graphs. As a main result of this paper we prove characterization theorems for the class of languages representable by labeled graphs and extend the results about state minimization for finite automata to these graphs. For a given language we prove necessary and sufficient conditions under which the language can be accepted by a graph. The construction is a generalization of the classical Myhill-Nerode construction of finite automata theory. For a given labeled graph we construct minimal complete deterministic graph which is equivalent to the original graph and give reachable upper and lower bounds on the size of the minimal graph.

1. G. Dudek, M. Jenkin, E. Milios, D. Wilkes. Map validation and robot self-location in a graph-like world // Robotics and autonomous systems. - 1997. - Vol. 22(2). - P. 159-178.
2. C. Baier, J.-P. Katoen. Principles of Model Checking - Cambridge: MIT Press, 2008. - 975 p.

Absolute G-retracts and their applications

N.M Pyrch

Ukrainian Academy of Printing, Lviv, Ukraine
pnazar@ukr.net

A subspace Y of a topological space X is called a G-retract of X if any continuous mapping from Y to topological group H admits continuous extension to Y. A topological space X is called an absolute G-retract if X is G-retract of any Tychonoff space Y containing X as closed subspace.

Theorem 1. *Let X be a Tychonoff space, A be its closed subspace such that spaces A and X/A are absolute G-retracts. Then X is absolute G-retract.*

Example 1. Applying Theorem 1 to the space $X = \{(0,t)|t \in [-1,1]\} \cup \{(t, sin(\pi/2t))|t \in (0,1]\}$ and its subspace $A = \{(1,t)|t \in [-1,1]\}$ we obtain an example of topological space being absolute G-retract and not being absolute G-retract.

Theorem 2. *Let X be a Tychonoff space, $A_1, ..., A_n$, $B_1, ..., B_n$ be its closed disjoint subspaces, which are absolute G-retracts and for every $i = 1, ..., n$ the spaces A_i and B_i are homeomorphic. Then free topological groups on spaces $X/\{A_1, ..., A_n\}$ and $X/\{B_1, ..., B_n\}$ are topologically isomorphic.*

Divisor function $\tau_3(w)$ in arithmetic progression

A. S. Radova

Odessa National University, Odessa, Ukraine
radova_as@mail.ru

Let $Z[i]$ be the ring of Gaussian integers and $k \geq 2$, $k \in N$. We define the divisor function $\tau_k(w)$, $w \in Z[i]$ as the coefficient of $N(w)^s$ in the Dirichlet series

$$Z^k(s) = \sum_{w}{}^{*} \frac{\tau_k(w)}{N(w)^s}, \ Res > 1$$

.

Over the ring of the Gaussian integers we constructed the asymptotic formula for summatory function of the divisor function $d_3(w)$ in an arithmetic progression $N(w) \equiv l \ (mod\ q)$ which is a non-trivial for $q \leq x^{2/7-\epsilon}$.

Theorem. Let l, q be the positive integers, $1 \leq l < q$, $(l,\ q) = 1$. Then for $x \to \infty$ we have

$$\sum_{\substack{N(w) \equiv l \,(mod\, q),\\ N(w) \leq x}} = \frac{x}{q^2} I(l,q) \prod_{p|q} (1 - \frac{1}{N(p)})^2 P_2(log\, x) +$$

$$+ \frac{x}{q^2} I(l,q) \prod_{p|q} (1 - \frac{1}{N(p)}) P_1(log\, x) + \frac{12x}{q^2} I(l,q) + O(x^{\frac{5}{7}+\epsilon}),$$

where $I(l,q)$-denotes the number of the solutions of $x^2 + ly^2 \equiv l \,(mod\, q)$,

$$I(l,q) = E(l,q) q \prod_{\substack{p^a || q,\\ p\ is\ odd}}{}' (1 - \frac{\chi_4(p)^{v_p(l,p^\alpha)+1}}{p} + (1 - \frac{1}{p}) \sum_{b=a-v_p\,((l,p^\alpha))}^{a-1} \chi_4(p^{a-b})),$$

$$E(l,q) = \begin{cases} 1, & \text{if } q\, is\, odd; \\ 1, & \text{if } q \equiv 2\, (mod\, 4); \\ 1, & \text{if } q \equiv 0\, (mod\, 4)\ and\ v_2(l) > v_2(q) - 2; \\ 2, & \text{if } q \equiv 0\, (mod\, 4)\ and\ l2^{-v_2(l)} \equiv 1\, (mod\, 4); \\ 0, & \text{if } q \equiv 0\, (mod\, 4)\ and\ l2^{-v_2(l)} \equiv 3\, (mod\, 4). \end{cases}$$

$P_j(w)$ are the poliminials of j^{th}-degree with the compuable coefficients, moreover these coefficients and O is a constant that does not depend on x, l, p.

Finite local nearrings with multiplicative Miller-Moreno group

I.Iu. Raievska, M.Iu. Raievska, Ya.P. Sysak

Institute of Mathematics of Nat. Acad. Sci. of Ukraine, Kyiv, Ukraine
raemarina@rambler.ru

Finite local nearrings with Miller-Moreno group of units are investigated. In [1] all multiplicative groups of finite nearfields which are Miller-Moreno group are classified. In [2] local nearrings of order 2^n (n is an integer) with Miller-Moreno group of units are considered.

Using the computer system GAP4.6 [3], the characterization of all finite local nearrings with Miller-Moreno multiplicative group are given in the following theorem.

Theorem 1. *Let R be a finite local nearring of order p^n whose group of units R^* is a Miller-Moreno group and L a subgroup of all non-invertible elements of R. If R is a nearfield, then either R^* is a quaternion group of order 8, or a non-abelian metacyclic group of order 24, 63 or 80. If R is not a nearfield then the following statements hold:*

(I) if $p \neq 2$, then $|R| = p^2$ for some Fermat's prime p, additive group R^+ is elementary abelian and there exists such non-invertible element $a \in R$, such that

$$R^+ = <i> + <a>,$$

where i is an identity of R, $a^2 = 0$ and $(ik)a = -ak$ for an arbitrary primitive root k modulo p. Moreover, for each Fermat's prime p there exists an unique local nearring of order p^2;

(II) if $p = 2$ and $|R : L| > 2$, then $n \geqslant 4$, R^+ is group of order 2^{2q} and its exponent does not exceed 4, where q is a prime number for each a number $2^q - 1$ is a Mersenne prime, R^* is group of order $2^q(2^q - 1)$ and L is elementary abelian 2-group with $xy = 0$ for all $x, y \in L$;

(III) if $p = 2$ and $|R : L| = 2$, then $n \leq 6$, in particulary, if $n = 6$ then R^* is non-metacyclic Miller-Moreno group of order 32 and of exponent 4.

1. Раєвська І.Ю. Майже-поля з неабелево спадковими мультиплікативними групами / І.Ю. Раєвська, М.Ю. Раєвська // Математичні студії. — 2010. — Т. 34, № 1. — С. 38–43.
2. Раєвська М.Ю. Про локальні майже-кільця з мультиплікативною групою Міллера-Морено / М.Ю. Раєвська, Я.П. Сисак // Укр. мат. журн. — 2012. — Т. 64, №6. — С. 811–818.
3. The GAP Group, Aechen, St Andrews. GAP — Groups, Algorithms and Programming, Version 4.6, 2013 (http://www.gap.dcs.st-and.ac.uk/ gap).

Correspondences of the semigroup of endomorphisms of an equivalence relation

E. A. Romanenko

Luhansk National Taras Shevchenko University, Luhansk, Ukraine
e.a.rom@mail.ru

Let ρ be a binary relation on a set X, $End(\rho)$ be a set of all endomorphisms of the relation ρ. An ordered pair (φ, ψ) of transformations φ and ψ of the set X is called an endotopism [1] if $(x, y) \in \rho$ implies $(x\varphi, y\psi) \in \rho$ for all $x, y \in X$. The set of all endotopisms of ρ is a semigroup relative to the operation of the direct product of transformations. This semigroup is called the semigroup of endotopisms of the relation ρ and it is denoted by $Et(\rho)$.

Let G be a universal algebra. If we consider a subalgebra of $G \times G$ as a binary relation on G, then the set $S(G)$ of all subalgebras of $G \times G$ is a semigroup relative to the operation of the composition of binary relations. Elements of this semigroup are called correspondences of the algebra G [2].

Lemma 1. *For any equivalence relation α on a set X the semigroup $Et(\alpha)$ is the correspondence of the semigroup $End(\alpha)$.*

Let α be an arbitrary equivalence relation on the set X. We define a small category K such that $Ob\,K = X/\alpha$ and $Mor(A, B)$ is the set of all mappings from A to B for all $A, B \in ObK$.

We designate by W the wreath product $\mathfrak{J}(X/\alpha)wrK$ of the symmetric semigroup $\mathfrak{J}(X/\alpha)$ with the small category K (see, e.g., [3]) and by K^2_* the full subcategory of category K^2 defined on $ObK^2_* = \{(A; A) | A \in ObK\}$.

Theorem 1. *Let α be an equivalence on a set X, K be the small category defined above. The correspondence $Et(\alpha)$ of the semigroup $End(\alpha)$ can be exactly represented as:*

1) the subdirect product of the monoid $(\Im\,(X/\alpha)\,wrK) \times (\Im\,(X/\alpha)\,wrK)$;
2) the wreath product $\Im\,(X/\alpha)\,wrK^2_*$ of $\Im\,(X/\alpha)$ with the small category K^2_*.

Corollary 1. *For any equivalence relation α on a finite set X we have*

$$|Et(\alpha)| = \sum_{\varphi \in \Im(X/\alpha)} \Big(\prod_{A \in X/\alpha} |A\varphi|^{|A|} \Big)^2.$$

In addition, we study such correspondences of the semigroup of all endomorphisms of an equivalence relation as the monoid of all strong endotopisms and the group of all autotopisms of the given equivalence relation.

1. Popov B. V. Endotopism semigroups of an μ-ary relation, Uch. zap. LGPI im. A.I. Gercen, v. 274 (1965), 184-201 (In Russian).
2. Kurosh A. G., General algebra (lectures 1969-70 school year). – M.: Nauka, 1974, P. 110 (In Russian).
3. Knauer U., Nieporte M. Endomorphisms of graphs. I. The monoid of strong endomorphisms. Arch. Math., Vol. 52 (1989), 607 – 614.

Greatest common left divisor of matrices one of which is disappear

A.M. Romaniv

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics NAS of Ukraine, Lviv, Ukraine
romaniv_a@ukr.net

Let R be a commutative elementary divisor domain with $1 \neq 0$ and A, B be a $n \times n$ matrices over R. There exists invertible matrices P_A , Q_A , such that

$$P_A A Q_A = E, \text{ where } E = diag(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k, 0, \ldots, 0),\ \varepsilon_i \,|\varepsilon_{i+1}\ ,\ i = 1, \ldots, k-1.$$

The matrix E called the canonical diagonal form for matrix A and P_A , Q_A are called left and right transforming matrices for matrix A .

Denote by $\mathbf{P}_A$ the set of all left transforming matrices for matrix A .

The matrix B is a left divisor of the matrix A, if $A = BC$. The matrix D is a common left divisor of the matrices A and B, if $A = DA_1$ and $B = DB_1$. Moreover, the matrix D is a greatest common left divisor of the matrices A and B, if the matrix D is divided into every other common left divisor of the matrices A and B (by notation $(A, B)_l$).

The method for finding the greatest common left divisor of the matrices A and B, which based on the results of E. Cahen [2] and A. Chatelet [3] was proposed by C.C. MacDuffee [1] in 1933.

Theorem 1. *Let*

$$A \sim diag(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n), \quad \varepsilon_i \,|\varepsilon_{i+1}\ , \quad i = 1, \ldots, n-1;$$

$$B \sim diag(1, 1, \ldots, 1, \delta), \quad P_B P_A^{-1} = \|s_{ij}\|_1^n, \text{ where } P_A \in \boldsymbol{P}_A,\ P_B \in \boldsymbol{P}_B.$$

Then the greatest common left divisor of the matrices A and B has the form

$$(A, B)_l = P_B^{-1}\Phi,$$

where

$$\Phi = diag(1, \ldots, 1, \varphi), \quad \varphi = ((\varepsilon_n, \delta), \varepsilon_1 s_{n1}, \ldots, \varepsilon_{n-1} s_{n.n-1})\,.$$

Corollary 1. *The sets of transforming matrices $(A, B)_l$ and B connected to each other by the following relations:*

1. $\boldsymbol{P}_{(A,B)_l} = \boldsymbol{G}_\Phi P_B$,
2. $\boldsymbol{P}_B \subset \boldsymbol{P}_{(A,B)_l}$.

1. MacDuffe C.C. Matrices with elements in a principal ring // Bull.Amer. Math. Soc. – 1933. – 39. – P. 570-573. ISSN 1088-9485
2. Cahen E., (1914). Théorie des Nombres.
3. Chatelet A., (1924). Groupes Abéliens Finis.

2-state ZC-automata generating free groups

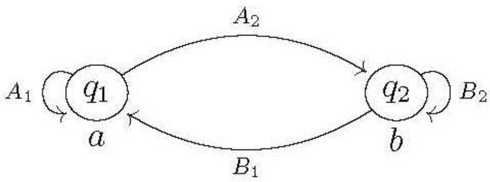
N.M. Rusin

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
n.m.rusin89@gmail.com

Let Z be a countable alphabet, identified with the ring $\mathbb{Z}$ of integers. A permutational automaton $\mathcal{A} = \langle \mathsf{Z}, Q, \varphi, \psi \rangle$ is called ZC-automaton [1], if in any inner state $q \in Q$ the output function ψ_q realizes a shift on some integer c_q:

$$\psi_q(z) = z + c_q, z \in \mathsf{Z}.$$

Consider 2-state ZC-automata with states q_1 and q_2. Such an automaton $\mathcal{A}$ is determined by two partitions of the set Z, $\mathsf{Z} = A_1 \bigcup A_2$ i $\mathsf{Z} = B_1 \bigcup B_2$, and by integers a and b (see Fig. 1). Hence each 2-state ZC-automaton can be uniquely determined as the quadruple $\langle A_1, B_1, a, b \rangle$, where $A_1, B_1 \subset \mathsf{Z}$, $a, b \in \mathsf{Z}$.



2-state ZC-automaton

Fig. 1

Define an automaton $\mathcal{A}$, specified by a quadruple $\langle A_1, \mathsf{Z} \setminus A_1, 1, 0 \rangle$ such that $A_1 \subset \mathsf{Z}$ is determined as follows. Each natural number $n \in \mathbb{N}$ can be uniquely represented in the form $n = \frac{k(k+1)}{2} + r$ for some $k > r \geqslant 0$. Define a map $f : \mathsf{Z} \longrightarrow \mathbb{N}$ by

$$f(z) = \begin{cases} 0, & \text{if } z = 0; \\ k, & \text{if } |z| = \frac{k(k+1)}{2} + r, k > r \geqslant 0. \end{cases}$$

Then $z \in \mathsf{Z}$ belongs to A_1 if and only if $f(z)$ is even.

Theorem 1. *The group generated by transformations determined by the automaton $\mathcal{A}$ in its states q_1 and q_2 is a free group of rank two.*

The proof of Theorem 1 is based on the dual automaton approach [2]. The described construction can be generalized.

1. Andriy Oliynyk, Vitaly Sushchanskiy, The Groups of ZC-Automaton Transformations. Siberian Mathematical Journal. vol. 51, no. 5, pp. 879 - 891, 2010.
2. Maria Vorobets, Yaroslav Vorobets, On a Free Group of Transformations Defined by an Automaton. Geometriae Dedicata. vol. 124, no. 1, pp. 237-249, 2007.

Conjugacy in finite state wreath powers of finite permutation groups

A.V. Russyev

Kyiv Mohyla Academy, Kyiv, Ukraine
andrey.russev@gmail.com

Let A be a finite set of cardinality $m \geqslant 2$. Consider a finite group G acting faithfully on the set A. In other words, the permutation group (G, A) is a subgroup of the symmetric group $Sym(A)$. In the sequel we assume that the groups act on the sets from the right and denote by a^g the result of the action of a group element g on a point a.

Denote by $W^\infty(G, A)$ the infinitely iterated wreath product of (G, A). The group $W^\infty(G, A)$ consists of permutations of the infinite cartesian product X^∞ given by infinite sequences of the form

$$\mathsf{g} = [\, g_1, g_2(x_1), \ldots, g_n(x_1, \ldots, x_{n-1}), \ldots],$$

where $g_1 \in G$ and $g_n(x_1, \ldots, x_{n-1}) : A^{n-1} \to G$, $n \geqslant 2$. An element g acts on a point

$$\bar{a} = (a_1, a_2, \ldots, a_n, \ldots) \in A^\infty$$

by the rule

$$\bar{a}^{\mathsf{g}} = (a_1^{g_1}, a_2^{g_2(a_1)}, \ldots, a_n^{g_n(a_1,\ldots,a_{n-1})}, \ldots).$$

Let $\mathsf{g} = [\, g_1, g_2(x_1), g_3(x_1, x_2), \ldots] \in W^\infty(G, A)$ and $\bar{a} = (a_1, \ldots, a_n) \in A^n$ for some $n \geqslant 1$. Define an element $\mathsf{rest}(\mathsf{g}, \bar{a}) \in W^\infty(G, A)$ as

$$\mathsf{rest}(\mathsf{g}, \bar{a}) = [\, h_1, h_2(x_1), h_3(x_1, x_2), \ldots],$$

where $h_k(x_1, \ldots, x_{k-1}) = g_{n+k}(a_1, \ldots, a_n, x_1, \ldots, x_{k-1})$, $k \geqslant 1$. The element $\mathsf{rest}(\mathsf{g}, \bar{a})$ is called the state of g at $\bar{a}$. Also we consider g as a state of itself.

Define the set $\mathcal{Q}(\mathsf{g}) = \{\mathsf{rest}(\mathsf{g}, \bar{a}) : \bar{a} \in A^n, n \geqslant 1\} \cup \{\mathsf{g}\}$ of all states of g. Let

$$FW^\infty(G, A) = \{\mathsf{g} \in W^\infty(G, A) : |\mathcal{Q}(\mathsf{g})| < \infty\}.$$

This set forms a subgroup of $W^\infty(G, A)$ that is called the finite state wreath power of the permutation group (G, A).

Theorem 2. *Arbitrary elements of finite order of the group $FW^\infty(G, A)$, conjugated in the group $W^\infty(G, A)$, are conjugated in the group $FW^\infty(G, A)$ as well.*

Radical subrings in the field of Q of rational numbers

Yu.M. Ryabuchin, L.M. Ryabuchina

Dnestruniversity, Tiraspol, Moldova
loretar@mail.ru

We will show that nonzero radical subrings it is possible precisely and all fully to describe (we will notice that all of them do not contain 1).

Theorem 1. *Let S be the subring in Q, maximal among all which do not contain* 1*. Then there is an only thing simple p,$p \geq 2$, such that* $S = \dfrac{mp}{1-np}$

Theorem 2. *For a nonzero radical ring R there is a unique natural number $n \geq 2$ such that* $R = R_n = \dfrac{mn}{1-kn}$, *$m, k$ from Z.*

For the proof of the theorems we will apply the construction of "the attached multiplication" investigated by V.A. Andrunakiyevich [2] and results from works [1,3].

1. A.S.Gotseridze, G.N.Ermakova. The description maximum subrings without 1 of a field of rational numbers ,Materials IV of the International scientific and practical conference: "Improvement of mathematical education in the organizations of the general education, averages and the highest professional institutions", Publishing house of Dnestruniversity, Tiraspol, 2006,p.328-330.
2. V.A.Andrunakiyevich. Semi-radical rings., M.1948,p.129-178.
3. Yu.M.Ryabukhin. The structural theory and the theory of radicals -basic researches. Academicianul Vladimir Andrunachievici., Kishinev,2009,p.149-177.

Zero divisors in the semigroups of infinite dimensional special triangular matrices

Olena Ryabukho

Donbass State Pedagogical University
rom.olenaa@gmail.com

An infinite right and down matrix a_{ij} is called upper triangular matrix, if $a_{ij} = 0$, $i < j$. The sequence of elements a_{ii} is called the main diagonal of this matrix. The semigroups and algebras of infinite matrices including triangular over fields were studied in the works by A.K. Sushkevych [1]–[4]. One of the aspects of the study was to investigate left and right zero divisors in such semigroups. The aim of this report is the transference of some results by A.K. Sushkevych to the semigroups of infinite triangular matrices over commutative rings.

Infinite triangular matrix A over the commutative ring R is called special if some (possibly all) of the elements of the main diagonal are zero divisors in R. In the semigroup $T_0(R)$ of all special infinite triangular matrices over the ring R are defined such subsemigroups:

– $T_{fin}(R)$ matrices in which only finite number of diagonal elements are zero divisors;

– $T_{inf}(R)$ matrices in which only finite number of diagonal elements are not zero divisors;

– $T_{bin}(R)$ matrices in which the set of zero divisors and non-zero divisors on the main diagonal are.

Theorem 1. *1. Each element $T_0(R)$ is the right divisor of zero.*

2. Each element $T_{fin}(R)$ end $T_{inf}(R)$ is the left divisor of zero.

3. The semigroup $T_{bin}(R)$ may contain the elements which are not the left zero divisors.

An example of such infinite triangular matrix over the ring of integers is given by A.K. Sushkevich.

1. Sushkevich A.K. Generalized groups of special matrices. Notes of Scientific and Research Institute of Mathematics and Mechanics and Kharkov Mathematics Society. Chapter 4. (1939), t.16.
2. Sushkevich A.K. On the Problem of Some types of Groups of Infinite Matrices. The same Notes. (1948), t.19.
3. Sushkevich A.K. On the Type of Algebras of Infinite Matrices. The same Notes. (1949), t.21.
4. Sushkevich A.K. On Infinite Algebra of Triangular Maatrices. The same Notes. (1950), t.22.

Linear groups with a spacious family of G-invariant subspaces

A.V. Sadovnichenko

Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
sadovnichenko.lit@rambler.ru

Let F be a field, A a vector space over F and $\mathbf{GL}(F, A)$ a group of all F-automorphisms of A. If G is a subgroup of $\mathbf{GL}(F, A)$ then a subspace B of A is called G-invariant, if $bx \in B$ for every $b \in B$ and every $x \in G$. If B is a subspace of A, then B has the largest G-invariant subspace $\mathrm{Core}_G(B)$, which called the *G-core of B*. A subspace B is called almost G-invariant, if $\dim_F(B/\mathrm{Core}_G(B))$ is finite.

We consider a specific approach in studying of infinite dimensional groups. This approach is based on the notion of invariance of action of a group G. The study of infinite dimensional linear groups having very big family of G-invariant subspaces could be fruitful [1, 2].

The next result here is following:

Theorem 1. *Let G be a subgroup of $\mathbf{GL}(F, A)$. Suppose that every subspace of A is almost G-invariant. Then A includes an FG-submodule C satisfying the following conditions:*

(i) $\dim_F(A/C)$ is finite;

(ii) every subspace of C is G-invariant.

1. Kurdachenko L.A, Sadovnichenko A.V., Subbotin I.Ya. On some infinite dimensional groups. Central European Journal of Mathematics, **7**, no. 2, (2009), P. 178-185.
2. Kurdachenko L.A, Sadovnichenko A.V, Subbotin I. Ya. Infinite dimensional linear groups with a large family of G-invariant subspaces. Commentationes Mathematicae Universitatis Carolinae, **51**, no. 4, (2010), P. 551-558

On totally ω-composition formations with limited soluble c_{∞}^{ω}-defect

I.N. Safonova

International Sakharov Environmental University
in.safonova@mail.ru

All groups considered are finite. We use terminology and notations from [1].

For any class of groups $\mathfrak{X}$, $c_{\infty}^{\omega}\mathrm{form}\mathfrak{X}$ denotes the totally ω-composition formation generated by the class of groups $\mathfrak{X}$, i.e. $c_{\infty}^{\omega}\mathrm{form}\mathfrak{X}$ is the intersection of all totally ω-composition formations containing $\mathfrak{X}$. For every totally ω-composition formations $\mathfrak{M}$ and $\mathfrak{H}$, we set $\mathfrak{M} \vee_{\infty}^{\omega} \mathfrak{H} = c_{\infty}^{\omega}\mathrm{form}(\mathfrak{M} \cup \mathfrak{H})$. With respect to the operations $\vee_{\infty}^{\omega}$ and $\cap$ the set c_{∞}^{ω} of totally ω-composition formations forms a modular lattice. Let $\mathfrak{S}$ be the formation of soluble groups. The length of the lattice $\mathfrak{F}/_{\infty}^{\omega}\mathfrak{F} \cap \mathfrak{S}$ of totally ω-composition formations $\mathfrak{X}$ with $\mathfrak{F} \cap \mathfrak{S} \subseteq \mathfrak{X} \subseteq \mathfrak{F}$ is called *an* $\mathfrak{S}_{\infty}^{\omega}$-*defect* (or *a soluble* c_{∞}^{ω}-*defect*) of the totally ω-composition formation $\mathfrak{F}$. A totally ω-composition formation $\mathfrak{F}$ is called $\mathfrak{S}_{\infty}^{\omega}$-*critical* (or *a minimal totally* ω-*composition non-soluble formation*) if $\mathfrak{F} \not\subseteq \mathfrak{H}$ but all proper totally ω-composition subformations of $\mathfrak{F}$ are contained in $\mathfrak{S}$. Let $\{\mathfrak{F}_i | i \in I\}$ be the set of all proper totally ω-composition subformations of $\mathfrak{F}$, $\mathfrak{X} = c_{\infty}^{\omega}\mathrm{form}(\cup_{i \in I}\mathfrak{F}_i)$. Then $\mathfrak{F}$ is called *an irreducible* totally ω-composition formation if $\mathfrak{F} \neq \mathfrak{X}$ and $\mathfrak{F}$ is called *a reducible* totally ω-composition formation if $\mathfrak{F} = \mathfrak{X}$.

Theorem 1. *Let $\mathfrak{F}$ be a reducible totally ω-composition formation. Then a soluble c_{∞}^{ω}-defect of $\mathfrak{F}$ equals k if and only if one of the following statements is satisfied:*

1) $\mathfrak{F} = \mathfrak{H} \vee_{\infty}^{\omega} \mathfrak{M}$ where $\mathfrak{H}$ is an irreducible totally ω-composition formation of soluble c_{∞}^{ω}-defect t, $1 \leqslant t \leqslant k-1$ and $\mathfrak{M}$ is a totally ω-composition formation of soluble c_{∞}^{ω}-defect $k-1$ such that $\mathfrak{M} \cap \mathfrak{H}$ is a maximal totally ω-composition subformation of $\mathfrak{H}$;

2) $\mathfrak{F} = \mathfrak{H} \vee_{\infty}^{\omega} \mathfrak{M}$ where $\mathfrak{H}$ is an irreducible totally ω-composition formation of soluble c_{∞}^{ω}-defect k, $\mathfrak{M}$ is a soluble totally ω-composition formation and $\mathfrak{M} \not\subseteq \mathfrak{H}$.

Corollary 1. *Let $\mathfrak{F}$ be a non-soluble totally ω-composition formation. Then a soluble c_{∞}^{ω}-defect of $\mathfrak{F}$ equals 1 if and only if $\mathfrak{F} = \mathfrak{H} \vee_{\infty}^{\omega} \mathfrak{M}$, where $\mathfrak{H}$ is a minimal totally ω-composition non-soluble formation, $\mathfrak{M}$ is a soluble totally ω-composition formation.*

In particular, any soluble totally ω-composition subformation of $\mathfrak{F}$ is contained in $\mathfrak{M} \vee_{\infty}^{\omega} (\mathfrak{H} \cap \mathfrak{S})$ and if $\mathfrak{F}_1$ is a non-soluble totally ω-composition subformation of $\mathfrak{F}$ then $\mathfrak{F}_1 = \mathfrak{H} \vee_{\infty}^{\omega} (\mathfrak{F}_1 \cap \mathfrak{S})$.

Corollary 2. *Let $\mathfrak{F}$ be a reducible totally ω-composition formation. Then a soluble c_{∞}^{ω}-defect of $\mathfrak{F}$ equals 2 if and only if one of the following statements is satisfied:*

1) $\mathfrak{F} = \mathfrak{H}_1 \vee_{\infty}^{\omega} \mathfrak{H}_2 \vee_{\infty}^{\omega} \mathfrak{M}$, where $\mathfrak{H}_i$ is a minimal totally ω-composition non-soluble formation $(i = 1, 2)$, $\mathfrak{H}_1 \neq \mathfrak{H}_2$, $\mathfrak{M} \subseteq \mathfrak{S}$;

2) $\mathfrak{F} = \mathfrak{H} \vee_{\infty}^{\omega} \mathfrak{M}$, where $\mathfrak{H}$ is an irreducible totally ω-composition formation of soluble c_{∞}^{ω}-defect 2, $\mathfrak{M} \subseteq \mathfrak{S}$, and $\mathfrak{M} \not\subseteq \mathfrak{H}$.

1. A.N.Skiba, L.A.Shemetkov Multiply L-composition formations of finite groups. Ukr. math. zh., v. 52, N 6, 2000, p. 783-797.

On the unit group of a commutative group ring

Mohamed A. Salim

UAE University - Al-Ain, United Arab Emirates
MSalim@uaeu.ac.ae

Let $V(\mathbb{Z}_{p^e}G)$ be the group of normalized units of the group algebra $\mathbb{Z}_{p^e}G$ of a finite abelian p-group G over the ring $\mathbb{Z}_{p^e}$ of residues modulo p^e with $e \geq 1$. The abelian p-group $V(\mathbb{Z}_{p^e}G)$ and the ring $\mathbb{Z}_{p^e}G$ are applicable in coding theory, cryptography and threshold logic (see [1, 4, 5, 7]).

In the case when $e = 1$, the structure of $V(\mathbb{Z}_pG)$ has been studied by several authors (see the survey [2]). The invariants and the basis of $V(\mathbb{Z}_pG)$ has been given by B. Sandling (see [6]). In general, $V(\mathbb{Z}_{p^e}G) = 1 + \omega(\mathbb{Z}_{p^e}G)$, where $\omega(\mathbb{Z}_{p^e}G)$ is the augmentation ideal of $\mathbb{Z}_{p^e}G$. Clearly, if $z \in \omega(\mathbb{Z}_{p^e}G)$ and $c \in G$ is of order p, then $c + p^{e-1}z$ is a nontrivial unit of order p in $\mathbb{Z}_{p^e}G$. We may raise the question whether the converse is true, namely does every $u \in V(\mathbb{Z}_{p^e}G)$ of order p have the form $u = c + p^{e-1}z$, where $z \in \omega(\mathbb{Z}_{p^e}G)$ and $c \in G$ of order p?

We obtained a positive answer to this question and applied it for the description of the group $V(\mathbb{Z}_{p^e}G)$ (see [3]). Our research can be considered as a natural continuation of Sandling's results.

In general, when $e \geq 2$, the structure of the abelian p-group $V(\mathbb{Z}_{p^e}G)$ is still not well studied.

1. N.N. Aizenberg, A.A. Bovdi, E.I. Gergo, and F.E. Geche. Algebraic aspects of threshold logic. *Cybernetics*, 2:26–30, 1980.
2. A.A. Bovdi. The group of units of a group algebra of characteristic p. *Publ. Math. Debrecen*, 52(1-2):193–244, 1998.
3. V.A. Bovdi, M.A. Salim. On the unit group of a commutative group ring. *Submitted for publication*, p.1–10, 2012.
4. B. Hurley and T. Hurley. Group ring cryptography. *Int. J. Pure Appl. Math.*, 69(1):67–86, 2011.
5. T. Hurley. Convolutional codes from units in matrix and group rings. *Int. J. Pure Appl. Math.*, 50(3):431–463, 2009.
6. R. Sandling. Units in the modular group algebra of a finite abelian p-group. *J. Pure Appl. Algebra*, 33(3):337–346, 1984.
7. W. Willems. A note on self-dual group codes. *IEEE Trans. Inform. Theory*, 48(12):3107–3109, 2002.

Topological generation of just-infinite branch groups

I.O. Samoilovych

National Taras Shevchenko University of Kyiv, Kyiv, Ukraine
samoil449@gmail.com

A profinite group is just-infinite if all nontrivial closed normal subgroups have finite index. Every profinite just-infinite group is either a branch group or it contains an open normal subgroup which is isomorphic to the direct product of a finite number of copies of some hereditarily just-infinite profinite group [1]. Lucchini constructed in [2] examples of finitely generated, but not positively finitely generated just-infinite groups, answering to the question of Pyber. All these examples are hereditarily just-infinite [3]. I construct just-infinite branch groups with the above properties as an infinitely iterated permutational wreath product of powers of simple groups. Let $\{s_i\}_{i\geq 1}$ and $\{k_i\}_{i\geq 1}$ be two integer sequences. Define the group

$$W = \varprojlim G_i \wr \ldots \wr G_1, \qquad G_i = (Alt(s_i))^{k_i}.$$

Theorem 1. *There exist sequences $\{s_i\}_{i\geq 1}$ and $\{k_i\}_{i\geq 1}$ such that the group W is a finitely generated, but not positively finitely generated, just-infinite branch group.*

1. Rostislav I. Grigorchuk (2000). Just infinite branch groups. In New Horizons in pro-p Groups. Prog. Math., Vol. 184, 121–179. ISBN 0-8176-4171-8.
2. Andrea Lucchini (2004). A 2-generated just-infinite profinite group which is not positively generated. Isr. J. Math., Vol. 141, 119–123.
3. Colin Reid (2011). On a construction of A. Lucchini. Preprint, avilable at http://arxiv.org/abs/1106.4423.

On the average order of some function in an arithmetic progression

O. Savastru

Odessa I.I.Mechnikov National University
sav_olga@bk.ru

Let $A_k(x)$ denote the number of integer solutions of equation $u^2+v^2=n^k$, $n \leqslant x$, $k \geqslant 2$. Then

$$A_k(x) = \sum_{n \leqslant x} r(n^k),$$

where $r(n)$ is the number of ways to write the positive integer n as a sum of two squares. The asymptotic formula for $A_k(x)$ is obtained in [1]. Fischer K. [2], Recknagel W. [4], Kühleitner M. and Nowak W. [3] investigated the case, when $k=3$.

Let $A_3(x,l,q)$ denote the number of integer triples (u,v,n) on the circle cone $u^2+v^2=n^3$, $n \leqslant x$, $n \equiv l \pmod q$, where $l,q \in \mathbb{N}$.

We infer

Theorem 1. *Let $l,q \in \mathbb{N}$, $0 < l \leqslant q$, $(l,q)=1$. Then for large real x*

$$A_3(x,l,q) = A_1(q)x\log x + A_0(q)x + O\left(x^{\frac{1}{2}}\tau(q)\log^5 x\right)$$

where $A_1(q), A_0(q)$ are the computable constants, which depend from q.

1. Бабаев Г., Распределение целых точек на алгебраических поверхностях, Душанбе,1966.
2. Fischer K.H., Über die Anzahl der Gitterpunkte auf Kreisen mit quadratfreien Radienquadraten. Arch.Math. 33 (1979),150-154.
3. Kühleitner M., Nowak W., The average number of solution of the diophantic equation $u^2+v^2=w^3$ and related arithmetic functions. Acta Math. Hung. 104 (2004),225-240.
4. Recknagel W. Varianten des Gaußschen Kreisproblems. Abh. Math. Sem. Univ. Hamburg 59 (1989),183-189.

Topological, Metric and Fractal Properties of the Set of Incomplete Sums of One Class Positive Series

I. Savchenko

National Pedagogical Dragomanov University
igorsav4enko@rambler.ru

We consider the numerical serie

$$s = \sum_{n=1}^{\infty} a_n,$$

which satisfies the following conditions:

1) $a_n + a_{n+1} = \lambda \cdot r_n \Leftrightarrow a_{n+2} = \frac{a_n}{1+\lambda}$, where $0 < \lambda \in \mathbb{R}$, $r_{n+1} = \sum\limits_{k=n+1}^{\infty} a_n$;

2) $0 < a_{n+1} \leq a_n \Rightarrow t \leq \frac{a_2}{a_1} < 1$, де $t = \frac{1}{1+\lambda}$.

Definition 1. The set

$$A_s = \left\{ x : x = \sum_{n=1}^{\infty} x_n \cdot a_n,\ x_n \in \{0,1\} \right\}.$$

is called the set of incomplete sums of the serie (1).

We study the topological, metric and fractal properties of the set A_s.

Theorem 1. *The set A_s has the following properties:*

1) if $t \in \left(0, \frac{1}{4}\right)$, then it is a nowhere dense, is of zero Lebesgue measure;

2) if $t \in \left[\max\{\frac{a_1-a_2}{2a_1}, \frac{a_2}{a_1+2a_2}\},\ 1\right)$, then it is a segment $\left[0, \frac{a_1+a_2}{1-t}\right]$.

3) if $t \in \left(0,\ \min\{\frac{a_1-a_2}{2a_1}, \frac{a_2}{a_1+2a_2}\}\right]$, then Hausdorff-Besicovitch dimension is equal to

$$\alpha_0(A_s) = -\log_t 4.$$

Theorem 2. *If $\frac{a_1}{2} < a_2 < a_1$ and $\frac{1}{4} \leq t = \frac{a_1-a_2}{a_1+a_2} < \max\{\frac{a_1-a_2}{2a_1}, \frac{a_2}{a_1+2a_2}\}$, then A_s is a nowhere dense set, is of zero Lebesgue measure, Hausdorff-Besicovitch dimension is equal to $\alpha_0(A_s) = -\log_t 4$.*

Theorem 3. *If $\frac{a_1}{2} < a_2 < a_1$ and $\frac{1}{4} \leq t = \frac{a_2}{a_1} < \max\{\frac{a_1-a_2}{2a_1}, \frac{a_2}{a_1+2a_2}\}$, then A_s is a nowhere dense set, is of zero Lebesgue measure.*

1. M. V. Pratsiovytyi, Fractal approach to investigations of singular probability distributions, National Pedagogical Univ., Kyiv, 1998, 296 p.

On the maximality by strong containment for Fitting classes of partially soluble groups

N.V. Savelyeva

Brest State University named after A.S. Pushkin
natallia.savelyeva@gmail.com

All groups considered are finite. In definitions and notation we follow [1].

Recall that a normally hereditary class of groups $\mathfrak{X}$ is called a Fitting class if it is closed under the products of normal $\mathfrak{X}$-subgroups. An $\mathfrak{X}$-injector of a group G is a subgroup V of G with the property that $V \cap K$ is an $\mathfrak{X}$-maximal subgroup of K for all subnormal subgroups K of G. If $\mathfrak{X} \neq \emptyset$ then a unique maximal normal $\mathfrak{X}$-subgroup of an arbitrary group G is called its $\mathfrak{X}$-radical and denoted by $G_{\mathfrak{X}}$.

For a Fitting class $\mathfrak{X}$ let $\pi(\mathfrak{X})$ be a set of all prime divisors of all $\mathfrak{X}$-groups, and let $\mathfrak{S}$ ($\mathfrak{S}^{\pi(\mathfrak{X})}$) stand for the class of all soluble ($\pi(\mathfrak{X})$-soluble) groups. Then $\mathfrak{X}\mathfrak{S}$ ($\mathfrak{X}\mathfrak{S}^{\pi(\mathfrak{X})}$) is the class of all groups G such that quotient groups $G/G_{\mathfrak{X}}$ are soluble ($\pi(\mathfrak{X})$-soluble). Note that, the existence and conjugacy of injectors of soluble groups established in [2] was later extended for $\mathfrak{X}\mathfrak{S}$- and $\mathfrak{X}\mathfrak{S}^{\pi(\mathfrak{X})}$-groups (see the main result of [3] and the theorem 2.5.3 in [4] correspondingly). Recall that a Fitting class $\mathfrak{X}$ is called:

(i) strongly contained in a Fitting class $\mathfrak{Y}$ (this is denoted by $\mathfrak{X} \ll \mathfrak{Y}$), if in each group G an $\mathfrak{X}$-injector of G is contained in a $\mathfrak{Y}$-injector of G;

(ii) maximal by strong containment in a Fitting class $\mathfrak{Y}$ (this is denoted by $\mathfrak{X} \ll \cdot\mathfrak{Y}$), if $\mathfrak{X} \ll \mathfrak{Y}$ and the condition $\mathfrak{X} \underset{\neq}{\overset{\ll}{}} \mathfrak{M} \underset{\neq}{\overset{\ll}{}} \mathfrak{Y}$, where $\mathfrak{M}$ is a Fitting class, always implies $\mathfrak{M} \in \{\mathfrak{X}, \mathfrak{Y}\}$.

In the class $\mathfrak{S}$ of all soluble groups it was established [5] that a Fitting class $\mathfrak{X}$ is maximal by inclusion among the Fitting subclasses of a Fitting class $\mathfrak{Y}$ if for all $\mathfrak{Y}$-groups G there exists a prime p such that $\mathfrak{X}$-injectors of G have index 1 or p.

The mentioned results [3] and [4] allow to set a similar result for strongly contained Fitting classes of partially soluble groups. In particular, it is proved

Theorem 1. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be Fitting classes, $\mathfrak{X} \ll \mathfrak{Y}$ and $\mathfrak{Y} \subseteq \mathfrak{X}\mathfrak{S}$ ($\mathfrak{Y} \subseteq \mathfrak{X}\mathfrak{S}^{\pi(\mathfrak{X})}$). If there exists a prime p ($p \in \pi(\mathfrak{X})$) such that in every group $G \in \mathfrak{Y}$ its $\mathfrak{X}$-injector has index 1 or p, then $\mathfrak{X}$ is maximal by strong containment in $\mathfrak{Y}$.*

1. Doerk K., Hawkes T. Finite soluble groups. Berlin–New York, Walter de Gruyter, 1992. – 891 p.
2. Fischer, B. Injektoren endlicher auflösbarer Gruppen / B. Fischer, W. Gaschütz, B. Hartley // Math. Z. – 1967. – Bd. 102, No. 5. – S. 337-339.
3. Sementovskiy V.G. Injectors of finite groups // Investigation of of normal and subgroup structure of finite groups. – Minsk, Nauka i tehnika, 1984. – P. 166-170 (in Russian).
4. Guo, W. The Theory of Classes of Groups / W. Guo. – Sc. Press Kluwer Acad. Public, 2000. – 258 p.
5. Bryce, R. A. Maximal Fitting classes of finite soluble groups / R. A. Bryce, J. Cossey // Bull. Austral. Math. Soc. – 1974. – Vol. 10. – P. 169-175.

Injectors for Fitting sets

M.G. Semenov, N.T. Vorob'ev

P.M. Masherov Vitebsk State University
mg-semenow@mail.ru

All groups considered are finite. All terminology and notations are standard and follows [1] and [2].

Fischer, Gaschütz, Hartley [3] proved that any solvable group G possesses exactly one conjugacy class of $\mathfrak{F}$-injectors for any Fitting class $\mathfrak{F}$. Extension of the above result [3] for a Fitting set of partially soluble group was first obtained by L.A. Shemetkov [4] (for solvable case see also [5]). We have found a new unique conjugacy of $\mathscr{F}$-injectors in an arbitrary π-solvable group G for any π-saturated Fitting set $\mathscr{F}$.

We say that Fitting set $\mathscr{F}$ of group G is *π-saturated* if it verifies $H \in \mathscr{F}$ whenever $O^{\pi'}(H) \in \mathscr{F}$ for every subgroup H of group G. Group $O^{\pi'}(H)$ represents the smallest normal subgroup N of H such that H/N is a π'-group.

Theorem 1. *Let G be a π-solvable group and $\mathscr{F}$ be a π-saturated Fitting set of G. Then G has unique conjugacy of $\mathscr{F}$-injectors.*

Group G is said [2] to be: *π-closed* if G contains normal Hall π-subgroup; *π-special* if G contains normal nilpotent Hall π-subgroup.

Corollary 1. *Any π-solvable group possesses exactly one conjugacy class of π-closed injectors.*

Corollary 2. *Any π-solvable group G possesses π-special injectors which are conjugate in G.*

Corollary 3. *[5, 6] Let $\mathscr{F}$ be a Fitting set of solvable group G. Then G has unique conjugacy of $\mathscr{F}$-injectors.*

Corollary 4. *[3] Let $\mathfrak{F}$ be a Fitting class and G be a solvable group. Then G has unique conjugacy of $\mathfrak{F}$-injectors.*

1. K. Doerk, T. Hawkes, Finite Soluble Groups, Walter De Gruyter, Berlin–New York, 1992.
2. Л. А. Шеметков, Формации конечных групп, Наука, М., 1978.
3. B. Fischer, W. Gaschütz, B. Hartley, "Injektoren endlicher auflosbarer Gruppen", Math. Z., 102 (1967), 337–339.
4. Л. А. Шеметков, "О подгруппах π-разрешимых групп", Конечные группы, 1975, 207–212.
5. W. Anderson, "Injectors in finite solvable groups", Journal of algebra, 36(1975), 333–338.
6. I. Hawthorn, "The existence and uniqueness of injectors for fitting sets of solvable groups", Proc. Amer. Math. Soc., 126 (1998), 2229–2230.

Fibonacci graceful labeling of some special class of graphs

M.F. Semenyuta, D.A. Petreniuk

Kirovograd Flight Academy of National Aviation University, Kirovograd, Ukraine
V.M. Glushkov Institute of Cybernetics of NAS of Ukraine
marina_semenyuta@mail.ru, petrenewk@rambler.ru

The notion of graceful labeling was first introduced by Alexander Rosa in 1967. A graph $G = (V, E)$ with $|V| = p$ and $|E| = q$ is graceful, if there exists a vertex labeling f by distinct integers from the set {0, 1, 2,..., q}, which generates an edge labeling $f^*(u, v) = |f(u) - f(v)|$, where u, v are vertexes of G, so that all the edges have distinct labels. A natural development of the concept of graph $G = (V, E)$ gracefulness can be considered, which is the case when the edge labeling f^* is a bijection from the edge set to the first q numbers of an arbitrary sequence $\{a_i\}$. In [1] sequence $\{a_i\}$ consists of Fibonacci numbers. In this paper we use the definition of Fibonacci graceful labeling that has been introduced in [2]. Function f is called Fibonacci graceful labeling of graph G with q edges, if f is injection from the set $V(G)$ to the set $\{0, 1, 2, \ldots, F_q\}$ (where Fq is the q^{th} Fibonacci number), and f induces an edge labeling $f^*(u, v) = |f(u) - f(v)|$ which is a bijection from the set $E(G)$ to the set $\{F_1, F_2, \ldots, F_q\}$. Graph that has a Fibonacci graceful labeling is called Fibonacci graceful graph. The general problem of characterization of all Fibonacci graceful graphs that was stated in [2] is still open. It is known that all trees are Fibonacci graceful graphs. Kathiresen and Amutha [3] have proved that K_n is Fibonacci graceful if and only if $n \leq 3$; if G is Eulerian and Fibonacci graceful then $q \equiv 0(mod 3)$; every path P_n of length n is Fibonacci graceful; P_n^2 is a Fibonacci graceful graph; caterpillars are Fibonacci graceful. One possible way to characterize Fibonacci graceful graphs is to find a complete graph list, such that G is Fibonacci graceful if and only if G does not contain a subgraph isomorphic to any graph from that list. This approach seems to be complicated since gracefulness is not a particular characteristic, but a global one. Nevertheless, the following theorem imposes restrictions on the class of Fibonacci graceful graphs.

Theorem 1. *If every edge of graph $G = (V, E)$ belongs to any two simple cycles that have lengths more than two each and do not have any other common elements except for the edge mentioned, then graph G is not Fibonacci graceful.*

Corollary 1. *For any non-negative integers $m \geq 3$ and $n \geq 3$ graphs $C_m \times C_n$, $C_m[C_n]$, $P_m[P_n]$, $C_m[P_n]$, and $P_m[C_n]$ are not Fibonacci graceful.*

1. Koh K. M., Lee D. G., Tan T. Fibonacci trees// SEA bull.math.-1978-Vol.2, pp. 45-47.
2. Bange D. W., Barkauskas A. E. Fibonacci graceful graphs// Fibonacci quarterly.-1983-Vol. 21, № 3, pp. 174-188
3. Sridevi R., Navaneethakrishnan S., Nagarajan K. Super Fibonacci graceful labeling of some special class of graphs.// International J. Math. Combin.-2011-Vol.1, pp. 59-72.

Multiplicative functions over $\mathbb{Z}[i]$ weighted by Kloosterman sums modules

S. Sergeev

Odessa I. I. Mechnikov National University
sorathss@gmail.com

Let $\mathbb{Q}(i)$ be the field of Gaussian numbers. $\mathbb{Q}(i) = \left\{a + bi | a, b \in \mathbb{Q}, i^2 = -1\right\}$. By G we denote ring of Gaussian integers, $G = \{a + bi | a, b \in \mathbb{Z}\}$. Consider Kloosterman norm sum

$$\tilde{K}(\alpha; \beta; q) = \sum_{x,y \in G} e^{2\pi i Re\left(\frac{\alpha x + \beta y}{q}\right)}$$

where $\alpha, \beta \in G, q \in \mathbb{N}, q > 1$.

In [2] was obtained non-trivial estimate on $K(\alpha; \beta; q)$.

Our objective is to obtain an asymptotic formula for the sums of Kloosterman sums, weighted by multiplicative function of special kind over G:

$$\sum_{N(\omega) \leq x} f(\omega) \tilde{K}(1, \omega)$$

where $f(\omega)$ is associated with Z-function of order 2 in terms of Iwaniec-Kowalski ([1], §5.1) over ring G.

We obtain asymptotic formula for functions $f(\omega)$ that are Dirichlet convolution of completely multiplicative functions $g_1(\omega), g_2(\omega)$, where $|g_1(\omega)| = |g_2(\omega)| = 1$

1. H. Iwaniec and E. Kovalski. Analytic number theory, Amer. Math Soc. Colloquium Publ., 53, 2004
2. S. Varbanets, The norm Kloosterman sums over $\mathbb{Z}[i]$, Anal. Probab. Methods Number Theory, (ed. A. Larrincicas, E. Manstavicins), Vilnins, 2007, 225-239.

Absolute Extensors in Asymptotic Category

Yu. Shatskyi

Ivan Franko National University of Lviv, Lviv, Ukraine
yurashac@gmail.com

The notion of absolute extension in asymptotic category is considered. It is provided the condition, execution of which shows that the metric space is an absolute extensor in this category.

Definition 1. [2] The **asymptotic category** $\mathcal{A}$ is a category in which objects are metric spaces and morphisms are proper asymptotically Lipschitz maps.

Definition 2. [1] An object Y of a category $\mathcal{A}$ is its **absolute extensor,** $Y \in AE(\mathcal{A})$, if for any object X in $\mathcal{A}$, its subobject (A, i) and morphism $f : A \to Y$ there exists a proper asymptotically Lipschitz map $\overline{f} : X \to Y$ such that $\overline{f} \circ i = f$.

The following theorem help us to prove the main result of the paper.

Theorem 1. [2] *$\mathbb{R}^n{}_+ \in AE(\mathcal{A})$ and $\mathbb{R}^n_+ \in AE(\widetilde{\mathcal{A}})$ for all n.*

Here is the condition of the absolute extension in asymptotic category.

Theorem 2. *Let we have the metric space $(\mathbf{T}^{n+1}, \widehat{d})$,*

$$\mathbf{T}^{n+1} = \{(x_1, \ldots, x_,y) |\ y \geq \sum_{i=1}^{n} t(x_i)\},$$

where function t is continuous, positive definite, monotonic, and even.

It is an absolute extensor in the category $\mathcal{A}$ if for all $x, x' \in \mathbb{R}^{n+1}_+$ the following condition holds:
there is a constant s such that

$$\max_{i=\overline{1,n}}\{|\mathrm{sgn}(x_i)t^{-1}(|x_i|) - \mathrm{sgn}(x'_i)t^{-1}(|x'_i|), \max_{i=\overline{1,n}}\{|x_i - x'_i|\}\} \leq \max_{i=\overline{1,n}}\{|x_i - x'_i|, s\}. \qquad 1$$

Example 1. Let we have the metric space $(\mathbf{P}^{n+1}, \widehat{d})$,

$$\{\mathbf{P}^{n+1} = \{(x_1, \ldots, x_,y) |\ y \geq \sum_{i=1}^{n} x_i^2\}.$$

For all $x, x' \in \mathbb{R}^{n+1}_+,\ x = (x_1, \ldots, x_n, y),\ x' = (x'_1, \ldots, x'_n, y')$,

$$\max_{i=\overline{1,n}}\{|\mathrm{sgn}(x_i)\sqrt{|x_i|} - \mathrm{sgn}(x'_i)\sqrt{|x'_i|}\ |, \max_{i=\overline{1,n}}\{|x_i - x'_i|\}\} \leq \max_{i=\overline{1,n}}\{|x_i - x'_i|, 1\}$$

Thus, $s = 1$, condition (1) and the Theorem 2 hold for $\mathbf{P}^{n+1}$. That is why $\mathbf{P}^{n+1} \in AE(\mathcal{A})$.

1. M. Sawicki: *Absolute Extensors And Absolute Neighborhood Extensors In Asymptotic Categories*
2. A. N. Dranishnikov: *Asymptotic Topology*, Uspekhi Mat. Nauk, Vol. 55, No 6(336) (2000), 71—116

Reduction of polynomial matrices by semiscalar equivalent transformations

B.Z. Shavarovskii

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics
shavarb@iapmm.lviv.ua

The notion of semiscalar equivalence of polynomial matrices is well known [1] (see also [2], [3]). In this work the structure of polynomial matrices in connection with their reducibility by semiscalar equivalent transformations to simple forms is considered. The classes of polynomial matrices are singled out for which canonical forms with respect to the above transformations are indicated. We use this tool to construct a canonical form with respect to simultaneous similarity transformation for the collections of coefficients corresponding to the polynomial matrices This seems to be a rather complicated problem in the general case. The problem on simultaneous similarity of one type of collection of square matrix over the field complex numbers is reduced to the problem on the special quasi-diagonal equivalence of rectangular matrices, corresponding to these collections. It is shown that it is possible to find a collection of square matrices according to the arbitrary matrix from a class of specially quasi-diagonal equivalent matrices, corresponding to it. We try to find (complete) system of invariants with respect to semiscalar equivalence for polynomial matrices with some constraints on the properties of the their Smith forms. In this work a problem of reduction of rather wide classes of polynomial matrices to direct sum of summands has been solved. This enabled to reduce complex problems of semiscalar equivalence and similarity of matrices to analogous problems for matrices of lover dimension.

1. Kazimirs'kii P.S. and Petrychkovych V.M. Equivalence of Polynomial Matrices // In Theoretical and Applied Problems of Algebra and Differential Equations . Kiev: Naukova Dumka, 1977. - P. 61-66 (in Ukrainian).
2. Baratchart L. Un theoreme de factorisation et son application a la representation des systemes cuclique causaux // C. R. Acad. Sci. Paris Ser. 1. Math. - 1982. - 295, No 3. - P. 223-226.
3. Dias da Silva J.A, Laffey T.J. On simultaneous similarity of matrices and rela-ted questions // Linear Algebra Appl. - 1999. - 291. - P. 167-184.

On common properties of Bezout domains and rings of matrices over them

V. Shchedryk

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics NAS of Ukraine
shchedrykv@ukr.net

C.C. MacDuffee [1] has presented the method, due in essence to E. Cahen [2] and A. Chatelet [3], for finding the right greatest common divisor (g.c.d.) of two given matrices with elements in a principal ideal ring. With the rather severe restriction that both matrices are nonsingular, it has been found the left least common multiple (l.c.m.) of this matrices. B.M. Stewart [4] weaker these restriction requiring that the greatest common right divisor is nonsingular. In this note we remove these restrictions and generalize this result to matrices over commutative Bezout domains.

Let R be a commutative Bezout domain i.e. commutative domain in which all finitely generated ideals are principal and $D \in M_n(R)$. Denote by $Ann^r(D)$ the set of right annihilator of the matrix D :

$$Ann^r(D) = \{Q \in M_n(R) | DQ = \mathbf{0}\}.$$

Denote by $(A, B)_l$ and $[A, B]_r$ the left g.c.d. and right l.c.m. of matrices A and B, respectively.

Theorem 1. *Let $A, B \in M_n(R)$. There exists an invertible matrix $\left\| \begin{matrix} U & M \\ V & N \end{matrix} \right\|$ such that*

$$\left\| \begin{matrix} A & B \end{matrix} \right\| \left\| \begin{matrix} U & M \\ V & N \end{matrix} \right\| = \left\| \begin{matrix} D & \mathbf{0} \end{matrix} \right\|,$$

where $D = (A, B)_l$ and $Ann^r(D) \subseteq Ann^r(V)$.

Theorem 2. *The matrix $\left\| \begin{matrix} A & B \\ \mathbf{0} & B \end{matrix} \right\|$ is right associate to the matrix $\left\| \begin{matrix} (A, B)_l & \mathbf{0} \\ * & [A, B]_r \end{matrix} \right\|$.*

Corollary 1. *The product of nonzero diagonal elements of the right Hermite normal form of A and B coincides with the product of nonzero diagonal elements of the right Hermite normal form of $(A, B)_l$ and $[A, B]_r$.*

Corollary 2. $det(AB) = det(A, B)_l det[A, B]_r$.

1. C.C. MacDuffee Matrices with elements in a principal ideal ring Bull. Amer. Math. Soc. 39 (1933), 564-584.
2. E. Cahen Théorie des Nombres, vol. I, 1914.
3. A. Chatelet Groupes Les Groupes Abéliens Finis et les Modules de Points Entiers. Paris and Lille, 1925. 221 pp.
4. B.M. Stewart A note on least common left multiples
   Bull. Amer. Math. Soc. 55 (1949), 587-591.

On Boolean matrix idempotents

O. Shchekaturova

Saratov State University, Saratov, Russia
ot-vna@mail.ru

Let $\mathbf{B}_{\mathbf{1}\times\mathbf{1}}^{\mathbf{n}\times\mathbf{n}}$ be the set of matrices of order from 1×1 to $n\times n$ over an arbitrary Boolean algebra. The dual partial operations of multiplication $\sqcap$ and $\sqcup$ are defined naturally. The $(m\times k)$-matrix $C = A\sqcap B \in \mathbf{B}_{\mathbf{1}\times\mathbf{1}}^{\mathbf{n}\times\mathbf{n}}$ with the elements $C_j^i = \bigcup_{t=1}^n (A_t^i \bigcap B_j^t)$ is called *the conjunctive product* of an $(m\times s)$-matrix $A = (A_j^i)$ and an $(s\times k)$-matrix $B = (B_j^i)$ $(1 \leqslant m, s, k \leqslant n)$. We dually define *the disjoint product* $A\sqcup B$: $A\sqcup B = (A'\sqcap B')'$.

We consider the idempotents of the partial finite matrix semigroups $\mathbf{B}_{\mathbf{1}\times\mathbf{1}}^{\mathbf{n}\times\mathbf{n}}$ under the introduced operations of multiplication $\sqcap$ and $\sqcup$. All of the idempotent matrices are divided into primary and secondary idempotents. We restrict our attention to secondary idempotents. They are obtained by a special procedure, which uses the operations of multiplication, transposition, and closure operation.

It is known [1] that secondary idempotents are concerned with solvability of matrix equations and with Green's relations on partial semigroups of Boolean finite matrices. We prove, that reflexive transitive closure of a square matrix is a secondary idempotent, which is generated by a certain (not necessarily square) Boolean matrix, and vice versa. The divisibility of reflexive transitive closures in partial finite matrix semigroups over an arbitrary Boolean algebra was also considered by Poplavski in [2].

We show the disposition of secondary idempotents (reflexive transitive closures) in relation to two-sided ideals of the partial semigroup $\mathbf{B}_{\mathbf{1}\times\mathbf{1}}^{\mathbf{n}\times\mathbf{n}}$.

1. *Поплавский В.Б.* О приложениях ассоциативности дуальных произведений алгеры булевых матриц / В.Б. Поплавский // Фундаментальная и прикладная математика. – 2012. – Т. 17, № 4. – С. 181—192.
2. *Поплавский В.Б.* Об идемпотентах алгебры булевых матриц / В.Б. Поплавский // Изв. Сарат. ун-та. Нов. сер. Сер. Математика. Механика. Информатика. – 2012. – Т. 12, № 2. – С. 26—33.

Orthogonality of parastrophes of alinear quasigroups

V. Shcherbacov

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova
`scerb@math.md`

Definition 1. A binary groupoid (Q, A) such that in the equality $A(x_1, x_2) = x_3$ knowledge of any two elements from the elements x_1, x_2, x_3 (not necessarily distinct) uniquely specifies the remaining one, is called a quasigroup [2].

Definition 2. Binary groupoids (Q, A) and (Q, B) are called orthogonal if the system of equations

$$\begin{cases} A(x, y) = a \\ B(x, y) = b \end{cases}$$

has an unique solution (x_0, y_0) for any fixed pair of elements $a, b \in Q$.

Definition 3. From Definition 1 it follows that with a given binary quasigroup (Q, A) it is possible to associate $(3! - 1)$ others, so-called parastrophes of quasigroup (Q, A) [3, p. 230], [1, p. 18]:

$$\begin{array}{l} A(x_1, x_2) = x_3 \Longleftrightarrow A^{(12)}(x_2, x_1) = x_3 \Longleftrightarrow \\ A^{(13)}(x_3, x_2) = x_1 \Longleftrightarrow A^{(23)}(x_1, x_3) = x_2 \Longleftrightarrow \\ A^{(123)}(x_2, x_3) = x_1 \Longleftrightarrow A^{(132)}(x_3, x_1) = x_2. \end{array}$$

A quasigroup $(Q, \cdot)$ with the form $x \cdot y = I\varphi x + I\psi y + a$, where $(Q, +)$ is a group, $\varphi, \psi \in Aut(Q, +)$, $Ix = -x$ for all $x \in Q$, is called an alinear quasigroup. Below $J_t x = t + x - t$ for all $x \in Q$.

Theorem 1. *For an alinear quasigroup (Q, A) of the form $A(x, y) = I\varphi x + I\psi y + c$ over a group $(Q, +)$ the following equivalences are true:*

1. $A \perp A^{12} \Longleftrightarrow$ *the mapping $(\psi^{-1}\varphi - J_t\varphi^{-1}\psi)$ is a permutation of the set Q for any $t \in Q$;*
2. $A \perp A^{13} \Longleftrightarrow$ *the mapping $(\varphi - J_{\psi t}J_c)$ is a permutation of the set Q for any $t \in Q$;*
3. $A \perp A^{23} \Longleftrightarrow$ *the mapping $(\varepsilon + I\psi J_t)$ is a permutation of the set Q for any $t \in Q$;*
4. $A \perp A^{123} \Longleftrightarrow$ *the mapping $(\psi^2 - \varphi J_{\psi^{-1}c})$ is a permutation of the set Q;*
5. $A \perp A^{132} \Longleftrightarrow$ *the mapping $(\psi - \varphi^2)$ is a permutation of the set Q.*

1. V.D. Belousov. *Foundations of the Theory of Quasigroups and Loops.* Nauka, Moscow, 1967. (in Russian).
2. V.D. Belousov. *n-Ary Quasigroups.* Stiintsa, Kishinev, 1971. (in Russian).
3. Sh. K. Stein. On the foundations of quasigroups. *Trans. Amer. Math. Soc.*, 85(1):228–256, 1957.

Ternary derivations of semisimple Jordan superalgaebras over a field of characteristic zero

A.I. Shestakov

Novosibirsk, Russia
shestalexy@mail.ru

Ternary derivations and related to them generalized derivations were studied in various classes of algebras, see for example [1]-[6]. In the present work ternary and generalized derivations of semisimple Jordan superalgebras over an algebraically closed field of characteristic zero are described.

Let A be a superalgebra over a field Φ, i. e. $A = A_0 + A_1$ is a Z_2-graded algebra.

A triple $\Delta = (D, F, G)$ of homogeneous linear mappings $D, F, G \in \mathrm{End}(A)$ is a *ternary derivation* of a superalgebra A, provided that for all $x, y \in A$

$$D(xy) = F(x)y + (-1)^{deg(x)deg(G)}xG(y).$$

The first component D of ternary derivation (D, F, G) is also called a *generalized derivation*. Ternary derivation generalizes ordinary derivation with $D = F = G$, and δ-derivation with $F = G = \delta D$, $\delta \in \Phi$, (look for ex. [7]-[9]).

Note, that the following triples are obviuosly ternary derivations in any superalgebra A:

$$\Delta = (\phi + \psi + D^0,\ \phi + D^0,\ \psi + D^0), \tag{1}$$

where ϕ, ψ are arbitrary elements in the centroid of superalgebra A, and D^0 is an arbitrary ordinary derivation in A, such as $deg\ \phi = deg\ \psi = deg\ D^0$.

The derivations given above, i. e. of the form (1) are called *standart*; and their first components are called standart generalized derivations as well.

We describe ternary and generalized derivations of semisimple Jordan superalgebras over an algebraically closed field of characteristic zero. The main result states that under certain restrictions every ternary(generalized) derivation in these superalgebras is standart. The restriction is related with the simple Jordan superalgebra of bilinear superform f on two-dimensional odd vector space V:

$$J(V, f) = \Phi \cdot 1 + V_0 + V_1 \ \text{ with } \ V_0 = 0,\ V_1 = V,\ \ dim\ V = 2;$$

all even ternary derivations of this superalgebra are standart and all odd ternary derivation are discribed by the following form:

$$\{\, \Delta_v \mid \Delta_v(1) = (v, \frac{1}{2}v, \frac{1}{2}v),\ \Delta_v(x) = (\frac{1}{2}f(x, v), f(x, v), f(x, v))\ \ \forall\, x \in V \,\}_{v \in V}.$$

This is the only case, ternary(generalized) derivation of a simple Jordan superalgebra is not standart. Therefore, if a semisimple Jordan superalgebra over an algebraically closed field of characteristic zero doesn't have the direct summands of this type, then every it's ternary(generalized) derivation is standart.

1. Jimenez-Gestal C., Perez-Izquierdo J. M. *Ternary derivations of finite-dimensional real division algebras.* Linear Algebra Appl., **428** (2008), №8-9.

2. Bresar M. *On the distance of the composition of two derivations to the generalized derivations.* Glasgow Math. J. 33 (1991).
3. Komatsu H., Nakajima A. *Generalized derivations of associative algebras.* Quaest. Math., **26** (2003), №2, 213–235.
4. Leger G., Luks E. *Generalized derivations of Lie Algebras.* J. of Algebra, **228** (2000), 165–203.
5. Zhang R., Zhang Y. *Generalized derivations of Lie superalgebras.* Comm. Algebra, **38** (2010), №10, 3737–3751.
6. Шестаков А. И., *Тернарные дифференцирования сепарабельных ассоциативных и йордановых алгебр*, Сиб. матем. журн., 53:5 (2012), 1178–1195. A. I. Shestakov, *Ternary derivations of separable associative and Jordan algebras*, Siberian Math. J., 53:5 (2012), 943–956.
7. Филиппов В. Т. *О δ-дифференцированиях альтернативных первичных и мальцевских алгебр.* Алгебра и Логика, **39** (2000).
8. Кайгородов И. Б. *О δ-супердифференцированиях простых конечномерных йордановых и лиевых супералгебр.* Алгебра и Логика, **49** (2010), №2.
9. Желябин В.Н., Кайгородов И.Б. *О δ-супердифференцированиях простых супералгебр йордановой скобки.* Алгебра и Анализ, **23** (2011), №4, 40–58.

On convex subgroups of pl-groups

Elena E. Shirshova

Moscow Pedagogical State University
shirshova.elena@gmail.com

Suppose G is a partially ordered group (*po*-group), and $G^+ = \{x \in G | \, e \leqslant x\}$. A subgroup M of a *po*-group G is said to be *convex* if $a \leqslant g \leqslant b$ imply $g \in M$ for any $a, b \in M$ and $g \in G$. An *o-ideal* is a convex directed (see [1]) normal subgroup of *po*-group. Elements $a, b \in G^+$ of a *po*-group G are said to be *almost orthogonal* if $c \leqslant a, b$ imply $c^n \leqslant a, b$ for any $c \in G$ and any integer $n > 0$. G is an *$\mathcal{AO}$-group* if each $g \in G$ has a representation $g = ab^{-1}$ for some almost orthogonal elements a and b of G^+. G is called a *Lex-extension of a convex normal subgroup M by the po-group G/M*, if each strictly positive element in G/M consists entirely of positive elements of G.

Theorem 1. *Suppose G is an $\mathcal{AO}$-group, N is its o-ideal, and M is an o-ideal of the group N. If N is the Lex-extension of M, then the set $S = \bigcup_{x \in G} x^{-1}Mx$ is an o-ideal of the group G, and N is the Lex-extension of the group S. If T is the set-theoretic intersection of all convex directed subgroups $K \subseteq N$, where $K \nsubseteq S$, then T is an o-ideal of G.*

A *po*-group G is an *interpolation group* if whenever $a_1, a_2, b_1, b_2 \in G$ and $a_1, a_2 \leqslant b_1, b_2$, then there exists $c \in G$ such that $a_1, a_2 \leqslant c \leqslant b_1, b_2$. An interpolation $\mathcal{AO}$-group is called a *pl*-group. Positive elements u and v of a *po*-group G are said to be *a-equivalent* if there exist some integers $m > 0$ and $n > 0$ such that $u \leqslant v^m$ and $v \leqslant u^n$. In a *po*-group G, for any element $a \in G^+$ $(a \neq e)$, there is the lowest convex directed subgroup $[a]$, where $a \in [a]$.

Theorem 2. *Let G be a pl-group, and let a and $b \in G^+$ be some almost orthogonal elements of G. If M is the unique value of the element a in the group $[a]$, then the following assertions hold:*

1. *$[a]$ is the Lex-extension of the group M by the group $[a]/M$;*
2. *if $u, v \in [a] \setminus M$, then the elements u and v are comparable and a-equivalent.*

1. L. Fuchs, *Partially Ordered Algebraic Systems*, Pergamon Press, Oxford-London-New York-Paris; Addison-Wesley, Reading, Mass.-Palo Alto, Calif.-London, 1963.

Derived and nilpotent π-lengths of finite π-soluble groups

O. A. Shpyrko

The Branch of MSU in Sevastopol, Sevastopol, Ukraine
shpyrko@mail.ru

Only finite groups are considered. All used concepts and notations correspond to accepted in [1]. Let π be a set of primes. A set of all primes which do not belong to π is denoted by π' and a set of all primes that divides the order of G group is designated by $\pi(G)$. The growing (π', π)-series of group G is called a series:

$$1 = P_0 \leqslant N_0 < P_1 < N_1 < \cdots < P_i < N_i < \ldots,$$

where $N_i/P_i = O_{\pi'}(G/P_i), P_{i+1}/N_i = O_\pi(G/N_i),\ i = 0, 1, 2, \ldots$. Here $O_{\pi'}(X)$ and $O_\pi(X)$ are the largest normal π'– and π–subgroups of the group X, respectively. If group G is a π-soluble group, then equality $N_k = G$ for some natural integer k is performed. The smallest natural integer k with such property is called a π-length of π-soluble group G and is defined by $l_\pi(G)$.

A nilpotent π-length of a π-soluble group G is defined as follows. Let $P_0^n = 1$, $N_i^n/P_i^n = O_{\pi'}(G/P_i^n)$, $P_{i+1}^n/N_i^n = O_\pi^n(G/N_i^n) = F(G/N_i^n)$, $i = 0, 1, 2, \ldots$. Here $F(X)$ is a Fitting subgroup of group X. The smallest value k for which in (π', π^n)-series

$$1 = P_0^n \leqslant N_0^n < P_1^n < N_1^n < \cdots < P_i^n < N_i^n < \ldots$$

the equality $N_k^n = G$ is realized, is called a nilpotent π-length of π-soluble group G and is denoted by $l_\pi^n(G)$.

A derived π-length of π-soluble group G is determined as the smallest number abelian π-factors among all subnormal (π', π^a)-series of G with π'-factors or abelian π-factors and is defined by $l_\pi^a(G)$. It's clear that the inequality $l_\pi(G) \leqslant l_\pi^n(G) \leqslant l_\pi^a(G)$ for any π-soluble group G is true and in case when π-Hall subgroup is abelian the relation $l_\pi(G) = l_\pi^n(G) = l_\pi^a(G) \leqslant 1$ is correct.

For unit group E $l_\pi^a(E) = l_\pi^n(E) = l_\pi(E) = 0$ will be assumed. In case $\pi = \{p\}$ we have an equality: $l_p(G) = l_\pi(G) = l_\pi^n(G) = l_\pi^a(G)$. In what follows under $l_\pi^*(G)$ we will mean either π-length $l_\pi(G)$ or nilpotent π-length $l_\pi^n(G)$ or derived π-length $l_\pi^a(G)$ of the group G.

Theorem 1. *Let G be a π-soluble group, K be a normal subgroup of G and $l_\pi^*(K) = k$, $l_\pi^*(G/K) = t$, then $l_\pi^*(G) \leqslant k + t$.*

1. Monakhov V.S. The introduction in theory of finite groups and it's classes // Minsk: The hiher school. 2006.

The properties of admissible sequences of indecomposable serial rings with Noetherian diagonal

V.V. Shvyrov

Luhansk Taras Shevchenko National University, Luhansk, Ukraine
slavik_asas@mail.ru

The properties of admissible sequences for indecomposable serial rings with Noetherian diagonal and nilpotent prime radical were studied. The formula for finding of amount of such sequences for cases $c_1 \in \{1, 2, 3\}$, where c_1 is the first element of admissible sequence, were obtained.

Definition 1. Any sequence which satisfies the following inequalities is called an admissible sequence.

$$2 \leq c_i \leq c_{i-1} + 1, i = 2, \ldots, n, \; c_1 \leq c_n + 1.$$

Admissible sequences arise up at the study of Kupisch series of serial rings, see [1]. Such sequences satisfy some natural restrictions which allow to calculate their number. Using admissible sequences and some results from work N. M. Gubareni, V. V. Kirichenko [2] it possible classify of indecomposable serial rings with Noetherian diagonal and nilpotent prime radical.

Definition 2. Let A, B is the serial indecomposable rings with Noetherian diagonal and nilpotent prime radical. $s_A = pl_1, \ldots, pl_n$ and $s_B = pl_1, \ldots, pl_m$ is corresponding admissible sequences of rings A and B. We will suppose, that the ring A is equivalent by Kupisch to ring B, if $n = m$ and $s_A = s_B$, and such equivalence will be named the Kupisch equivalence.

Theorem 1. *The number of equivalence classes for Kupisch equivalence of indecomposable serial rings with Noetherian diagonal and nilpotent prime radical, the prime quiver of which contains n vertex is equal:*

a) C_{n-1}, *if* $p_n = 1$*;*

b) C_n, *if* $p_n = 2$*;*

c) $\frac{3n}{n+2} C_n$, *if* $p_n = 3$,

where $p_1, \ldots, p_n$ *is an admissible sequence, which define the equivalence,* C_n *denotes the* n*-th Catalan number.*

1. F.W. Anderson, K. R. Fuller. *Rings and Categories of Modules.* — Springer values, New York and Berlin, 2nd ed., 1992. — 385 p.
2. N. M. Gubareni, V. V. Kirichenko. Serial Rings with T-nilpotent Prime Radical // *Algebras and Representations*

Lipschitz type inequality and universality of periodic zeta-functions

D. Šiaučiūnas

Siauliai University
siauciunas@fm.su.lt

Let $H(D)$, $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$, be the space of analytic functions on D. We say that the operator $F : H^2(D) \to H(D)$ belongs to the class $Lip(\beta_1, \beta_2)$, $\beta_1, \beta_2 > 0$, if the following hypotheses are satisfied:

1° For any compact set $K \subset D$ with connected complement and each polynomial $p = p(s)$, there exists an element $(g_1, g_2) \in F^{-1}\{p\} \subset H^2(D)$ such that $g_1(s) \neq 0$ on K;

2° For any compact set $K \subset D$ with connected complement, there exist a constant $c > 0$ and compact sets $K_1, K_2 \subset D$ with connected complements such that

$$\sup_{s \in K} |F(g_{11}(s), g_{12}(s)) - F(g_{21}(s), g_{22}(s)))| \leq c \sup_{1 \leq j \leq 2} \sup_{s \in K_j} |g_{1j}(s) - g_{2j}(s)|^{\beta_j}$$

for all $(g_{j1}, g_{j2}) \in H^2(D)$, $j = 1, 2$.

In the report, we consider the universality of the function $F(\zeta(s; \mathfrak{a}), \zeta(s, \alpha; \mathfrak{b}))$, where $\zeta(s; \mathfrak{a})$ and $\zeta(s, \alpha; \mathfrak{b})$, $s = \sigma + it$, are the periodic and periodic Hurwitz zeta-functions, respectively, defined, for $\sigma > 1$, by the series

$$\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s} \qquad \text{and} \qquad \zeta(s, \alpha; \mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b_m}{(m + \alpha)^s},$$

and by analytic continuation elsewhere. Here $\mathfrak{a} = \{a_m\}$ and $\mathfrak{b} = \{b_m\}$ are periodic sequences of complex numbers, and $0 < \alpha \leq 1$ is a fixed parameter. Denote by $\text{meas}\{A\}$ the Lebesgue measure of a measurable set $A \subset \mathbb{R}$. Then we have the following result.

Theorem 1. *Suppose that the α is a transcendental number, and that $F \in Lip(\beta_1, \beta_2)$. Let $K \subset D$ be a compact set with connected complement, and $f(s)$ be a continuous function on K which is analytic in the interior of K. Then, for every $\varepsilon > 0$,*

$$\liminf_{T \to \infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0, T] : \sup_{s \in K} |F(\zeta(s + i\tau; \mathfrak{a}), \zeta(s + i\tau, \alpha; \mathfrak{b})) - f(s)| < \varepsilon \right\} > 0.$$

The details and other results can be found in [1].

1. D. Korsakienė, V. Pocevičienė and D. Šiaučiūnas. On universality of periodic zeta-functions, *Šiauliai Math. Seminar* (submitted).

Basis of multilinear part of variety $\widetilde{\mathbf{V}}_3$ of Leibniz algebras

Skoraya T.V.

Ulyanovsk state University
skorayatv@yandex.ru

The characteristic of basic field Φ will be equal to zero. All not defined concepts can be found in the book [1].

The generalization of concept of Lie algebras is Leibniz algebra, which is defined by identity $(xy)z \equiv (xz)y + x(yz)$ and, probably, for the first time appeared in work [2].

Let $T = \Phi[t]$ be the ring of polynomials of a variable t. Consider three-dimensional Heisenberg algebra H with basis $\{a, b, c\}$ and multiplication $ba = -ab = c$, the product of other basic elements equally to zero. Let's turn a ring of polynomials T into the right module of algebra H, in which basic elements of algebra H affect on the right a polynomial f from T as follows: $fa = f', fb = tf, fc = f$, where f' is private derivative of a polynom f on a variable t. Consider the direct sum of vector spaces H and T with multiplication by the rule: $(x+f)(y+g) = xy+fy$, where x, y are from H; f, g are from T. Designate it a symbol $\widetilde{H}$. It is possible to be convinced by direct check that $\widetilde{H}$ is a Leibniz algebra.

The algebra $\widetilde{H}$ generates the variety $\widetilde{\mathbf{V}}_3$ of Leibniz algebras. This variety is analog of well-known variety $\mathbf{V}_3$ of Lie algebras. Earlier, in work [3], concerning variety $\widetilde{\mathbf{V}}_3$ it was proved that it has almost polynomial growth, in work [4] there were defined its multiplicities and colength.

In 1949 A. I. Maltsev proved that in the case when the main field has the zero characteristic, any identity is equivalent to system of multilinear identities. Therefore in this case all information on variety contains in space of multilinear elements of degree n from variables $x_1, x_2, \ldots x_n$.

Let $\mathbf{V}$ is some variety of linear algebras. The space of its multilinear elements we will designate $P_n(\mathbf{V})$.

Theorem. *The set of elements of the form*

$$\theta(i, i_1, ..., i_m, j_1, ..., j_m) = x_i(x_{i_1}x_{j_1})(x_{i_2}x_{j_2})...(x_{i_m}x_{j_m})x_{k_1}x_{k_2}...x_{k_{n-2m-1}},$$

where $i_s < j_s, s = 1, 2, ..., m, i_1 < i_2 < ... < i_m, j_1 < j_2 < ... < j_m, k_1 < k_2 < ... < k_{n-2m-1}$, *form basis of space* $P_n(\widetilde{V}_3)$.

Work is performed with financial support of RFPF (project code 12-01-33031).

1. Giambruno A., Zaicev M. Polynomail identities and Asymptotic Methods // American Mathematical Society, Providence, RI: Mathematical Surveys and Monographs. — 2005. — V. 122. — P. 352
2. Blokh A.M. A generalization of the concept of a Lie algebra // Russian original Sov. MAth., Dokl. 6 (1965) 1450-1452 (1966); translation from Dokl. Akad. Nauk SSSR. 165 (1965), 471—473.
3. Mishchenko S.P. Varieties of linear algebras with almost polynomial growth // Polynomial identities and combinatorial methods. Pantelleria, 2001. P. 383—395.
4. Skoraya T.V. The structure of multilinear part of variety $\widetilde{\mathbf{V}}_3$ // Scientific notes of the Oryol state university. — 2012. —№6(50). — P.203—2012.

Identities of parastrophic distributivity

Fedir M. Sokhatsky

University "Ukraina", Vinnytsia Institute of Economics and Social Sciences
sokha@ukr.net

Let $(Q;\cdot,/,\backslash)$ be a quasigroup. A *left translation* L_a, *right translation* R_a and *middle translation* M_a are defined by the following relationships:

$$L_a(x) := a\cdot x, \qquad R_a(x) := x\cdot a, \qquad M_a(x) := y \;\Leftrightarrow\; x\cdot y = a.$$

A quasigroup is called left (right, middle) distributive if all its left (right, middle) translations are its automorphisms, i.e., if the left (right, middle) distributivity

$$x\cdot yz = xy\cdot xz \qquad \Big(yz\cdot x = yx\cdot zx, \qquad (yz)\backslash x = (y\backslash x)\cdot(z\backslash x)\Big) \tag{$*$}$$

holds. A quasigroup is called distributive if it is left and right distributive simultaneously.

Theorem 1. *Any two identities from* $(*)$ *imply the third one.*

Corollary. *If a quasigroup is distributive, then all its translations are automorphisms of every of its parastrophes.*

Theorem 2. *A quasigroup* $(Q;\cdot)$ *is distributive if and only if there exists a Comutative Moufang Loop* $(Q;+)$ *and its commute automorphisms* φ, ψ *such that*

$$x\cdot y = \varphi x + \psi y, \qquad x + (y+z) = (\varphi x + y) + (\psi x + z).$$

This theorem is some refinement of the corresponding Belousov's theorem:

Theorem 8.1 [1]. *Every distributive quasigroup is isotopic to a Commutative Moufang Loop.*

1. Belousov V.D., Foundations of the Theory of Quasigroups and Loops, Nauka, 1967, 222.

Tractability of algebraic function fields in one variable over pseudoglobal field

L. Stakhiv

Ivan Franko National University of Lviv, Lviv, Ukraine
mlm2010@i.ua

By a pseudoglobal field k we mean an algebraic function field in one variable over a pseudofinite [1] constant field. Let $chark \neq 2$. A field k is called tractable if for every nonzero $a_i, b_i \in k$, $i = 1, 2, 3$, whenever the quaternion algebra $(a_i, b_j/k)$ is split for all $i \neq j$ and $(a_1, b_1/k) \cong (a_2, b_2/k) \cong (a_3, b_3/k)$, then $(a_i, b_i/k)$ is split. The purpose of this paper is to extend some of the results of I. Han [2] on the tractability of algebraic function fields in one variable over global field to the case of pseudoglobal fields.

For a field k, let $Br(k)'$ be the subgroup of $Br(k)$ consisting of all elements of order relatively prime to n when $char(k) = n$, and $Br(k)' = Br(k)$ when $char(k) = 0$. Define $P(k) = \{p | p$ is a prime spot of $k\}$. Then K is a function field of some curve C where C is a geometrically irreducible curve defined over k. Conversely, the function field $k(C)$ of the curve C is an algebraic function field in one variable over k. Assume now that a curve C has a k-rational point, that is, $C(k) \neq \varnothing$.

For each $p \in P(k)$, denote by $\hat{k_p}$ the completion of k with respect to p. Let $\hat{k_p}(C)$ be the function field of $C_p = C \times_k \hat{k_p}$. Consider the map $\phi : Br(C) \longrightarrow \prod_{p \in P(k)} Br(C_p)$ In order to identify the kernel of this map, let us also consider the map $\phi_K(BrK) \longrightarrow \prod_{p \in P(k)} Br(\hat{k_p}(C))$. Let $\phi'_K = \phi_K|_{Br(K)'}$ and likewise let $\phi' = \phi|_{Br(C)'}$. Then we have

Theorem 1. *For the map ϕ'_K and ϕ' as above $ker(\phi'_K) \cong ker(\phi')$.*

Let $\bar{k}$ be the separable closure of k, and $\bar{k_p}$ the separable closure of the completion $\hat{k_p}$. Let J and J_p denote the Jacobians of $\bar{C} = C \times_k \bar{k}$ and of $\bar{C_p} = C \times_k \bar{k_p}$, respectively. Let $G = Gal(\bar{k}/k)$ and $Gp = Gal(\bar{k_p}/\hat{k_p})$ be the absolute Galois groups of k and $\hat{k_p}$, respectively.The Shafarevich–Tate group Ш(J) of the Jacobian J is defined as follows: Ш$(J) = ker(H^1(G, J) \xrightarrow{\psi} \prod_{p \in P} H^1(G_p, J_p)$.

Theorem 2. *Let k be a tractable pseudoglobal field and let C be a curve over k with $C(k) \neq \varnothing$. If ${}_2$Ш$_*(J) = 0$, then K is tractable.*

1. J.Ax. The elementary theory of fields. Ann. Math. 1968. V. 88, N 2, P. 239–271.
2. I. Han. Tractability of algebraic function fields in one variable of genus one over global fields. Journal of Algebra 317 (2007) 796–812

Parafunctions and partition polynomials

S. Stefluk

Vasyl Stefanyk Precarpathian National University
ljanys_89@mail.ru

The article deals with relations between one class of partition polynomials and parafunctions of triangular matrices (tables) and linear recurrence relations.

Introduced by Bell in [1] the concept of partition polynomials is widely used in discrete mathematics.They arise in number theory [2], algebra (the theory of symmetric polynomials), combinatorics [3] (e.g. expression of the sum of divisors of the natural number through unordered partitions of the natural number), differentiation of composite functions (Faa di Bruno's formula) [4] etc.

Theorem 1. *Let polynomials $y_n(x_1, x_2, \ldots, x_n), n = 0, 1, \ldots$ be given by some recurrent equations*

$$y_n = x_1 y_{n-1} - x_2 y_{n-2} + \ldots + (-1)^{n-2} x_{n-1} y_1 + (-1)^{n-1} a_n x_n y_0,$$

where $y_0 = 1$, then fair the equality

$$y_n = \left\langle \begin{array}{cccc} a_1 x_1 & & & \\ a_2 \frac{x_2}{x_1} & x_1 & & \\ \vdots & \ldots & \ddots & \\ a_n \frac{x_n}{x_{n-1}} & \ldots & \frac{x_2}{x_1} & x_1 \end{array} \right\rangle,$$

$$y_n = \sum_{\lambda_1 + 2\lambda_2 + \ldots + n\lambda_n = n} (-1)^{n-k} \left(\sum_{i=1}^{n} \lambda_i a_i \right) \frac{(k-1)!}{\lambda_1! \lambda_2! \cdot \ldots \cdot \lambda_n!} x_1^{\lambda_1} x_2^{\lambda_2} \cdot \ldots \cdot x_n^{\lambda_n},$$

holds, where $k = \lambda_1 + \lambda_2 + \ldots + \lambda_n$.

1. *Bell E.T.* Partition polynomials. — Ann. Math. 29, 1927, p. 38 – 46.
2. *Fine N.J.* Sums over partitions.— Report of the Institute in the Theory of Numbers, Boulder, 1959, p. 86-94.
3. *Riordan J.* Kombinatornye tozhdestva. (Russian) [Combinatorial identities] Translated from the English by A. E. Zhukov. "Nauka", Moscow, 1982. 256 pp.
4. *Riordan J.* An Introduction to Combinatorial Analysis, Wiley, New York, 1958.

On wild groups over local factorial rings

M. V. Stoika

Uzhgorod National University
stoyka_m@yahoo.com

The problem of describing the tame and wild finite groups G over a field K is completely solved in [1]. For a ring K, this problem is completely solved in the cases when K is a ring of p-adic numbers or a complete discrete valuation ring or a ring of formal power series with P-adic coefficients (see [2]–[6]). In many other cases the problem is solved when there are constraints on groups or rings. We consider the case when G is a 2-group and K is local factorial rings of characteristic 0.

1. V. M. Bondarenko, Ju. A. Drozd. The representation type of finite groups // Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI), 1977, 71, P. 24–41 (in Russian).
2. P. M. Gudivok. Modular and integral representations of finite groups // Dokl. Akad. Nauk SSSR 1974, 214, P. 993–996 (in Russian).
3. P. M. Gudivok. Representations of finite groups over a complete discrete valuation ring // Trudy Mat. Inst. Steklov, 1978, 148, P. 96–105 (in Russian).
4. E. Dieterich. Group rings of wild representation type // Math. Annn., 1983, 266, N1, P. 1–22.
5. P. M. Gudivok, V. M. Oros, A. V. Roiter. Representations of finite p-groups over a ring of formal power series with integer p-adic coefficients // Ukrain. Mat. Zh., 1992, 44, N6, P. 753–765 (in Russian).
6. V. M. Bondarenko, P. M. Gudivok. Representations of finite p-groups over a ring of formal power series with integer p-adic coefficients // Infinite groups and related algebraic structures, Akad. Nauk Ukrainy, Inst. Mat., Kiev, 1993, P. 5–14 (in Russian).

On posets with nonnegative quadratic Tits form

M.V. Styopochkina

Zhytomyr National Agroecological University
StMar@ukr.net

In [1] the author together with V. M. Bondarenko proved that a poset is critical with respect to the positivity of the Tits form if and only if it is minimax isomorphic to a critical poset of Kleiner, and as a consequence described all such posets (the minimax equivalence and the minimax isomorphism was introduced by V. M. Bondarenko in [2]). Similar results were obtained by the same authors for posets that are critical with respect to the nonnegativity of the Tits form [3, 4]; in this case, instead of the critical posets of Kleiner one needs to take the supercritical posets of Nazarova.

Recently we received new results on posets with nonnegative Tits form.

1. V. M. Bondarenko, M. V. Stepochkina, *(Min, max)-equivalence partially ordered sets and quadratic Tits form*, Zb. Pr. Inst. Mat. NAN Ukr., 2, N3 (2005), pp. 18-58 (in Russian).
2. V. M. Bondarenko, *On (min, max)-equivalence of posets and applications to the Tits form*, Bull. of the University of Kiev (series: Physics and Mathematics), N1 (2005), pp. 24-25.
3. V. M. Bondarenko, M. V. Stepochkina, *(Min, max)-equivalence of posets, and nonnegative Tits forms*, Ukr. Mat. Zh., 60, N9 (2008), pp. 1157-1167 (in Russian); translation in Ukrainian Math. J.,60, N9 (2008), pp. 1349-1359.
4. V. M. Bondarenko, M. V. Stepochkina, *Description of partially ordered sets that are critical with respect to the nonnegativity of the quadratic Tits form*, Ukr. Mat. Zh., 61, N5 (2009), pp. 611-624 (in Russian); translation in Ukrainian Math. J.,61, N5 (2009), pp. 734-746.

Negative-ternary-quinary representation of real numbers

Yu.Yu. Sukholit

Dragomanov National Pedagogical University, Kyiv, Ukraine
freeeidea@ukr.net

Let A= {0,1,2,3,4} be the alphabet of classical quinary number system.

Lemma 1. *For any number $x \in (0;2)$ there exists a sequence (β_k), $\beta_k \in A$, such that*

$$x = \frac{3}{2} + \sum_{k=1}^{\infty} \frac{\beta_k}{(-3)^k} = \frac{3}{2} - \frac{\beta_1}{3} + \frac{\beta_2}{3^2} - \frac{\beta_3}{3^3} + \frac{\beta_4}{3^4} - \ldots \equiv \Delta_{\beta_1\beta_2\ldots\beta_k\ldots}. \tag{1}$$

Example 1. $1 = \Delta_{1(04)}$, $\frac{1}{3} = \Delta_{3(04)}$, $2 = \Delta_{(04)}$.

A representation of the number x in the form (1) is called a *negative-ternary expansion* with two extra digits 3 and 4 or *negative-ternary-quinary expansion.* Reduced (symbolic) record $x \equiv \Delta_{\beta_1\beta_2\ldots\beta_k\ldots}$ of equality (1) is called a *negative-ternary-quinary representation*, or shorter: $5_{-3}-$*representation* of the number x.

Lemma 2. *The numbers $\Delta_{c_1\ldots c_m\alpha_1\alpha_2b_1b_2\ldots b_n\ldots}$ and $\Delta_{c_1\ldots c_m\gamma_1\gamma_2b_1b_2\ldots b_n\ldots}$ are equal if and only if $3(\gamma_1 - \alpha_1) = \gamma_2 - \alpha_2$.*

Theorem 1. *The numbers $\Delta_{\alpha_1\alpha_2\ldots\alpha_n\ldots}$ and $\Delta_{\beta_1\beta_2\ldots\beta_n\ldots}$ are equal if and only if $5_{-3}-$representation of the first number can be obtained from $5_{-3}-$representation second "string" replace pairs of consecutive numbers into pairs that are equivalent:*

$$\begin{array}{llll} \overline{00} = \overline{13}; & \overline{20} = \overline{33}; & \overline{01} = \overline{14}; & \overline{21} = \overline{34}; \\ \overline{10} = \overline{23}; & \overline{30} = \overline{43}; & \overline{11} = \overline{24}; & \overline{31} = \overline{44}. \end{array}$$

Corollary 1. *The numbers, which in the $5_{-3}-$number system can be represented as*

$$\Delta_{i\underbrace{2\ldots2}_{m}},\ i \in A, m \in N;$$

$$\Delta_{0(i+3)\underbrace{2\ldots2}_{m}} \text{ and } \Delta_{4(i-1)\underbrace{2\ldots2}_{m}},\ i \in \{0,1\}, m \in Z_0,$$

with a single $5_{-3}-$representation.

Theorem 2. *Almost all (in the sense of Lebesgue measure) numbers $x \in (0;2)$ have a continuum set of formally different representations.*

Definition 1. Let $(c_1c_2\ldots c_m)$ be a fixed set of figures of set {0,1,2,3,4}. A set $\Delta_{c_1c_2\ldots c_m} = \{x : x = \Delta_{c_1c_2\ldots c_m\beta_{m+1}\beta_{m+2}\ldots}, (\beta_{m+n}) \in A^\infty\}$ is called the 5_{-3}-adic cylindrical set of rank m with the base $c_1c_2\ldots c_m$.

In the report offers: criterion of rationality and irrationality numbers in his $5_{-3}-$representation; its relationship with ternary-quinary representation; results of research geometry $5_{-3}-$representation of numbers (geometric meaning of digits of number; properties of cylinders, half-cylinder, tail sets etc), and various applications.

On the Gorenstein tiled orders

A. Surkov

Taras Shevchenko National University of Kyiv
artyoms@i.ua

All the necessary theoretical information on Gorenstein tiled orders can be found in [1], [2]. We use the notation of [1], [2].

Let $\Lambda = \{\mathcal{O}, \mathcal{E}(\Lambda) = (\alpha_{ij})\}$ be a reduced Gorenstein tiled order with Kirichenko's permutation σ, let $\sigma = \sigma_1 \cdots \sigma_s$ be a decomposition of permutation σ into a product of cycles.

Then two-sided Peirce decomposition of Λ has the form

$$\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \cdots & \Lambda_{1s} \\ \Lambda_{21} & \Lambda_{22} & \cdots & \Lambda_{2s} \\ \cdots & \cdots & \cdots & \cdots \\ \Lambda_{s1} & \Lambda_{s2} & \cdots & \Lambda_{ss} \end{pmatrix}, \tag{1}$$

where Λ_{kk} are cyclic Gorenstein tiled orders and

$$\frac{\sum\limits_{k=1}^{|\langle\sigma_i\rangle|} \alpha_{k\sigma_i(k)}}{|\langle\sigma_i\rangle|} = t \geq 1. \tag{2}$$

for all $i = 1, \ldots s$

Theorem 1. *Let $\Lambda_{11}, \Lambda_{22}, \ldots, \Lambda_{ss}$ be cyclic Gorenstein tiled orders with Kirichenko's permutations $\sigma_1, \sigma_2, \ldots, \sigma_s$. If the condition (2) holds, then there is reduced Gorenstein tiled order of the form (1) with Kirichenko's permutation σ, where $\sigma = \sigma_1 \sigma_2' \cdots \sigma_s'$ is the decomposition of permutation σ into a product of cycles with do not intersect, $\sigma_k'(l) = \sigma_k(l - (|\langle\sigma_1\rangle| + \cdots + |\langle\sigma_{k-1}\rangle|)) + (|\langle\sigma_1\rangle| + \cdots + |\langle\sigma_{k-1}\rangle|)$.*

1. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 1, Series: Mathematics and Its Applications, **575**, Kluwer Acad. Publish., 2004. xii+380pp.
2. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 2, Series: Mathematics and Its Applications, **586**. Springer, Dordrecht, 2007. xii+400pp.

D-monomial Groups and Interval Exchange Transformations

V. Sushchanskyy

Silesian University of Technology
vitaliy.sushchanskyy@polsl.pl

The presented work is joint with Bogdana Oliynyk.

Let $M_n(G) = G \wr S_n$ be the group of monomial permutations over a group G. Group $M_n(G)$ can be constructed as a group of monomial matrices of rank n over the group ring ZG. For every $k \in \mathbb{N}$ we define the k–diagonal embedding $f_k : M_n(G) \to M_{kn}(G)$ be the rule

$$f_n(a) = \underbrace{a \oplus a \oplus ... \oplus a}_{k\text{–times}},$$

where $\oplus$ denotes the Kronecker sum of matrices.

For every divisible sequence $\langle n_1, n_2, ...\rangle$, where $n_i | n_{i+1}$ and $\frac{n_{i-1}}{n_i} = k_i$ for $i = 1, 2, ...$ we define the direct spectrum

$$\langle M_{n_i}(G), f_{k_i}\rangle_{i\in\mathbb{N}},$$

which we call the *diagonal spectrum*. The limit group of an arbitrary diagonal spectrum is called the *D-monomial group over group G*. For every group G the family of D-monomial groups over G contains continuum many pairwise nonisomorphic groups. In the talk we investigate the properties and structure of this family of groups. We also discuss a number of applications of D-monomial groups, in particular in the theory of IET-groups, in the theory of automorphisms of free groups of infinite rank and others.

1. O. Ore, "Theory of monomial groups", Trans. Amer. Math. 51 (1942) 15-64.

On recursive derivatives and cores in loops

P. Syrbu

Moldova State University
syrbuviv@yahoo.com

The recursive derivatives and recursively differentiable quasigroups arises in the theory of recursive codes [1]. Let $Q(A)$ be a n-ary gruppoid and $n, k \in N$, $n \geq 2$. The operation $A^{(k)}$, where:

$$A^{(k)}(x_1^n) = A(x_{k+1}^n, A^0(x_1^n), \dots, A^{k-1}(x_1^n)), \text{ if } k < n,$$
$$A^{(k)}(x_1^n) = A(A^{(k-n)}(x_1^n), \dots, A^{(k-1)}(x_1^n)) \text{ if } k \geq n,$$

is called the k-recursive derivative of A. By definition, $A^{(0)} = A$.

A n-ary quasigroup operation A is called reccursively s-differentiable if its k-recursive derivatives $A^{(0)}, A^{(1)}, \dots, A^{(s)}$ are n-ary quasigroups. For $n = 2$, denoting A by " $\cdot$ " and $A^{(k)}$ by $(\overset{k}{\triangle})$, the recursive derivatives of A are:

$$x \overset{0}{\triangle} y = x \cdot y,\ x \overset{1}{\triangle} y = y \cdot (xy),\ \dots,$$
$$x \overset{k}{\triangle} y = (x \overset{k-2}{\triangle} y) \cdot (x \overset{k-1}{\triangle} y), \forall k \geq 2.$$

Recursively differentiable quasigroups are partially studied in [2,3]. The notion of a core was defined by R.H. Bruck for Moufang loops and lately was generalized by V. Belousov for quasigroups. If $(Q, \cdot)$ is a quasigroup, then the grouppoid $(Q, +)$, where $x + y = x \cdot (y \backslash x)$, for every $x \in Q$, is called the core of $(Q, \cdot)$. Connections between cores and 1-recursive derivatives are studied in the present work for some classes of loops (LIP, left alternative, middle Bol).

Proposition 1. *A left alternative quasigroup $(Q, \cdot)$ is recursively 1-differentiable if and only if the mapping $x \to x \cdot x$ is a bijection on Q.*

Corollary 1. *A left Bol loop $(Q, \cdot)$ is recursively 1-differentiable if and only if the mapping $x \to x \cdot x$ is a bijection.*

Corollary 2. *Finite groups of odd order are recursively 1-differentiable.*

Proposition 2. *In every LIP-loop the core is an isotope of the 1-recursive derivative.*

Corollary 3. *A LIP-loop $(Q, \cdot)$ is recursively 1-differentiable if and only if its core is a quasigroup.*

Proposition 3. *If a middle Bol loop $(Q, \cdot)$ is recursively 1-differentiable then every its loop isotope is recursively 1-differentiable.*

1. E. Couselo, S. Gonzales, V. Markov, A. Nechaev, Recursive MDS-codes and recursively differentiable quasigroups. Diskret. Mat. **10** (1998), no.2, 3-39. (Russian)
2. V. Izbash, P. Syrbu, Recursively differentiable quasigroups and complete recursive codes. Commentat. Math. Univ. Carol. **45** (2004), 257-263.
3. P. Syrbu, On recursively differentiable binary quasigroups. XI Int. Seminar "Discrete Mathematics and Applications", Moscow, MGU, 2012. 430-433. (Russian)

On semisimple matrices and representations of semigroups

O. M. Tertychna

Vadym Hetman Kyiv National Economic University
olena-tertychna@mail.ru

Let I be a finite set without 0 and J a subset in $I \times I$ without the diagonal elements (i,i), $i \in I$. Let $S(I,J)$ denotes the semigroup with zero generated by e_i, $i \in I \cup 0$, with the following defining relations:

1) $e_0^2 = e_0$, $e_0 e_i = e_i e_0 = e_0$ for any $i \in I$, i.e. $e_0 = 0$ is the zero element;

2) $e_i^2 = e_i$ for any $i \in I$;

3) $e_i e_j = 0$ for any $(i,j) \in J$.

The set of all semigroups of the form $S(I,J)$ is denoted by $\mathcal{I}$. Every semigroup $S(I,J) \in \mathcal{I}$ is called *a semigroup generated by idempotents with partial null multiplication* (see, e.g., [1], [2]).

We call a quadratic matrix A over a field k *α-semisimple*, where $\alpha \in k$, if $(A - \alpha E)$ is similar to the direct sum of some invertible and zero matrices. If A is α-semisimple for all $\alpha \in k$, then it is obviously semisimple in the classical sense.

We study 0-semisimple matrices associated with matrix representations of a finite semigroup S from $\mathcal{I}$ over any field k. In particular, we proved the following theorem (see [3]).

Theorem. *Let $S = S(I,J)$ be a finite semigroup from $\mathcal{I}$ and R a matrix representation of S. Then, for any $\alpha_i \in k$, where i runs over I, the matrix $\sum_{i \in I} \alpha_i R(e_i)$ is 0-semisimple.*

The results were obtained together with V. M. Bondarenko.

1. V. M. Bondarenko, O. M. Tertychna. On semigroups genetated by idempotents with partial null multiplication // Transactions of Institute of Mathematics of NAS of Ukraine, 2006, vol. 3, N 4, P. 294–298 (In Russian).
2. V. M. Bondarenko, O. M. Tertychna. On tame semigroups generated by idempotents with partial null multiplication // Algebra Discrete Math., 2008, N 4, P. 15–22.
3. V. M. Bondarenko, O. M. Tertychna. On 0-semisimplicity of linear hulls of generators for semigroups generated by idempotents // Algebra Discrete Math., 2012, vol. 14, N 2, P. 168–173.

On graded Cohen–Macaulay algebras of wild Cohen–Macaulay type

O. Tovpyha

Institute of Mathematics, National Academy of Sciences, Kyiv, Ukraine
tovpyha@gmail.com

It is a joint work with Yuriy Drozd.

Let $\mathbf{k}$ be an infinite field of arbitrary characteristic. We consider a graded $\mathbf{k}$-algebra $R = \bigoplus_{i=0}^{\infty} R_i$ such that $R_0 = \mathbf{k}$ and R is generated in degree 1, i.e. $R = \mathbf{k}[R_1]$, and suppose that R is Cohen–Macaulay of Krull dimension d.

Definition 1. R is called *strictly Cohen–Macaulay infinite (strictly CM-infinite)* if it has a strict family of graded maximal Cohen-Macaulay modules over the polynomial algebra $\mathbf{k}[x]$.

Definition 2. R is called *algebraically Cohen–Macaulay wild (or briefly CM-wild)* if for any finitely generated $\mathbf{k}$-algebra Λ there is a strict family of graded maximal Cohen–Macaulay R-modules over Λ.

We refer to [2] for the definition of strict families of Cohen-Macaulay modules.

Theorem 1. *Let $\mathbf{y} = (y_1, y_2, \ldots, y_d)$ be an R-sequence, where y_i is a graded element of degree $m_i > 0$, $m = \sum_{i=1}^{d} m_i$ and $\overline{R} = R/\mathbf{y}$. Consider a graded component $\overline{R}_c$, where $c > m - d + 1$.*

1) If $\dim \overline{R}_c > 1$, then R is strictly CM-infinite.

2) If $\dim \overline{R}_c > 2$, then R is CM-wild.

If R is a graded coordinate ring of algebraic variety $X \subset \mathbb{P}^n$, i.e. $X = \mathrm{Proj} R$, then X is called *arithmetically Cohen–Macaulay (ACM)* and the category of graded maximal Cohen–Macaulay R-modules is equivalent to the category of *arithmetically Cohen–Macaulay (ACM)* sheaves over X [1]. We say that X is *strictly ACM-infinite (ACM-wild)* if R is *strictly CM-infinite (CM-wild)*.

Corollary 1. *A hypersurface of degree $e \geq 4$ in $\mathbb{P}^n$ is strictly ACM-infinite. If $n \geq 2$ it is CM-wild.*

Corollary 2. *Let $X \subset \mathbb{P}^n$ be a complete intersection of codimension k defined by polynomials $f_1, f_2, \ldots, f_k$ of degrees $\deg f_j = d_j > 1$. If $k \geq 3$ or $d_i \geq 3$ for dome i, then X is ACM-wild.*

1. M. Casanellas and R. Hartshorne, *Gorenstein biliaison and ACM sheaves*, Journal of Algebra **278** (2004), 314–341.
2. Y. Drozd, *Cohen–Macaulay Modules over Cohen–Macaulay Algebras*, Representation Theory of Algebras and Related Topics, CMS Conference Proceedings. **19** (1996), 25–53.

Inversive congruential generator of PRN's of second order

Tran The Vinh, S.P. Varbanets

I.I. Mechnikov Odessa National University
ttvinhcntt@yahoo.com.vn, varb@sana.od.ua

Let $p > 2$ be a prime number. For $m \geq 3$, we denote by $\mathbb{Z}_{p^m}$ the residue ring modulo p^m. In our talk we present some distribution properties of a generalization of pseudorandom number generator first introduced in [1] and defined by a recursion

$$y_{n+1} \equiv a y_n^{-1} + b \pmod{p^m}, \tag{1}$$

where $y_n y_n^{-1} \equiv 1 \pmod{p^m}$ if $(y_n, p) = 1$, and $(a, p) = 1$, $b \equiv 0 \pmod{p}$, an initial value y_0 is co-prime to p.

Let $y_0, y_1 \in \{0, 1, \ldots, p^{m-1}\}$, $(y_0, p) = (y_1, p) = 1$, and let $(a_0, p) = 1$, $a_1 \equiv 0 \pmod{p}$.

Consider the sequence $\{y_n\}$ produced by recurrence

$$y_{n+1} \equiv \frac{a}{a_0 y_{n-1} + a_1 y_n} + b \pmod{p^m}. \tag{2}$$

Using the work [2] we study the sequence of PRNs $\left\{\frac{y_n}{p^m}\right\}$ produced by (2) and infer non-trivial estimation for the discrepancy D_N of points $x_0, x_1, \ldots, x_{N-1}$, where $x_n = \frac{y_n}{p^m}$.

1. Eichenauer-Hermann J. and Topuzöglu A., On the period of congruential pseudorandom number sequences generated by inversions, J.Comput. Appl. Math., 31(1990), 87–96.
2. Varbanets P. and Varbanets S., Generalizations of Inversive Congruential Generator, Analytic and Probabilistic Methods in Number Theory, Proc. of the Int. Conf. in Honour of J. Kubilius, Palanga, Lithuania, 4-10 September 2011, 2012, 265-282.

Pseudo-random numbers produced by norm residue

Tran The Vinh, E. Vernigora

I.I. Mechnikov Odessa National University
ttvinhcntt@yahoo.com.vn, verlin@ukr.net

Consider the quadratic expansion $\mathbb{Q}(\sqrt{-d})$, $d > 0$ is a square-free number.

Let p be a prime number such that $\left(\frac{-d}{p}\right) = 1$. Denote $\mathbb{Z}^* \left[\sqrt{-d}\right]_{p^m}$ is the multiplicative group of inverse elements modulo p^m over $\mathbb{Z}\left[\sqrt{-d}\right]$.

In our talk we study the distribution of elements of the sequence $\{\Re z_n\}$, $n = 0, 1, \ldots$, where

$$z_n \equiv z_{n-1}^{\ell} \pmod{p^m}, n = 1, 2, \ldots$$

with the initial value z_0, $(z_0, p)1$.

On the assumption $e^{p-1} \not\equiv 1 \pmod{p^2}$ we show that the sequence $\left\{\frac{\Re z_n}{p^m}\right\}$ is an equidistributed on $[0, 1)$.

This investigation can be considered as generalization of [1].

1. John B. Friedlander, J. Hansen and I. E. Shparlinski, On the Distribution of the Power Generator Modulo a Prime Power, Proc. DIMACS workshop on unusual applications of number theory 2000, Amer. Math. Soc., (2004), 71–79.

Structural decompositions in module theory and their constraints

Jan Trlifaj

Univerzita Karlova, Praha
trlifaj@karlin.mff.cuni.cz

Some of the classic theorems of module theory yield existence, and even uniqueness, of direct sum decompositions: the Krull-Schmidt type theorems, Kaplansky theorems, etc. These theorems apply to few classes of (not necessarily finitely generated) modules – many other classes of interest do not provide for decompositions into direct sums of indecomposable, or small, submodules.

However, there do exist more general structural decompositions that are almost ubiquitous. The trick is to replace (infinite) direct sums by transfinite extensions.

For example, taking direct sums of copies of the group $\mathbb{Z}_p$, we obtain all $\mathbb{Z}_p$-modules. Their sole isomorphism invariant is the vector space dimension. Transfinite extensions of copies of $\mathbb{Z}_p$ yield the much richer class of all abelian p-groups whose (Ulm-Kaplansky) isomorphism invariants are known only in the totally-projective case.

Over the past decade, numerous classes $\mathcal{C}$ of modules have been shown to be deconstructible, that is, expressible as transfinite extensions of small modules from $\mathcal{C}$. The point is that each deconstructible class is precovering, hence it provides for approximations of modules. By choosing appropriately the class $\mathcal{C}$, we can tailor these approximations to the needs of various particular structural problems, [3].

In this talk, we will concentrate on limits of this theory given by the set-theoretic constraints on existence of structural decompositions.

In 2003, Eklof and Shelah [2] proved that it is consistent with ZFC that the class of all Whitehead groups is not precovering, and hence not deconstructible. The latter fact is not provable in ZFC, because it is also consistent that all Whitehead groups are free. More recently, it has been shown in ZFC that the class $\mathcal{F}$ of all flat Mittag-Leffler modules is deconstructible, iff the underlying ring R is perfect, [4]. Moreover, $\mathcal{F}$ is not precovering in case R is countable and non-perfect, [1], [5].

Suprisingly, the latter results are not exceptional, but represent a general phenomenon. We will present a new proof of the main result from [4] using trees on cardinals and their decoration by Bass modules that reveals a connection of this phenomenon to infinite dimensional tilting theory: given a tilting module T, we can form the category $\mathrm{Add}(T)$ of all direct summands of direct sums of copies of T – these are the T-free objects. We then consider the class $\mathcal{L}(T)$ of all locally T-free modules. It turns out that if $\mathrm{Add}(T)$ is not pure-split, then often $\mathcal{L}(T)$ is not precovering, [6].

For example, if $T = R$ (the regular representation), then $\mathrm{Add}(R)$ is the class of all projective modules, which is pure-split, iff R is right perfect. Then $\mathcal{L}(R) = \mathcal{F}$, and we recover the results from [1], [4], and [5] mentioned above.

The non-precovering phenomenon can be traced much further on, e.g., to countable finite dimensional hereditary algebras A: if $T = L$ (the Lukas tilting module), then the class $\mathcal{L}(L)$ of all locally Baer modules is precovering, iff A has finite representation type, [6].

1. S.Bazzoni; J.Šťovíček, *Flat Mittag-Leffler modules over countable rings*, Proc. Amer. Math. Soc. **140** (2012), 1527 – 1533.
2. P.C.Eklof; S.Shelah, *On the existence of precovers*, Illinois J. Math. **47** (2003), 173 – 188.

3. R.Göbel; J.Trlifaj, *Approximations and Endomorphism Algebras of Modules*, 2nd revised and extended edition, GEM **41**, De Gruyter, Berlin/Boston 2012.
4. D.Herbera; J.Trlifaj, *Almost free modules and Mittag-Leffler conditions*, Advances in Math. **229**(2012), 3436 – 3467.
5. J.Šaroch; J.Trlifaj, *Kaplansky classes, finite character, and $\aleph_1$-projectivity*, Forum Math. **24**(2012), 1091-1109.
6. A.Slávik; J.Trlifaj, *Approximations and locally free modules*, preprint, arXiv:1210.7097.

Groups with supplemented subgroups

M.V. Tsybanyov

Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
tsybanyov@rambler.ru

Definition 1. Let G be a group, a subgroup $B < G$ is called a supplement to subgroup A in group G if $AB = G$. In this case it is said, that the subgroup A is supplemented in the group G.

This concept had been introduced by P. Hall [1], he called the above mentioned subgroup B as a partial complement of the subgroup A. It is obvious that Definition 1 differs from a concept of supplement used in finite groups [2].

Definition 2. A supplement B of a subgroup A in group G is called minimal if for any subgroup $B_1 < B$ the product $AB_1 \neq G$.

A requirement of existence of a supplement to every non-unit subgroup of a group is strong enough. You can see it from the following propositions.

Proposition 1. *If every non-unit periodical subgroup of a group G is supplemented in G, then this group is primitively factorized (definition cf. [3]).*

Proposition 2. *Let G be an infinite group. Every infinite subgroup from G have a minimal supplement in G if and only if group G is F-factorized (definition cf. [4]).*

The author considered the question of existence of non-supplemented infinite subgroups in a group and received the following results.

Theorem 1. *Let abelian group A with group of operators G be a direct sum $A = A_1 \oplus A_2 \oplus ... \oplus A_n \oplus ...$ of infinite set of finite indecomposable G-isomorphic G-admissible subgroups, the orders of which are not prime. If factor-group $G/C_G(A)$ is non-cyclic, then there exists such an infinite subgroup of A, which doesn't have G-admissible supplement in A.*

1. Hall P. The construction of soluble groups. J. Reine Angew. Math. - 1940, 182, N.3/4, 206-214.
2. Shemetkov L.A. Formations of finite groups. - M.:Nauka - 1978, 272 p.
3. Gorchakov U.M. About primarily factorable groups. Ukr. Matem. J. - 1962, XIV., 3-9.
4. Tsybanyov M.V. Soluble F-factorized groups. Sib. Matem. J. - 1980, 21, N.2, 202-208.

The Maximal Subgroups of the Semigroup of the Correspondences of the Finite Group

Tetyana Turka

Donbass State Pedagogical University
tvturka@mail.ru

Let G be a universal algebra. If a subalgebra of $G \times G$ is considered as a binary relation on G, then the multitude $S(G)$ of all the subalgebras from $G \times G$ is the semigroup relatively de Morgan's product ratios. The semigroup $S(G)$ is called *a semigroup of correspondence* of algebra G.

The problem of the study of the semigroups of correspondence has been set by Kurosh O.G. [1].

We examine the question of the structure of the maximal subgroups of the semigroup of the correspondences $S(G)$, when G is a finite group. In this case, the elements of the semigroup $S(G)$ can be specified by the fives $(H_1, G_1, H_2, G_2, \varphi)$, where $G \geq G_i > H_i$ $(i = 1, 2)$, and $\varphi : G_1/H_1 \to G_2/H_2$ is isomorphism [2].

For any idempotent $e \in S$ through G_e let us denote the maximal subgroup of S, for which e is a unit.

Theorem 1. *Let G be the finite group, and the element $e = (H_1, G_1, H_2, G_2, \varphi)$ of the semigroup of the correspondences of $S(G)$ is idempotent. Then*

$$G_e = \{(H_1, G_1, H_2, G_2, \psi\varphi) | \psi \in \mathrm{Aut}(G_1/H_1)\} =$$

$$= \{(H_1, G_1, H_2, G_2, \varphi\psi) | \psi \in \mathrm{Aut}(G_2/H_2)\}.$$

In particular, if $G_1 = G_2 = G$, $H_1 = H_2 = H$, $\varphi = \varepsilon$ is the identical automorphism, then

$$G_e = \{(G, H, G, H, \psi) | \psi \in \mathrm{Aut}(G/H)\}.$$

1. Kurosh A.G. General algebra (lectures of 1969-70). (Russian) — M.: Nauka, 1974.–160 p.
2. Ganyushkin O.G., Turka T.V. The order of the semigroups of the correspondences of the finite group //(Ukrainian) The Bulletin of the University of Kiev. Series: Mechanics and mathematics. — 2009. — № 3. — P. 9–13.

On induced representations of groups of finite rank

A.V. Tushev

Dnepropetrovsk National University, Dnepropetrovsk, Ukraine
tushev@member.ams.org

We recall that a group G has finite (Prufer) rank if there is an integer r such that each finitely generated subgroup of G can be generated by r elements; its rank $r(G)$ is then the least integer r with this property. A group G is said to have finite torsion-free rank if it has a finite series in which each factor is either infinite cyclic or locally finite; its torsion-free rank $r_0(G)$ is then defined to be the number of infinite cyclic factors in such a series.

A group G is said to be minimax if it has a finite series each of whose factor is either cyclic or quasicyclic. The set SpG of all prime numbers p such that the minimax group G has a p-quasicyclic factor is said to be the spectrum of the group G. It follows from results of [1] that any finitely generated metabelian group of finite rank is minimax.

Theorem 1. *Let G be a finitely generated metabelian group of finite Prufer rank, let k be a field of characteristic $p > 0$, such that $p \notin SpG$ and let M be an irreducible kG-module such that $C_G(M) = 1$. If the group G is not nilpotent-by-finite then here are a subgroup $H \leqslant G$ and an irreducible kH-submodule $U \leqslant M$ such that $M = U \otimes_{kH} kG$ and $r_0(H) \leqslant r_0(G)$*

Corollary 1. *Let G be a finitely generated metabelian group of finite Prufer rank, and let k be a field of characteristic $p > 0$, such that $p \notin SpG$. Suppose that G is an extension of an abelian group A by a cyclic group $< g >$. If the group G is not nilpotent-by-finite then every faithful irreducible representation of G over k is induced from an irreducible representation of the group A over k*

The corollary spreads some results of [2] to the case of fields of nonzero characteristic. An example constructed by Wehrfritz in [3] shows that the restriction on characteristic $p > 0$ of the field k ($p \notin SpG$) is essential.

1. Ph. Hall On the finiteness of certain soluble groups , Proc. London Math. Soc.- 1959.- V. 9.- P. 595-622.
2. A.V. Tushev Spectra of conjugated ideals in group algebras of abelian groups of finite rank and control theorems , Glasgow Math. J. - 1996. - 38. - P. 309-320.
3. B.A.F. Wehrfritz Groups whose irreducible representations have finite degree , Math. Proc. Camb. Phil. Soc.- 1981.- V. 90.- P. 411-421.

On Reducibility of Monomial Matrixes over Commutative Local Rings

A. Tylyshchak, M. Bortos

Uzhgorod National University
alxtlk@tn.uz.ua, bortosmaria@gmail.com

The task of similarity of matrixes over rings instead over fields is not so investigated and had been solved only for square matrices of small degree over some principle ideal rings (see [1]-[4]). Moreover, the knowledge of all irreducible matrices of any degree over a commutative ring R with identity is also still far from complete. The problem of reducibility of the matrix

$$M(t, s_1, \dots, s_n) = \begin{pmatrix} 0 & \dots & 0 & t^{s_n} \\ t^{s_1} & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & t^{s_{n-1}} & 0 \end{pmatrix}$$

of arbitrary order n over the ring R is considered ($t \in R$, $s_1, \dots, s_n \geqslant 0$). The problem of reducibility of the matrix $M(t, 0, \dots, 0, 1, \dots, 1)$ over the ring R investigated in [5].

Theorem 1. *Let R be a commutative ring with identity, $t \in R$, n be a positive integer, $s_1, \dots, s_n \geqslant 0$. If $(n, \sum_{i=1}^n s_i) > 1$ then the matrix $M(t, s_1, \dots, s_n)$ is reducible.*

Theorem 2. *Let R be a commutative local ring and the Jacobson radical of ring R be a principle ideal with generator t where $t^2 \neq 0$, n be a positive integer, $s_1, \dots, s_n \in \{0, 1, 2\}$, $n < 5$. The matrix $M(t, s_1, \dots, s_n)$ is reducible if and only if $(n, \sum_{i=1}^n s_i) > 1$.*

It has been shown that founded criterion is not hold if $n = 5$.

Proposition 1. *Let R be a commutative local ring and the Jacobson radical of ring R be a principle ideal with generator t where $t^2 \neq 0$, $t^3 = 0$. Then the matrix $M(t, 0, 0, 2, 2, 2)$ is reducible over R. (Here $n = 5$, $\sum_{i=1}^n s_i = 6$.)*

1. A. Pizarro. Similarity Classes of 3×3 Matrices over a Discrete Valuation Ring. Linear Algebra and Its Applications. 1983, vol. 54, 29–51.
2. V. N. Shevchenko, S. V. Sidorov. On the similarity of second-order matrices over the ring of integers. (Russian) Izv. Vyssh. Uchebn. Zaved. Mat. 2006, no. 4, 57–64; translation in Russian Math. (Iz. VUZ) 50 (2006), no. 4, 56—63.
3. A. Prasad, P. Singla, and S. Spallone. Similarity of matrices over local rings of length two. `http://arxiv.org/pdf/1212.6157.pdf`.
4. H. Appelgate and H. Onishi. The similarity problem for 3×3 integer matrices. Linear Algebra Appl., 42:159–174, 1982.
5. A. A. Tylyshchak, R. F. Dinis. On Reducibility of Matrixes of Some Type over Commutative Local Principle Ideals Rings. Naukovij Visnik Uzhgorod Universitatu, ser. mat. and inform., 23 №1 (2012), 57–62.

Quasiidentities of nilpotent free A-loop and of nilpotent free Moufang loop

V. Ursu, A. Covalschi

Institute of Matematics and Computer Science Simion Stoilow of the Romanian Academy,
Technical University of Moldova
State Pedagogical University Ion Creangă of Moldova
Vasile.Ursu@imar.ro, Alexandru.Covalschi@yahoo.com

In this paper the loops are investigated as algebras of signature $< \cdot, /, \backslash, 1 >$ where the following identities $x(x\backslash y) = x\backslash(xy) = (y/x)x = (yx)/x = y$, $1x = x1 = x$ are true where 1 is called unit of loop. If all the internal substitutions of loop L are automorphism then it is called A-loop [1]. The loop where the identity $xy \cdot zx = x(yz \cdot x)$ is true is called Moufang loop [2].

Let K be a class of loops. The loop L is called free in class K or K-free if: a) $F \in K$; b) There is a subset $X \subseteq F$ so that $F =< X >$ and for any loop $L \in K$ any application $\varphi_0 : F \to L$ can be extended up to a homomorphism $\varphi : F \to L$. The elements of set X are called free generators, but the order of X is the rang of free loop F in class K. If K is made of all nilpotent A-loops (respectively Moufang nilpotent loops) of nilpotent class $\leq n$, then we'll state that K free loop F is nilpotent (or n-nilpoten) free loop.

Theorem 1. *A nilpotent free A-loop of any rang has an infinite and independent base of quasiidentities but the quasirang of the quasivariety generated by it is infinite.*

Theorem 2. *A nilpotent free Moufang loop has an infinite and independent base of quasiidentities but the quasirang of the quasivariety generated by it is infinite.*

1. R.H. Bruck, L.J. Paige. Loops whose inner mappings are automorphisms. Ann. of Math., 1956, 68, 308 - 323.
2. O. Chein, H.O. Pflugfelder, J.D.H. Smith. Quasigroups and Loops: Theory and applications. Berlin: Helderman Verlag, 1990.

Distribution of solutions of congruence on two variables

P. Varbanets

I.I. Mechnikov Odessa National University
varb@sana.od.ua

Let $f(x,y)$ be a polynomial with integer coefficients, p be a prime, m be a positive integer, $J_{p^m}(f)$ be the set of solutions in $\mathbb{Z}^2_{p^m}$ of the congruence

$$f(x,y) \equiv 0 \pmod{p^m} \tag{1}$$

and

$$R(T_1,T_2) = \left\{(x,y) \in \mathbb{Z}^2_{p^m},\ 1 \le x \le T_1 \le p^m,\ 1 \le y \le T_2 \le p^m\right\}.$$

Our talk is attached to the problem of construction of asymptotic formula for the numbers of solutions of (1) contained in $R(T_1,T_2)$.

We consider two type of the congruence (1):

(i) the solutions of (1) allow the p-adic description;

(ii) the polynomial $f(x,y)$ is quasi-homogeneous.

Generalized twisted Kloosterman sum over $\mathbb{Z}[i]$

S. Varbanets

I.I. Mechnikov Odessa National University
varb@sana.od.ua

In [1] we considered two type of general Kloosterman sums over $\mathbb{Z}[i]$

$$K_\chi(\alpha,\beta;k;\gamma)=\sum_{\substack{x \pmod{\gamma}\\(x,\gamma)=1}}\chi(x)\exp\left(\pi i Sp\frac{\alpha x^k+\beta x'^k}{\gamma}\right),$$

where α, β, $\gamma\in\mathbb{Z}[i]$, χ is a multiplicative character modulo γ.

$$\widetilde{K}(\alpha,\beta;h,q;k)=\sum_{\substack{x,y\in\mathbb{Z}[i]\\x,y \pmod{q}\\N(xy)\equiv h(mod\,q)}} e_q\left(\frac{1}{2}Sp(\alpha x^k+\beta y^k)\right),$$

where $\alpha,\ \beta\in\mathbb{Z}[i],\ h,q\in\mathbb{N},\ (h,q)=1$.

We call $K(\alpha,\beta;k;\gamma,\chi)$ the general power Kloosterman sum and $\widetilde{K}(\alpha,\beta;h,q;k)$ call the norm Kloosterman sum.

Let a modulus $q_1\in\mathbb{Z}^+$, and let χ be a Dirichlet character modulo q_1. Over the ring of Gaussian integers $G=\mathbb{Z}[i]$ we define the following generalized twisted Kloosterman sum with the multiplicative function χ for any q, $q\equiv 0 \pmod{q_1}$:

$$K_\chi(\alpha,\beta;\gamma;q)=\sum_{\substack{x,y\in G_q\\xy\equiv\gamma \pmod{q}}}\overline{\chi(N(x))}e^{2\pi i Sp\left(\frac{\alpha x+\beta y}{q}\right)}. \tag{1}$$

Note that χ is not generally a Dirichlet character modulo q, because it can be happened that $\chi(N(x))\neq 0$ when $(x,q)\neq 1$.

In the special case where $\gamma=1$ and $q_1=q$ we obtain the twisted Kloosterman sum with a character χ defined by

$$K_\chi(\alpha,\beta;q)=\sum_{\substack{x,y\in G_q^*\\xy\equiv 1 \pmod{q}}}\overline{\chi(x)}e^{\pi i Sp\left(\frac{\alpha x+\beta y}{q}\right)}. \tag{2}$$

We proved the following main theorem.

Theorem 1. *Let α, β, γ be the Gaussian integers, $q>1$ be a positive integer, χ be a Dirichlet character modulo q_1, $q_1|q$. Then the following estimate*

$$|K_\chi(\alpha,\beta;\gamma;q)|\leq\overline{\tau}(\gamma)\overline{\tau}(q)\sqrt{N(\alpha\gamma,\beta\gamma,q)}q^m$$

holds.

1. Varbanets S., The Norm Kloosterman Sums Over Z [i], Anal. Probab. Methods Number Theory, 2007. p.225-239.

Fundamental domains of crystal sets group of symmetries of hyperbolic geometry H^2

Varekh N.V., Gerasimova O.I., Dyshlis O.A., Tsibaniov M.V.

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
doozer@ua.fm

Definition 1. Geometry is a couple $\Gamma = (X, G)$, where X is a small manifold, G is a maximal group, which makes transitive operation on X and also any point of X stabilizer is compact.

Definition 2. Geometry $\Gamma = (X, G)$ and geometry $\Gamma' = (X', G')$ are equal if diffeomorfism X into X' exists and translates action of group G into action of group G'.

In this paper fundamental domains of crystal sets group of symmetries of Lobachevski two-dimentional geometry H^2 and also comformal models of geometry H^2 corresponding decompositions are found. They are physical models of two-dimentional quasicrystals.

The result of investigation is

Theorem 1. *Arbitary crystal set of two-dimentional Lobachevsky geometry H^2 fundamental domain of group of symmetry is a triangle with two zero angles and third angle $2\pi/n$, where n is one of the numbers: 8,10,12.*

1. П.Скотт. Геометрии на трехмерных многообразиях. – М. Мир., 1986 – 168с.

On $\mathfrak{F}$-modular subgroups of finite groups

V.A. Vasil'ev

Francisk Scorina Gomel State University
`vovichx@gmail.com`

Throughout this abstract, all groups are finite. We denote by $\mathfrak{N}$ the class of all nilpotent groups.

Let M, A be subgroups of a group G. We will say that M forms a modular pair (M, A) with A, if

(1) $\langle X, M \cap A\rangle = \langle X, M\rangle \cap A$ for all $X \leq G$ such that $X \leq A$, and

(2) $\langle M, A \cap Z\rangle = \langle M, A\rangle \cap Z$ for all $Z \leq G$ such that $M \leq Z$.

On the basis of this concept, we introduce the following

Definition 1. Let $\mathfrak{F}$ be a class of groups. A subgroup M of a group G is called $\mathfrak{F}$-modular, if (M, H) is a modular pair for all $H \leq G$ such that $H \in \mathfrak{F}$.

We received the following result.

Theorem 1. *Every maximal subgroup of a solvable group G is $\mathfrak{N}$-modular if and only if G is supersolvable and in every its non-Frattini chief factor H/K the automorphism group of the order not more than a prime is induced, i.e. $|G : C_G(H/K)|$ is a prime or 1.*

On permutizers of Sylow subgroups of finite groups

A.F. Vasil'ev, V.A. Vasil'ev, T.I. Vasil'eva

Francisk Skorina Gomel State University
Belarusian State University of Transport
formation56@mail.ru

All groups considered are finite. In the theory of groups the normalizer of a subgroup is a classical concept, about which there are many well-known results. For example, a group G is nilpotent if and only if $N_G(P) = G$ for every Sylow subgroup P of G.

A natural generalization of subgroup's normalizer is the concept of the permutizer of a subgroup introduced in [1].

Definition 1. [1, p. 27] Let H be a subgroup of a group G. The permutizer of H in G is the subgroup $P_G(H) = \langle x \in G \mid \langle x\rangle H = H\langle x\rangle\rangle$.

We need the following definitions from [2]:

A subgroup H of a group G is called $\mathbb{P}$-*subnormal* in G, if either $H = G$, or there exists a chain of subgroups $H = H_0 < H_1 < \cdots < H_{n-1} < H_n = G$ such that $|H_{i+1} : H_i|$ is a prime for every $i = 0, 1, \ldots, n-1$.

A group G is called w-*supersoluble*, if every Sylow subgroup of G is $\mathbb{P}$-subnormal in G. Denote by $\mathrm{w}\mathfrak{U}$ the class of all w-supersoluble groups.

Recall some properties of w-supersoluble groups [2].

Theorem 1. *[2] The following statements are true:*

1) Every w*-supersoluble group is Ore dispersive.*

2) The class $\mathrm{w}\mathfrak{U}$ *is a hereditary saturated formation and it has the local function* f *such that* $f(p) = (G \in \mathfrak{S} | \mathrm{Syl}(G) \subseteq \mathfrak{A}(p-1))$ *for every prime* p.

3) Every biprimary subgroup of a w*-supersoluble group is supersoluble.*

New characterization of w-supersoluble groups is received.

Theorem 2. *A group* G *is* w*-supersoluble if and only if* $P_U(P) = U$ *for every Sylow subgroup* P *of* G *and every subgroup* $U \geq P$.

1. Between Nilpotent and Solvable / H.G. Bray [and others]; edited by M. Weinstein. Passaic: Polugonal Publishing House, 1982.
2. A.F. Vasil'ev, T.I. Vasil'eva, V.N. Tyutyanov, On the finite groups of supersoluble type // Siberian Math. J. 2010. Vol. 51, No. 6. P. 1004–1012.

On factorial representation of real numbers and its modification

N. M. Vasylenko

National Pedagogical Dragomanov University
samkina_nata@mail.ru

It is known [3] that any real number $x \in [0, e]$ can be represented in the form

$$x = \sum_{k=1}^{\infty} \frac{a_k}{k!} \equiv \Delta_{\alpha_1 \alpha_2 \ldots \alpha_k \ldots}, \quad a_k \in \mathcal{A}_k = \{0, 1, \ldots, k\}. \tag{1}$$

A representation of the number $x \in [0, e]$ in the form (1) is called the *factorial representation*. Its basic properties are studied in [3].

In the talk, we study modified factorial representation of real numbers belonging to $[0, 1]$ and compare two mentioned representations.

In 1967 I. Niven proved [2] that any real number x can be represented in the form of the Cantor series [1], i.e.,

$$x = \alpha_0 + \sum_{n=1}^{\infty} \frac{\alpha_n}{b_1 b_2 \ldots b_n},$$

where $b_n > 1$, $\alpha_n \in \{0, 1, \ldots, b_n - 1\}$, $n \in \mathbb{N}$, $\alpha_0 \in \mathbb{Z}$.

For $\alpha_0 = 0$ and $b_n = n + 1$, we obtain representation of numbers in the form

$$x = \sum_{n=1}^{\infty} \frac{\alpha_n}{(n+1)!} \equiv \Delta^{!}_{\alpha_1 \alpha_2 \ldots \alpha_n \ldots}, \quad \alpha_n \in \mathcal{A}_n \equiv \{0, 1, \ldots, n\}. \tag{2}$$

Theorem 1. *Any real number $x \in [0, 1]$ can be represented in the form* (2).

A representation of the number $x \in [0, 1]$ in the form (2) is called the *modified factorial representation*.

It is natural to ask: "How many different modified factorial representations does any real number $x \in [0, 1]$ have?" In the talk we propose some theorems answering this question.

1. *Cantor G.* Über die einfachen Zahlensysteme // Zeitschrift für Mathematik und Physik, 1869. — **14**. — P. 121-128; also in Collected Works, Springer-Verlag, Berlin, 1932, P. 35-42.
2. *Niven I.* Irrational Numbers. — Washington DC: The Mathematical Association of America, 1967. — 161 p.
3. *Samkina N.M., Shkolnyi O.V.* Factorial number system and related probability distributions // Fractal Analysis and Related Problems. — 1998. — no. 2. — P. 157-165.

Cyclic self-similar groups

D. Vavdiyuk

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
super_qwertyk@ukr.net

Let $d \geqslant 2$ be a natural number. We consider a regular d-ary rooted tree T_d and fix a numeration of vertices, which start in the root. Then any automorphism g of the tree T_d can be uniquely expressed as:

$$g = (g_1, g_2, \ldots, g_d)\pi, \tag{1}$$

where $g_1, g_2, \ldots, g_d$ are some automorphisms of T_d and π is a permutation from the symmetric group S_d. These automorphisms are called first level states of the automorphism g. The nth level states are first level states of $(n-1)$th level states of the automorphism g. The automorphism g itself is the zero level state of g. Presentation (1) is called the wreath recursion of g.

A subgroup G of the automorphisms group T_d is called self-similar if all states of arbitrary automorphism $g \in G$ belong to G ([1, 2]).

Any cyclic self-similar group G can be generated by an element g having the following wreath recursion:

$$g = (g^{\alpha_1}, g^{\alpha_2}, \ldots, g^{\alpha_d})\pi,$$

where $\alpha_1, \ldots, \alpha_d \in \mathbb{Z}$, $\pi \in S_d$.

In this case we use the following notation:

1) $O_\pi(i) = \{j \in X | \pi^n(i) = j, n \in \mathbb{N}\}$ (the orbit of i in π).

2) $S(i) = \sum\limits_{j=1}^{|O_\pi(i)|} \alpha_{\pi^{j-1}(i)}$.

Theorem 1. *A group G is finite if and only if $S(i)$ is divisible by $|O_\pi(i)|$ for all $i \in X$. In this case $|G| = n$, where n is the order of the permutation π.*

Theorem 2. *1) Let g has finite number of states. Then*

a) G is finite or

b) $|S(i)| \leqslant |O_\pi(i)|$ for all $i = 1, \ldots, d$.

2) If $|S(i)| < |O_\pi(i)|$ for all $i = 1, \ldots, d$ then g has finite number of states.

1. Grigorchuk R. I., Nekrashevich V. V., Sushchanskii V. I. Automata, dynamical systems and groups, Proceedings of the Steklov Institute of Mathematics, 231, (2000), 128-203.
2. Nekrashevych. V. Self-similar groups, volume 117 of Mathematical Surveys and Monographs. Amer. Math. Soc., Providence, RI, 2005.

About the structure of multiplicative idempotent semirings

E. M. Vechtomov, A. A. Petrov

Vyatka State University of Humanities
vecht@mail.ru

We study the semirings with identity $xx = x$ (*multiplicative idempotent* semirings).

A *semiring* is an algebraic system $\langle S, +, \cdot \rangle$ such that $\langle S, + \rangle$ is a commutative semigroup, $\langle S, \cdot \rangle$ is a semigroup, rules of distributivity of multiplication with respect to addition hold. If there is a semiring S element 0, such that $x + 0 = 0 + x = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$, then we call S a *semiring with a zero* 0.

Multiplicative idempotent semiring with identity $x + x = x$ is called *idempotent*. A binary relation ρ on an arbitrary multiplicative idempotent semiring S, defined by the relation $x\rho y \Leftrightarrow x + x = y + y$, is a congruence on S, on which the factor-semiring S/ρ is idempotent.

A semiring S is called *extension of the family* semirings A_i $(i \in I)$ by means semiring of B, if there a congruence ρ on S such that $S/\rho \cong B$ and every class $[a_i]_\rho$ is subsemiring in S isomorphic to the corresponding semiring A_i.

Theorem 1. *Every multiplicative idempotent semiring S is an the extension of the family semirings with identity $x + x = y + y$ by means idempotent semirings.*

Study the structure of multiplicative idempotent semirings S with identity $x + x = y + y$. The element $\theta = x + x$ has the properties: $\theta + \theta = \theta$ and $x\theta = \theta x = \theta$ for all $x \in S$.

We introduce the congruence σ on S by the formula: $x\sigma y \Leftrightarrow x + \theta = y + \theta$.

A class $[\theta]_\sigma$ is a semiring with *constant addition* $(x + y = \theta)$ and subtractive ideal in S, so that the congruence of Bourne on it coincides with σ. Factor-semiring $S/\sigma \cong S + \theta$ is a Boolean ring.

A semiring S is called a 0-*extension* semiring A by means semiring B with zero 0 if on S there is a congruence ρ, such that $S/\rho \cong B$ and the inverse image of 0 for the canonical epimorphism $S \to S/\rho$ is isomorphic to A.

Theorem 2. *Any multiplicative idempotent semiring S with the identity $x + x = y + y$ is* 0-*extension multiplicative idempotent semiring with the constant addition by means a Boolean ring.*

Theorem 3. *Any multiplicative idempotent semiring with identity $x + xyx + xyx = x$ is extension of the family of Boolean rings by means idempotent semiring with identity $x + xyx = x$.*

Theorem 4. *Any idempotent semiring with identity $x + xyx = x$ is extension of the family of the semirings with identity $xyx = x$ by means distributive lattice.*

On finite groups with given embedding property of Sylow subgroups

A.S. Vegera

Francisk Skorina Gomel State University, Belarus
vegera.artem@gmail.com

We consider only finite groups. Properties of embedding Sylow subgroups in the group can, in many cases, determine the structure of the group itself. In [1] A.F. Vasil'ev initiated the research of the following problem. Let $\mathfrak{F}$ be a formation. What can be said about the structure of group G, if all its Sylow subgroups $\mathfrak{F}$-subnormal in G? Investigations on this issue have been continued in the work [2].

In 1978 O. Kegel [3] introduced the definition of $\mathfrak{F}$-reachable subgroups of group.

Definition 1. Let $\mathfrak{F}$ be a non-empty hereditary formation. A subgroup H of a group G is called K-$\mathfrak{F}$-subnormal ($\mathfrak{F}$-reachable [3]) subgroup of G (denoted H K-$\mathfrak{F}$-sn G), if there is a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$ such that or $H_{i-1} \triangleleft H_i$, or $H_i^{\mathfrak{F}} \subseteq H_{i-1}$, for $i = 1, \ldots, n$.

In [4] we introduced the definition of the class groups, whose all Sylow subgroups are K-$\mathfrak{F}$-subnormal. Further we generalize the definition of this class and investigate some of its properties.

Definition 2. Let $\mathfrak{F}$ be a non-empty formation and $\pi \subseteq \pi(\mathfrak{F})$. Denote the class groups $\overline{w}_\pi\mathfrak{F} = (G \mid \forall H \in \mathrm{Syl}_p(G), p \in \pi :\ H \text{ K-}\mathfrak{F}\text{-sn } G)$.

Lemma 1. *If $\mathfrak{F}$ is a hereditary formation, then $\overline{w}_\pi\mathfrak{F}$ is a hereditary formation.*

Lemma 2. *Let $\mathfrak{F}$ be a hereditary formation. Then:*

1) $\mathfrak{F} \subseteq \overline{w}_\pi\mathfrak{F}$.

2) $\mathfrak{N} \subseteq \overline{w}_\pi\mathfrak{F}$.

3) $\overline{w}_\pi(\overline{w}_\pi\mathfrak{F}) = \overline{w}_\pi\mathfrak{F}$.

Theorem 1. *Let $\mathfrak{F}$ be a hereditary saturated formation and $\tau = \pi(\mathfrak{F})$. Then $\overline{w}_\pi\mathfrak{F} \cap \mathfrak{G}_\tau$ is a hereditary saturated formation.*

1. A.F. Vasil'ev (1995). On the influence of primary $\mathfrak{F}$-subnormal subgroups on the structure of group. Problems in algebra, Vol. 8. P. 31–39.
2. A.F. Vasil'ev, T.I. Vasil'eva (2011). On finite groups with generalized subnormal Sylow subgroups. Problems of physics, mathematics and technics, № 4 (9). P. 86–91.
3. O.H. Kegel (1978). Untergruppenverbände endlicher Gruppen, die den Subnormalteilerverband echt enthalten. Arch. Math. Bd. 30, № 3. P. 225–228.
4. A.S. Vegera (2012). On saturated formations of finite groups, which determined by properties of embedding Sylow subgroups. Bulletin of the Francisk Skorina Gomel State University, № 6 (75). P. 154–158.

Additive arithmetical functions over matrix rings

I. Velichko

Odessa National University
velichko.pochta@gmail.com

Let $M_k(\mathbb{Z})$ be the ring of integer matrices of order k. We will call $P \in M_k(\mathbb{Z})$ a prime matrix, if

$$P = \begin{pmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ . & \dots & . & . & \dots & . \\ c_1 & \dots & c_{l-1} & p & \dots & 0 \\ . & \dots & . & . & \dots & . \\ 0 & \dots & 0 & 0 & \dots & 1 \end{pmatrix},$$

where $c_1, ..., c_{l-1} \in \mathbb{Z},\ 0 \le c_i < p,\ i = \overline{1, l-1}$, p is prime.

Denote as $w_p^{(k)}(A),\ A \in M_k(\mathbb{Z})$ the number of representations of matrix A as $A = PA_1$, where $A_1 \in M_k(\mathbb{Z})$. It is interesting to investigate asymptotic behavior of the sum

$$W_k(x) = \sum_{n \le x} w^{(k)}(n).$$

for cases $k > 2$.

Using facts about Riemann zeta-function and results of A. Ivic [1], the following theorem can be proved:

Theorem 1. *Let $N > 1$ be an arbitrary integer. Then there exist computable constants A, B, c_j ($A \ne 0$) such that*

$$W_3(x) = x^3(A \log\log x + B) + x^3 \sum_{j=1}^{N} c_j \log^{-j} x + O(x^3 \log^{-N-1} x).$$

Analogous results can be obtained for $k > 3$.

1. Aleksandar Ivić, Sums of products of certain arithmetical functions. Publications de l'institut mathematique, Nouvelle serie, tome 41 (55), 1987, pp. 31-41.
2. R. D. Dixon, On a generalized divisor problem, J. Indian Math. Soc. 28(1964), pp. 187-196.
3. Fugelo N. Average orders of divisor function of integer matrices in $M_3(\mathbb{Z})$. N. Fugelo, I. Velichko, Annales Univ. Sci. Budapest. 28(2008), pp. 261–270.

Non-periodic groups whose finitely generated subgroups are either permutable or weakly pronormal

T.V. Velichko

Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
etativ@rambler.ru

In the papers [1, 2, 3] the groups, whose subgroups are either subnormal or pronormal, have been considered. In the paper [4] the authors described the locally finite groups, whose finitely generated subgroups are either permutable or pronormal.

We consider some infinite groups whose finitely generated subgroups are either permutable or weakly pronormal. In the non-periodic case we need some additional restriction. We recall that a group G is called a generalized radical, if G has an ascending series whose factors are locally nilpotent or locally finite.

Let G be a group. A subgroup H is called weakly pronormal in G if the subgroups H and H^x conjugate in $H^{\langle x \rangle}$ for each element $x \in G$

Theorem 1. *Let G be a locally generalized radical group whose finitely generated subgroups are either weakly pronormal or permutable. If G is non-periodic then every subgroup of G is permutable.*

1. Legovini P. Gruppi finite i cue sottogruppi sono o subnormali o pronormali. Rendiconti del Seminario Matematico della Universita di Padova 58, (1977), p. 129-147.
2. Legovini P. Gruppi finite i cue sottogruppi sono o subnormali o pronormali. Rendiconti del Seminario Matematico della Universita di Padova 65, (1981), p. 47-51.
3. Kurdachenko L.A., Subbotin I.Ya., Chupordya V.A. On some near to nilpotent groups. Fundamental and Applied Mathematics 14(6), (2008), p. 121-134.
4. Kurdachenko L.A., Subbotin I.Ya., Ermolkevich T.V. Groups whose finitely generated subgroups are either permutable or pronormal. Central European Journal of Mathematics 3(4), (2011), p. 459-473.

Bounds for the quantifier depth in finite-variable logics: Alternation hierarchy

Oleg Verbitsky

Institute for Applied Problems of Mechanics and Mathematics, Lviv, Ukraine
overbitsky@googlemail.com

Given two structures G and H distinguishable in FO^k (first-order logic with k variables), let $A^k(G,H)$ denote the minimum quantifier alternation depth of a FO^k formula distinguishing G from H. Let $A^k(n)$ be the maximum value of $A^k(G,H)$ over n-element structures. We prove the strictness of the quantifier alternation hierarchy of FO^2 in a strong quantitative form, namely $A^2(n) > n/8-2$, which is tight up to a constant factor. For each $k \geqslant 2$, it holds that $A^k(n) > \log_{k+1} n - 2$ even over colored trees, which is also tight up to a constant factor if $k \geqslant 3$. For $k \geqslant 3$ the last lower bound holds also over uncolored trees, while the alternation hierarchy of FO^2 collapses even over all uncolored graphs.

This is a joint work with Christoph Berkholz and Andreas Krebs.

On the product of Lockett functors

E.A. Vitko, N.T. Vorob'ev

P.M. Masherov Vitebsk State University
alenkavit@tut.by vorobyovnt@tut.by

The notations used in this paper are standard [1, 2].

All groups considered below are finite.

Let $\mathfrak{X}$ be a nonempty Fitting class. A Fitting $\mathfrak{X}$-functor is a map f that assigns to each group $G \in \mathfrak{X}$ a nonempty set of its subgroups $f(G)$ such that the following conditions are satisfied:

(i) if α is an isomorphism of G onto $\alpha(G)$, then

$$f(\alpha(G)) = \{\alpha(X) : X \in f(G)\};$$

(ii) if $N \trianglelefteq G$, then

$$f(N) = \{X \cap N : X \in f(G)\}.$$

A Fitting $\mathfrak{X}$-functor is said to be hereditary if class $\mathfrak{X}$ is hereditary.

Let f and g be hereditary Fitting $\mathfrak{X}$-functors, then their product [2] is the map $f \circ g$ that assigns to each group $G \in \mathfrak{X}$ a nonempty set of subgroups $\{X : X \in f(Y) \text{ for some } Y \in g(G)\}$.

A Fitting $\mathfrak{X}$-functor is called a Lockett $\mathfrak{X}$-functor [3] provided that whenever $G \in \mathfrak{X}$ and $V \in f(G \times G)$, then

$$V = (V \cap (G \times 1)) \times (V \cap (1 \times G)).$$

Theorem 1. *If f and g are hereditary Lockett $\mathfrak{X}$-functors, then $f \circ g$ is a Lockett $\mathfrak{X}$-functor.*

1. Klaus Doerk, Trevor Hawkes (1992). Finite soluble groups. De gruyter expositions in mathematics 4. Berlin-New York: Walter de Gruyter. ISBN 3-11-012892-6.
2. Sergey F. Kamornikov, Mikhail V. Sel'kin (2003). Subgroup functors and classes of finite groups. Minsk: Belaruskaja navuka. (in Russian). ISBN 985-439-064-0.
3. Elena A. Vitko (2012). On the structure of the smallest element of the Lockett section of a π-soluble Fitting functor. Problems of Physics, Mathematics and Technics. 3: 48-57. (in Russian).

On normal π–solvable Fitting classes

S.N. Vorob'ev

Vitebsk State University of P.M. Masherov
belarus8889@mail.ru

We consider only finite groups. A class of groups $\mathfrak{F}$ is called a Fitting class if $\mathfrak{F}$ is closed under normal subgroups and if it is closed under products of normal $\mathfrak{F}$-subgroups. If π is a some set of prime numbers and $\mathfrak{F}\mathfrak{E}_{\pi'} = \mathfrak{F}$, then Fitting class $\mathfrak{F}$ is called π-saturated. Subgroup V of group G is called $\mathfrak{F}$-injector of G if $V \cap N$ is a $\mathfrak{F}$-maximal subgroup of G for any subnormal subgroup of group G.

A Fitting class $\mathfrak{X}$ is called a Fischer class [1], if $\mathfrak{F}$ is closed under subgroups of type PN, where P is some Sylow subgroup and N is a normal subgroup of $\mathfrak{X}$-group G.

Definition 1. [2] A Fitting class $\mathfrak{F}$ is called normal in injective class of groups $\mathfrak{X}$, if $\mathfrak{F} \subseteq \mathfrak{X}$ and for each group $G \in \mathfrak{X}$ $\mathfrak{F}$-injector of G is a normal subgroup of group G.

Theorem 1. *Let $\{\mathfrak{F}_i : i \in I\}$ be a set of π-saturated Fitting classes which is normal in the π-solvable Fischer class $\mathfrak{X}$ and $\mathfrak{F} = \cap_{i\in I}\mathfrak{F}_i$. Then $\mathfrak{F}$ is π-saturated Fitting class which is normal in $\mathfrak{X}$.*

If π is a set of all prime numbers and $\mathfrak{X}$ is a Fischer class of all solvable groups, then we have a famous Blessenohl-Gaschütz theorem [3].

1. B. Hartley (1969). On Fischer's dualization of formation theory. Proc. London Math. Soc. V.3, №2. P.193–207.
2. H. Laue (1977). Über nichtauflösbrer normalen Fittingklassen. J. Algebra. 45, №2. P.274–283.
3. D. Blessenohl, W.Gaschütz (1970). Über normalen Schunk und Fittingklassen. Math. Z. Vol.148, №1. P.1-8.

On regularity of semigroups of transformations that preserve an order and an equivalence

V. Yaroshevich

National Research University of Electronic Technology (MIET), Moscow, Russia
v-yaroshevich@ya.ru

Let $(X, \leqslant)$ be a poset with an equivalence relation E on it. Consider the following four types of mappings

$$\begin{aligned} T_E(X) &= \{\alpha \in T_X \mid \forall x, y \in X,\ (x,y) \in E\ \Rightarrow (x\alpha, y\alpha) \in E\}, \\ O_E(X) &= \{\alpha \in T_E(X) \mid \forall x, y \in X,\ x \leqslant y\ \Rightarrow x\alpha \leqslant y\alpha\}, \\ T_{E^*}(X) &= \{\alpha \in T_X \mid \forall x, y \in X,\ (x,y) \in E\ \Leftrightarrow (x\alpha, y\alpha) \in E\}, \\ O_{E^*}(X) &= \{\alpha \in T_{E^*}(X) \mid \forall x, y \in X,\ x \leqslant y\ \Rightarrow x\alpha \leqslant y\alpha\}. \end{aligned}$$

Eisenstadt [1] and later Adams and Gould [2] obtained the regularity conditions for $O_E(X)$ where $E = X \times X$. If $(X, \leqslant)$ is a finite chain then $O_{E^*}(X)$ is regular [3].

It is easy to see the order relation $\leqslant$ generates an equivalence relation $\lesssim$ on X. We say that $x \lesssim y$ iff $\exists u_1, u_2, \ldots, u_n \in X$ such that $x * u_1 * u_2 * \ldots * u_n * y$ where $* \in \{\leqslant, \geqslant\}$.

Definition 1. Suppose I is an arbitrary set and Y, Z are arbitrary and $Y \neq \varnothing$, $Z \neq \varnothing$, $Y \cap Z = \varnothing$. An Eisenstadt set is one of the following: (i) antichain; (ii) quasi-complete chain (see [2]); (iii) $L_I = \{a, c\} \cup \{b_i \mid i \in I\}$ where $a < b_i < c$ for all $i \in I$; (iv) $F_{Y,Z} = Y \cup Z$ where $y < z$ for $y \in Y$, $z \in Z$ and other pairs are not comparable; (v) $y_0 \in Y$, $z_0 \in Z$, $G_{Y,Z} = Y \cup Z$ where $y_0 < z$, $y < z_0$ for $y \in Y$, $z \in Z$, other pairs are not comparable; (vi) $C_6 = \{1, 2, 3, 4, 5, 6\}$ where $1 < 4$, $1 < 5$, $2 < 5$, $2 < 6$, $3 < 6$, $3 < 4$.

Theorem 1. *$O_E(X)$ is regular iff $(X, \leqslant)$ is an Eisenstadt set and either (i) $E = \{(x, y) \mid \forall x, y \in X\}$ or (ii) $E = \{(x, x) \mid \forall x \in X\}$.*

Theorem 2. *The following statements take place*

(i) if $(X, \leqslant)$ is an antichain then $O_{E^*}(X)$ is regular iff $|X/E| < \infty$;

(ii) if $(X, \leqslant)$ is a chain then $O_{E^*}(X)$ is regular iff $|X/E| < \infty$ and every $(A, \leqslant)$ with $A \in X/E$ is a quasi-complete chain;

(iii) if $X/\lesssim = X/E$ and $O_{E^*}(X)$ is regular then all $(B, \leqslant)$ with $B \in X/\lesssim$ are similar Eisenstadt sets (i.e. all $(B, \leqslant)$ are chains or all $(B, \leqslant)$ are antichains etc.);

(iv) if $O_{E^*}(X)$ is regular and $\exists B \in X/\lesssim$ and $\exists A \in X/E$ such that $B \subseteq A$ then B is a Eisenstadt set.

1. *Eisenstadt A. Ya.* On regular endomorphism semigroups of posets // Uch. zap. Leningradskogo gos. ped. instituta im. A. I. Herzena, 1968, Vol. 387, 3-11.
2. *Adams M. E., Gould M.* Posets whose monoids of order-preserving maps are regular // Order, 1989, 6, 195-201.
3. *Lun-Zhi Deng, Ji-Wen Zeng, Tai-Jie You* Green's relations and regularity for semigroups of transformations that preserve order and a double direction equivalence // Semigroup Forum, 2012, 84, 59–68.

One variety of linear algebras with fractional exponent

N. A. Yershova, M. V. Chigarkov

Ulyanovsk
yershova_na@mail.ru, m_chigarkov@mail.ru

Let a basic field have the characteristic equal to zero. Let $\mathbf{V}$ be a variety of non necessarily associative linear algebra and $F(X, \mathbf{V})$ be the relatively free algebra of the variety $\mathbf{V}$ of countable rank generated by $X = \{x_1, x_2, \dots\}$. The integer $c_n(\mathbf{V})$ is called the n-th codimension of $\mathbf{V}$. It is the dimension of space of polynomial elements in $x_1, x_2, ..., x_n$ of the relatively free algebra $F(X, \mathbf{V})$. If the limit of the sequence $\sqrt[n]{c_n(\mathbf{V})}$ exists, then it is called the exponent of $\mathbf{V}$, $\exp(\mathbf{V}) = \lim_{x\to\infty} \sqrt[n]{c_n(\mathbf{V})}$.

We need the linear algebra A which was constructed by S. P. Mishchenko and A. Giambruno (see [1]). This algebra is generated by one element a such that every word in A containing two or more subwords equal to a^2 must be zero. Let the operators L_a, R_a act on the elements of algebra A as follows $xL_a = ax$ and $xR_a = xa$. Now any element of degree n, $n \geqslant 3$, can be rewritten as $a^2 d(R_a, L_a)$, where $d(R_a, L_a)$ is the associative word degree $n-2$ from operators R_a, L_a. Construct an ideal I as the set of linear combinations of monomials $a^2 d(R_a, L_a)$, in which there is at least one subword $L^2{}_a$ in the word $d(R_a, L_a)$. The quotient algebra $B = A/I$ is the basic object for the formulation of our result.

Theorem 1. *Let $\mathbf{V}$ be a variety of linear algebra over the field of the zero characteristic generated by the algebra B. Then exponent of $\mathbf{V}$ is equal to the "golden ratio":*

$$\exp(\mathbf{V}) = \frac{1+\sqrt{5}}{2}.$$

1. Mishchenko S. P. Polynomial growth of the codimensions: A characterization / A. Giambruno, S. P. Mishchenko. // Proceedings of American Mathematical Society. - 2010.- №138.- P.853-859.

A sharp Bezout domain is an elementary divisor ring

B. Zabavsky

Ivan Franko National University of Lviv, Lviv, Ukraine
b_zabava@ukr.net

Only commutative ring with unity will be considered. We follow the notation of [1].

Gilmer [1] introduced the notion of a sharp domain via "property (#)". We say R has property (#) if for any two distinct subset M and N of $mspecR$ we have

$$\bigcap_{p\in M} R_p \neq \bigcap_{p\in N} R_p$$

A domain R is called a sharp domain if each overring of R has property (#). A domain R is called Bezout domain if every finitely generated ideal is principal.

A nonzero element a of a domain R is called adequate if for every element $b \in R$ one can find element $r, s \in R$ such that:

1) $a = r \cdot s$,

2) $rR + bR = R$,

3) for any $s' \in R$ such that $sR \subset s'R \neq R$ implies $s'R + bR$ is proper ideal.

A Bezout domain is called an adequate domain if all its nonzero elements are adequate [2].

Following Kaplansky [3] a ring R is said to be an elementary divisor ring if and only if every matrix over R is equivalent to a diagonal matrix.

Theorem 1. *A sharp Bezout domain is an elementary divisor ring.*

Theorem 2. *A sharp Bezout domain is an adequate domain if and only if each nonzero prime ideal is contained in a unique maximal ideal.*

1. R. Gilmer, Overring of Prufer domains, J. Algebra, 4 (1966), 331-340.
2. M. Larsen, W. Lewis, T. Shores, Elementary divisor rings and finitely presented modules, TAMS, 187 (1974),231-248.
3. I. Kaplansky, Elementary divisors and modules, TAMS 66 (1949), 464-491
4. B. Zabavsky, Fractionaly regular Bezout ring, Mat. Stud., 32 (2009), 70–80.

Weakly global dimension of finite homomorphic images of commutative Bezout domain

B. Zabavsky, S. Bilyavska

Ivan Franko National University of Lviv, Lviv, Ukraine
b_zabava@ukr.net, zosia_meliss@yahoo.co.uk

All rings are assumed to be commutative with unit. We follow the notation [1].

A domain is called a Bezout domain if every its finitely generated ideal is principal. A nonzero element a of a domain R is called adequate if for every elements $b \in R$ one can find element r, $s \in R$ such that

1) $a = r \cdot s$,
2) $rR + bR = R$,
3) for any $s' \in R$ such that $sR \subset s'R \neq R$ implies $s'R + bR \neq R$.

Theorem 1. *Let R be a commutative Bezout domain and $a \in R \setminus \{0\}$. If gl. w. dim(R/aR) is finite than a is adequate element. (gl. w. dim is the weakly global dimension).*

Theorem 2. *Let R be a commutative Bezout domain and $a \in R\setminus\{0\}$. Than $w.G.dim(R/aR) = 0$ (w. G. dim is the weakly Gorenstein pure dimension).*

Theorem 3. *A commutative domain R is Prufer domain if and only if $w.G.gl.dim(R/I) = 0$ for each non zero finitely generated ideal I.*

1. H. Holm, Gorenstein homological dimension J.Pure. Appl. Algebra, 189 (2004), 167–197
2. Zabavsky B. V., Bilyavska S. I., Every zero adequate ring is an exchanege ring, Fundamentalnaya and priklanaya mathematica, 17 (2011/2012), №3, 61–66.
3. Забавський Б. В., Білявська С. І, Слабка глобальна розмірність скінченних гомоморфних образів комутативної області Безу, Прикладні проблеми мех. і мат., 10 (2012), 71–74.

Full matrix over Bezout ring of stable range 2

B. Zabavsky, B. Kuznitska

Ivan Franko National University of Lviv, Lviv, Ukraine
b_zabava@ukr.net, danysja_@ukr.net

All rings considered will be commutative and have the identity. A ring is a Bezout ring if every its finitely generated ideal is principal. The ring of 2×2 matrices over R will be denoted by R_2. A matrix $A \in R_2$ is said to be a full matrix if $R_2 A R_2 = R_2$.

The set of 2×2 full matrices over R will be denoted by $F(R_2)$. We say that a matrix $A \in R_2$ admits diagonal reduction if there exist invertible matrices $P, Q \in R_2$ such that $PAQ = diag(d_1, d_2)$, where d_1 is a divisor of d_2. We say that a ring R has stable range 2 ($str(R) = 2$) if whenever $aR + bR + cR = R$, then there are $x, y \in R$ such that $(a + cx)R + (b + cy)R + cR = R$. We say that a ring R_2 has full stable range $1(str(F(R_2))$ if whenever $AR_2 + BR_2 = R_2$, where $A, B \in F(R_2)$ then there is $T \in F(R_2)$ such that $(A + BT)R_2 = R_2$.

Theorem 1. *Let R be a commutative Bezout ring of stable range 2 and $A, B \in F(R_2)$ such that $AR_2 + BR_2 = R_2$ and B admit diagonal reduction. Then there exists a full matrix $T \in F(R_2)$ such that $(A + BT)R_2 = R_2$.*

Theorem 2. *Let R_2 have a full stable range 1 $(st.r.(F(R_2) = 1)$ then $st.r.(R) = 2$.*

Theorem 3. *A stable range of R is equal 2 if and only if for every $a, b, c \in R$ such that $aR + bR + cR = R$ there exists $u, v, w \in R$ such that $au + bv + cw = 1$ and $uR + vR = R$.*

1. I. Kaplansky, Elementary divisors and modules, TAMS 66 (1949), 464–491.
2. H. Bass, Algebraic K-theory, W.A.Benjamin, Inc., New York-Amsterdam, (1968).
3. B. V. Zabavsky, Reduction of matrices over Bezout ring of stable rank at most 2, Ukrainian. Math. Zh. 55 (2003) №4,550–554.
4. Забавський Б. В., Петричкович В. М., Про стабільний ранг кілець матриць, Укр. мат. жур., т. 61 (2009) №11, 1575–1578.

Bezout morphic rings

B. Zabavsky, O. Pihura

Ivan Franko National University of Lviv, Lviv, Ukraine
b_zabava@ukr.net, pihuraoksana@mail.ru

All rings R are commutative with unity. For an element $a \in R$ the annihilator of a in R is denoted by $Ann(a)$. An element $a \in R$ is called morphic if $R/aR \cong Ann(a)$ as R-modules or equivalently if there exists $b \in R$ such that $aR = Ann(b)$ and $Ann(a) = bR$. A ring R is called morphic if each element in R is morphic [1]. If for each $0 \neq a \in R$ there exists a unique $b \in R$ such that $aR = Ann(b)$ and $Ann(a) = bR$. These rings are called uniquely morphic rings [2]. A domain R is called a Bezout domain if every finitely generated ideal is principal. Following Kaplansky [3] a ring R is said to be an elementary divisor ring if and only if every matrix over R is equivalent to a diagonal matrix. The ring with the property that all maximal ideals are annihilator are called Kasch rings [1].

Theorem 1. *Let R be a commutative Bezout domain. Then R/aR is a morphic ring for each nonzero element $a \in R$.*

Theorem 2. *Let R be a commutative Bezout domain. Then R/aR is a Kasch ring if and only if R is a domain in which every maximal ideal is principal.*

Theorem 3. *A Bezout uniquely morphic ring is an elementary divisor ring.*

1. W. K. Nicholson and E.Sanchez Campos, Rings with the dual of the isomorphism theorem, J. Algebra 271 (2004), 391–406.
2. M. Tamer Kosan, Tsiu-Kwen Lee, Yigiang Zhou, Uniquely morphic rings, J. Algebra 322 (2010), 1072–1085.
3. I. Kaplansky, Elementary divisors and modules, TAMS 66 (1949), 464–491.

On representations of semigroups of small orders

Ya. V. Zaciha

Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv
zaciha@mail.ru

Two semigroups are called *distinct* if they are neither isomorphic nor anti-isomorphic. The number of distinct semigroups $s(n)$ of order n is equal 1 for $n = 1$, 4 for $n = 2$, 18 for $n = 3$ (see [1]) and 126 for $n = 4$ (see [2]). For $n = 5, 6, 7, 8$, the number $s(n)$ is equal $1160, 15973, 836021, 1843120128$, respectively.

We study characteristic properties of semigroups of order $n < 5$, and also their matrix representations. These studies were carried out together with V. M. Bondarenko.

1. K. S. Carman, J. C. Harden, E. E. Posey, Appendix [to the authors' master's theses], University of Tennessee, about 1949.
2. G. E. Forsythe. SWAC computes 126 distinct semigroups of order 4 // Proc. Amer. Math. Soc., 1955, 6, P. 443–447.

Sets of numbers with restrictions on use of $\overline{Q_3}$–symbols

I.V. Zamriy

Dragomanov National Pedagogical University
irina-zamrij@yandex.ru

Let $Q_3 = \{q_0, q_1, q_2\}$ be a set of positive real numbers, where $q_0 + q_1 + q_2 = 1$.
Let Q_3–representation of real number x has the form:

$$x = \Delta^{Q_3}_{\underbrace{0\ldots0}_{a_1}\underbrace{1\ldots1}_{a_2}\underbrace{2\ldots2}_{a_3}\ldots\underbrace{0\ldots0}_{a_{3n-2}}\underbrace{1\ldots1}_{a_{3n-1}}\underbrace{2\ldots2}_{a_{3n}}\ldots}, \quad a_n \in \mathbb{Z}_0,$$

consider its modification:

$$x = \overline{\Delta}^{Q_3}_{a_1 a_2 \ldots a_n \ldots}. \tag{1}$$

Definition 1. Representation of real number $x \in [0,1]$ in the form (1) is called a modified $\overline{Q_3}$ -representation of real number x.

Theorem 1. *The set*

$$I = I[\overline{Q_3}, N_0 \setminus \{2\}] = \{x : x = \overline{\Delta}^{Q_3}_{a_1 a_2 \ldots a_n \ldots},\ \partial e\ a_k \neq 2\ \forall k \in N\}$$

is nowhere dense set of zero Lebesgue measure.

Theorem 2. *The set*

$$I = I[\overline{Q_3}, N_0 \setminus \{s\}] = \{x : x = \overline{\Delta}^{Q_3}_{a_1 a_2 \ldots a_n \ldots},\ \partial e\ a_k \neq s,\ s \geq 3,\ \forall k, s \in N\}$$

is nowhere dense set and its Lebesgue measure is

$$\lambda\left(I[\overline{Q_3}, N_0 \setminus \{s\}]\right) = 1 - \sum_{n=0}^{\infty} 3\cdot 2^n \left(\prod_{\alpha_k \in A} q^s_{\alpha_i} \cdot q^n_{\alpha_k}\right),$$

where all α_i – is either 0, or 1, or 2.

Definition 2. A set of numbers with restrictions on use of $\overline{Q_3}$-symbols is called a set

$$C[\overline{Q_3}, V] = \{x : x = \overline{\Delta}^{Q_3}_{a_1 a_2 \ldots a_n \ldots},\ a_n \in V \subseteq \mathbb{N}_0\}.$$

Theorem 3. *The set $C[\overline{Q_3}, V]$ of numbers with restrictions on use of $\overline{Q_3}$-symbols is:*
1) a segment $[0,1]$, if $V = \mathbb{N}_0$;
2) nowhere dense, if $V \neq \mathbb{N}_0$ and n – infinite set of values.

Theorem 4. *The Lebesgues measure of set $C[\overline{Q_3}, V]$ is calculated by one of the formulas*

$$\lambda(C[\overline{Q_3}, V]) = \prod_{k=1}^{\infty}\left(1 - \frac{\lambda(\overline{U}_k)}{\lambda(U_{k-1})}\right) \quad or \quad \lambda(C[\overline{Q_3}, V]) = \prod_{k=1}^{\infty} \frac{\lambda(U_k)}{\lambda(U_{k-1})},$$

where $U_0 = [0,1]$, U_k is union of cylinders of rank k, in internal points of which there are points of set C, $\overline{U}_k = U_{k-1} \setminus U_k$.

On Some Parametrization of Generalized Pellian Equations

R. Zatorsky

Precarpathian Vasyl Stefanyk National University
romazz@rambler.ru

The author finds some class of integral units of the field $\mathbb{Q}(\sqrt[n]{m})$ which is connected with the solution of the Diophantine equation of the form [1, 2]

$$\begin{vmatrix} s_0 & ms_{n-1} & ms_{n-2} & \cdots & ms_2 & ms_1 \\ s_1 & s_0 & ms_{n-1} & \cdots & ms_3 & ms_2 \\ s_2 & s_1 & s_0 & \cdots & ms_4 & ms_3 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ s_{n-2} & s_{n-3} & s_{n-4} & \cdots & s_0 & ms_{n-1} \\ s_{n-1} & s_{n-2} & s_{n-3} & \cdots & s_1 & s_0 \end{vmatrix} = \pm 1.$$

The main result is some parametrization of these equations for

$$n = 3, 5, 7, 9, 11.$$

There appears a number triangle

$$\begin{array}{rlllll} n=3: & 1 & & & & \\ n=5: & 1 & 2 & & & \\ n=7: & 1 & 3 & 5 & & \\ n=9: & 1 & 1 & 1 & 2 & \\ n=11: & 1 & 5 & 15 & 30 & 42 \end{array}.$$

The further study of this number triangle may elucidate the general solution of the Diophantine equation cited above.

1. *Делоне Б.Н., Фаддеев Д.К.* Теория иррациональностей третьей степени. М.:Изд-во АН СССР, 1940 г., 340 с.
2. *Wada H.* A Table o f Fundamental Units o f Purely Cubic Fields. Proc. Japan Acad., 46 (1970), p 1135-1140.

On equivalence of two forms associated with a quiver

M.V. Zeldich

National Taras Shevchenko University
zeldich@mail.ru

A simple and elementary combinatorial proof for canonical Z - equivalence of two quadratic (respectively, bilinear) integer forms naturally associated with a finite oriented graph (quiver) without loops and oriented cycles (see [4]) was done (this combinatorial statement has a homological interpretation in representation theory of finite dimensional algebras and also not complicated arise from [5]).

Moreover, present fact and its elementary combinatorial proof admits some generalization on the case of arbitrary finite quiver (possibly, with some loops and oriented cycles) when there is a canonical $Z[[h]]$ - equivalence for corresponding to these two forms their natural $Z[[h]]$ deformations according with the standard quiver paths category graduation (on the degrees of paths).

As an application of these results, we easily obtain a new simple proof for the famous author's theorem ([1],[2],[3]) about canonical Z - equivalence between quadratic form of partial order relation on a finite set (poset) and of the Tits quadratic form of corresponding to this poset Hasse quiver (in the case when the last contains no circuitous paths). Also, in the same case, we obtain the canonical Z - equivalence between (nonsymmetrical) bilinear form of partial order relation and of the (nonsymmetrical) bilinear Tits form of a quiver, which is dual (anti-isomorphic) to Hasse quiver of the poset.

1. M.V. Zeldich, On characteristic forms of partially ordered sets with simply connected Hasse graph, – Visn. Kyiv. un-tu (seriya: fiz.-matem. nauky), 2001, no 4, - pp. 36–44.
2. M.V. Zeldich, On the forms of relations of partial order on a finite set. – Proc. Ukrain. matem. congress – 2001. Section 1 "Algebra and Number Theory" pp. 62-70.
3. M.V. Zeldich, On the characteristic and multiply transitive forms of partially ordered sets — In: M.V. Zeldich ρ-exact partially ordered sets and characteristic forms. KNU im. Shevchenka, mekh.-mat. fakul'tet, Kyiv-2002, pp. 3-14.
4. M.V. Zeldich, On paths form and Tits form for a quiver. 4th International Algebraic Conference in Ukraine, August 4-9, 2003, Lviv. Abstracts, p. 243.
5. C.M. Ringel. Representation of K-Spaecies and Bimodules. Journal of algebra 41 (1976) , no. 2, pp. 269-302.

Unique quivers

A. Zelensky

Ivan Ohienka Kamianets-Podilskyi National University
zelik82@mail.ru

Denote by $M_n(\mathbb{Z})$ the ring of all square $n \times n$-matrices over the ring of integers $\mathbb{Z}$. Let $\mathcal{E} \in M_n(\mathbb{Z})$.

Definition 1. An matrix $\mathcal{E} = (\alpha_{ij})$ is called exponent matrix, if

(1) $\alpha_{ii} = 0$ for all $i = 1, \ldots, n$;

(2) $\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}$ for $1 \leq i, j, k \leq n$.

An exponent matrix $\mathcal{E}$ is said to be reduced, if $\alpha_{ij} + \alpha_{ji} > 0$, $i, j = 1, \ldots, n$; $i \neq j$.

Let $\mathcal{E} = (\alpha_{ij})$ be a reduced exponent matrix. Let $\mathcal{E}^{(1)} = (\beta_{ij})$, where $\beta_{ij} = \alpha_{ij}$ for $i \neq j$ and $\beta_{ii} = 1$ for $i = 1, \ldots, n$, and $\mathcal{E}^{(2)} = (\gamma_{ij})$, where $\gamma_{ij} = \min\limits_{1 \leq k \leq n} (\beta_{ik} + \beta_{kj})$.

Theorem 1. *[2] The matrix $[Q] = \mathcal{E}^{(2)} - \mathcal{E}^{(1)}$ is the adjacency matrix of a strongly connected simply laced quiver $Q = Q(\mathcal{E})$.*

Definition 2. The quiver $Q(\mathcal{E})$ is called the quiver of a reduced exponent matrix $\mathcal{E}$.

Definition 3. A strongly connected simply laced quiver is said to be admissible, if it is the quiver of a reduced exponent matrix.

Definition 4. A reduced exponent matrix $\mathcal{E} = (\alpha_{ij})$ is called Gorenstein if there exists a permutation σ of $\{1, 2, 3, \ldots, n\}$ such that $\alpha_{ik} + \alpha_{k\sigma(i)} = \alpha_{i\sigma(i)}$ for all $i, k = 1, \ldots, n$. A permutation σ of Gorenstein matrix $\mathcal{E}$ is called Kirichenko's permutation.

Definition 5. A permutation σ of Gorenstein matrix $\mathcal{E}$ is called Kirichenko's permutation.

Definition 6. An admissible quiver Q is said to be unique, if $Q = Q(\mathcal{E})$ only for Gorenstein matrices $\mathcal{E}$.

Theorem 2. *An admissible quiver with a loop in every vertex is unique if and only if it is a simply laced cycle.*

Theorem 3. *For any composite number $n \geqslant 4$ there exists unique quivers Q such that Q is not the cycle.*

1. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 1, Series: Mathematics and Its Applications, **575**, Kluwer Acad. Publish., 2004. xii+380pp.
2. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 2, Series: Mathematics and Its Applications, **586**. Springer, Dordrecht, 2007. xii+400pp.

On strongly prime acts over monoid

H. Zelisko

Ivan Franko National University of Lviv, Lviv, Ukraine
zelisko_halyna@yahoo.com

Many investigations of properties of semigroups and S-acts have been carried out in recent years. Many questions which appear here are similar to those which are being solved in the rings and modules theory.

The notion of strongly prime module was introduced by Beachy [1] and Handelman and Lawrence [2]. In [3] studied the strongly prime rings.

Let $Act - S$ be a cathegory of unitary and centered right acts over monoid S.

A functor $r : Act - S \to Act - S$ is called a preradical if for all right S-acts M and N and for any S-homomorphism $f : M \to N$ $r(M) \subseteq M$ and $f(r(M)) \subseteq r(N)$.

A preradical r is called a radical if $r(M/r(M)) = 0$ for any right act M.

Right act M is called r-torsion if $r(M) = M$, and M is called r-torsionfree if $r(M) = 0$.

If for any right S-act M $r_1(M) \subseteq r_2(M)$ then denote $r_1 \leq r_2$. Let R^N be a smallest preradical r, such that the right act N is r-torsion act. Then $R^N(M) = \sum_{\alpha\in J} f_\alpha(N)$, where f_α runs through all homomorphisms in $Hom(N, E(M))$, where $E(M)$ is the injective envelope of the right act M.

Definition 1. A nonzero right act M is called strongly prime if M is prime act and for each nonzero right subact $N \subseteq M$ and for each element $y \in M$ there exist elements $x_1, x_2, \ldots, x_n \in N$ such that $Ann(x_1, x_2, \ldots, x_n) \subseteq Ann(y)$.

Theorem 1. *For every nonzero right act M the following conditions are equivalent:*

1) M is a strongly prime act;

2) for any preradical r, either $r(M) = 0$ or $r(M) = M$;

3) M is contained in every completely invariant subact of act $E(M)$;

4) for each $y \in M$ and for $0 \neq x \in M$ there exist $s_1, s_2, \ldots, s_n \in S$ such that $Ann(xs_1, xs_2, \ldots, xs_n) \subseteq Ann(y)$.

1. Beachy, J., Some aspects of noncommutative localization, Noncommutative Ring Theory, LNM, vol 545, Springer-Verlag, 1975, 2-31.
2. Handelman, D., Lawrence, J., Strongly prime rings, Trans. Amer. Math. Soc. 211(1975), 209-223.
3. A.Kaucikas, R.Wisbauer, On strongly prime rings and ideals, Comm. Algebra, 28, 5461-5473, (2000).

On the Smith normal form of symmetric matrices over rings of polynomials with involution

V.R. Zelisko, M.I. Kuchma

Ivan Franko National University of Lviv, Lviv, Ukraine
Lviv Polytechnic National University Lviv, Ukraine
`zelisko_ vol@yahoo.com, markuchma@ukr.net`

Let involution ∇ be defined in a polynomial ring $\mathbf{C}[\mathbf{x}]$ in one of the possible ways [1]:

$$(\alpha)\ \Big(\sum_{i=1}^{m} a_i x^i\Big)^{\nabla} = \sum_{i=1}^{m} \bar{a}_i (-x)^i,$$

$$(\beta)\ \Big(\sum_{i=1}^{m} a_i x^i\Big)^{\nabla} = \sum_{i=1}^{m} a_i (-x)^i,$$

$$(\gamma)\ \Big(\sum_{i=1}^{m} a_i x^i\Big)^{\nabla} = \sum_{i=1}^{m} a_i x^i.$$

The involution ∇ transfer onto the matrix ring $M_n(\mathbf{C}[\mathbf{x}])$ as follows:

$$A(x)^{\nabla} = \|a_{ij}(x)\|^{\nabla} = \|a_{ji}(x)^{\nabla}\|.$$

It is studied the Smith normal form of symmetric matrices with different types of involution in $\mathbf{C}[\mathbf{x}]$.

Theorem 1. *For identical involution (γ) there exists a symmetric matrix $A(x)$ with arbitrary preassigned elementary divisors.*

Theorem 2. *For identical involutions (α) and (β) there exists a symmetric matrix $A(x)$ with arbitrary preassigned the Smith form S_A, which is satisfy the condition $(S_A)^{\nabla} = S_A$.*

Obviously, there are symmetric matrices $A(x)$ under the involutions (α) and (β) for which $(S_A)^{\nabla} \neq S_A$. Therefore, in this work are explored the system of roots of elementary divisors of the matrix $A(x)$, for which is performed $(S_A)^{\nabla} = S_A$.

These results are used in the study factorization $A(x) = B(x)C(x)B(x)^{\nabla}$ symmetric matrices over polynomial rings with involution [2].

1. B. D.Lyubachevskyy. Factorization of symmetric matrices with the elements from the ring with involution,I, Sib. Mat. Zh. 14 (1973), 337-356 (in Russian).
2. V.R.Zelisko, M.I.Kuchma. Factorization of symmetric matrices over rings of polynomials with involution. // Mathematical Methods and Physicomechanical Fields. - 1997. -40. пїЅ4. -пїЅ. 91-95. (in Ukrainian).

Representations of nodal finite dimensional algebras of type A

V. Zembyk

Institute of Mathematics of NASU, Kyiv, Ukraine
vaszem@rambler.ru

It is a joint work with Yuriy Drozd.

We consider finite dimensional algebras over an algebraically closed field $\mathbf{k}$.

An algebra $\mathbf{A}$ is called *nodal* if there is a hereditary algebra $\mathbf{H}$ such that $\mathbf{H} \supset \mathbf{A} \supset \operatorname{rad}\mathbf{H} = \operatorname{rad}\mathbf{A}$ and $\operatorname{length}_{\mathbf{A}}(\mathbf{H} \otimes_{\mathbf{A}} U) \leqslant 2$ for any simple $\mathbf{A}$-module U. We say that $\mathbf{A}$ is a *nodal algebra of type* A if $\mathbf{H}$ is Morita equivalent to $\mathbf{k}Q$, the path algebra of a Dynkin quiver of type A or $\tilde{A}$ [1].

A notion of *inessential gluings* is defined and it is proved that inessential gluings do not imply representation type. We call an algebra A *quasi-gentle* if it can be obtained from a gentle or skewed-gentle algebra [2] by a suitable sequence of inessential gluings.

Special classes of nodal algebras, called *exceptional* and *super-exceptional* are defined by means of quivers and relations. For these classes of algebras an explicit criterion of tameness is given. We call an algebra A *good exceptional* (*good super-exceptional*) if it is exceptional (respectively, super-exceptional) and not wild.

Theorem 1. *A non-hereditary nodal algebra of type A is representation finite or tame if and only if it is either quasi-gentle, or good exceptional, or good super-exceptional. In other cases it is wild.*

This theorem gives a complete description of tame nodal algebras of type A.

1. Y. A. Drozd and V. V. Kirichenko. Finite Dimensional Algebras. Vyshcha Shkola, Kiev, 1980. (English translation: Springer–Verlag, 1994.)
2. V. Bekkert, E. N. Marcos and H. Merklen. Indecomposables in derived categories of skewed-gentle algebras. Commun. Algebra 31, No. 6 (2003), 2615–2654.

Köthe's conjecture and selfinjective regular rings

Jan Žemlička

Univerzita Karlova v Praze
zemlicka@karlin.mff.cuni.cz

We say that a unital associative ring satisfies condition (NK) if it contains two nil right ideals whose sum is not nil. One among many equivalent formulation of the Köthe's conjecture [2] is the assertion that there exists no ring satisfies condition (NK).

Recall that a self-injective regular ring R is

1. Type I iff every nonzero right ideal contains nonzero abelian idempotent [1, 10.4],
2. Type II iff every nonzero right ideal contains nonzero directly finite idempotent [1, 10.8],
3. Type III iff it contains no nonzero directly finite idempotent.

Moreover R is Type I_f (Type II_f) provide it is Type I (Type II) and it is directly finite and it is Type I_∞ (Type II_∞) if it is directly infinite Type I (Type II). By [1, 10.22] every self-injective regular ring is uniquely a direct product of rings of Types $\mathcal{T} = \mathrm{I}_f$, I_∞, II_f, II_∞, III.

We will discuse consequences of the following result:

Theorem 1. *If a Koethe conjecture fails then there exists countable local subring satisfying (NK) of a suitable self-injective simple regular ring of type II_f.*

1. K. R. Goodearl: *Von Neumann Regular Rings*, London 1979, Pitman, Second Ed. Melbourne, FL 1991, Krieger.
2. G.Köthe. Die Struktur der Ringe, deren Restklassenring nach dem Radikal vollstanding irreduzibel ist. Math. Z. 32 (1930), 161–186.

On semiretractions of trioids

A.V. Zhuchok

Luhansk Taras Shevchenko National University, Luhansk, Ukraine
zhuchok_a@mail.ru

The notions of a trialgebra and a trioid were introduced by J.-L. Loday and M.O. Ronco [1] and investigated in different papers (see, for example, [1], [2]). Let us recall that a nonempty set T equipped with three binary associative operations $\dashv$, $\vdash$ and $\perp$ satisfying the following axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), \quad (x \vdash y) \dashv z = x \vdash (y \dashv z),$$

$$(x \dashv y) \vdash z = x \vdash (y \vdash z), \quad (x \dashv y) \dashv z = x \dashv (y \perp z),$$

$$(x \perp y) \dashv z = x \perp (y \dashv z), \quad (x \dashv y) \perp z = x \perp (y \vdash z),$$

$$(x \vdash y) \perp z = x \vdash (y \perp z), \quad (x \perp y) \vdash z = x \vdash (y \vdash z)$$

for all $x, y, z \in T$, is called a trioid. Trioids are a generalization of semigroups and dimonoids [3] while a trialgebra is just a linear analogue of a trioid. For a general introduction and basic theory see [1].

The following three definitions were first introduced in [4] for monoids.

A transformation τ of a semigroup S is called a left semiretraction, if

$$(xy)\tau = (x\tau\, y)\tau \tag{1}$$

for all $x, y \in S$. If instead of (1) the identity

$$(xy)\tau = (x\, y\tau)\tau \tag{2}$$

holds, then we say about a right semiretraction. If for τ the identities (1), (2) hold, then τ is called a (symmetric) semiretraction of S.

A transformation τ of a trioid $(T, \dashv, \vdash, \perp)$ will be called a left (right, symmetric) semiretraction, if τ is a left (right, symmetric) semiretraction of semigroups $(T, \dashv)$, $(T, \vdash)$ and $(T, \perp)$.

We give the general characteristic of semiretractions of trioids and show that the problem of the description of congruences on trioids is reduced to the description of semiretractions of trioids. Examples of left, right and symmetric semiretractions of trioids are given. We also present new trioid theoretical constructions for which characterize some symmetric semiretractions.

1. Loday J.-L. and Ronco M.O. *Trialgebras and families of polytopes*, Contemp. Math. **346** (2004), 369 – 398.
2. Zhuchok A.V. *Some congruences on trioids*, Journal of Mathematical Sciences **187** (2012), no. 2, 138 – 145.
3. Zhuchok A.V. *Dimonoids*, Algebra and Logic **50** (2011), no. 4, 323 – 340.
4. Usenko V.M. *Semiretractions of monoids*, Proc. Inst. Applied Math. and Mech. **5** (2000), 155 – 164 (In Ukrainian).

Symmetric dimonoids of transformations

Y.V. Zhuchok

Luhansk Taras Schevchenko National University, Luhansk, Ukraine
zhuchok_y@mail.ru

The notion of a dimonoid was introduced by Jean-Louis Loday in [1]. An algebra $(D, \dashv, \vdash)$ with two associative operations $\dashv$ and $\vdash$ is called a dimonoid if for all $x, y, z \in D$ the following conditions hold:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z),$$
$$(x \vdash y) \dashv z = x \vdash (y \dashv z),$$
$$(x \dashv y) \vdash z = x \vdash (y \vdash z).$$

If operations of a dimonoid coincide, then the dimonoid becomes a semigroup. Dimonoids and, in particular, dialgebras have been studied by many authors. More general information on dimonoids and examples of different dimonoids can be found, for example, in [1–4].

It is well-known that any semigroup can be embedded to the symmetric semigroup on some set. An analogue of Cayley's theorem for dimonoids was obtained in [3]. However, the symmetric dimonoid of all transformations of an arbitrary set was not constructed untill now. Here we construct such symmetric dimonoid (the transformation dimonoid) on an arbitrary set and define another more convenient dimonoid construction which is isomorphic to the transformation dimonoid. We show that the transformation dimonoid is a universal construction in the dimonoid theory in the sense that any dimonoid can be embedded into the transformation dimonoid on a suitable set. Also we describe the abstract characterization of the transformation dimonoid.

In addition, we construct the symmetric inverse dimonoid on an arbitrary set and study different properties of the defined dimonoid construction.

1. Loday J.-L., Dialgebras, In: Dialgebras and related operads, Lect. Notes Math. **1763**, Springer-Verlag, Berlin, 2001, 7–66.
2. Zhuchok A.V., Semilattices of subdimonoids, Asian-European Journal of Mathematics **4** (2011), no. 2, 359–371.
3. Zhuchok A.V., Dimonoids, Algebra and Logic **50** (2011), no. 4, 323–340.
4. Pirashvili T., Sets with two associative operations, Central European Journal of Mathematics **2** (2003), 169–183.

On automorphisms of the semigroup of order-decreasing order-preserving full transformation of the boolean

T. Zhukovska

Lesya Ukrainka East European National University, Lutsk, Ukraine
t.zhukovska@ukr.net

Let $(M, \leq)$ be a poset. A transformation $\alpha : M \to M$ is called *order-decreasing*, if $\alpha(x) \leq x$ for all $x \in M$. The set $\mathcal{F}(\mathcal{M})$ of such transformations is a semigroup with respect to the composition of transformations. A transformation α is called *order-preserving*, if for every $x, y \in M$, $x \leq y$ implies $\alpha(x) \leq \alpha(y)$. The set of such transformations forms a semigroup, which is denoted by $\mathcal{O}(\mathcal{M})$. The intersection $\mathcal{C}(\mathcal{M}) = \mathcal{F}(\mathcal{M}) \cap \mathcal{O}(\mathcal{M})$ is called the semigroup of *order-decreasing order-preserving* transformations of the M.

Many authors studied semigroups $\mathcal{F}(\mathcal{M})$, $\mathcal{O}(\mathcal{M})$ and $\mathcal{C}(\mathcal{M})$ in the case where the poset M is a finite chain. The analogues of these semigroups were studied also for all partial transformations or all partial injective transformations of the set M. These semigroups for other posets are studied much worse.

We consider a semigroup $\mathcal{C}(\mathcal{B}_n)$ were $\mathcal{B}_n$ is the set of all subsets of a n-element set naturally ordered by inclusion. Note that the semigroup $\mathcal{F}(\mathcal{B}_n)$ of order-decreasing transformations of the boolean $\mathcal{B}_n$ was studied in [3].

A number $h_\alpha = \sum_{A \in im(\alpha)} [\varnothing, A]$ is called *the height* of an element $\alpha \in \mathcal{C}(\mathcal{B}_n)$.

Theorem 1. *Every automorphism of the semigroup $\mathcal{C}(\mathcal{B}_n)$ preserves the height of elements.*

Theorem 2. *The automorphism group of the semigroup $\mathcal{C}(\mathcal{B}_n)$ is isomorphic to the symmetric group S_n.*

Note that the automorphism group of the semigroup $\mathcal{IO}_n$ of order-preserving injective transformations of n-element chain was described in [2].

1. *Ganyushkin O., Mazorchuk V.* Classical Finite Transformation semigroups. An Introduction. — Algebra and Applications. — London: Springer–Verlag, 2009. — **9**. — XII, 314 p.
2. *Ganyushkin O., Mazorchuk V.* On the structure of $\mathcal{IO}_n$. — Semigroup Forum. — 2003. — V.66. — P. 455–483.
3. *Stronska G. O.* The semigroup of the order-decreasing transformations of the set of all the subsets of a finite set. — Bulletin of the University of Kiev. Series: Physics & Mathematics. — 2006. — № 2. — P. 57–62.

On the sum of elements of exponent matrix

V. Zhuravlev

Taras Shevchenko National University of Kyiv
vshur@univ.kiev.ua

All the necessary theoretical information on tiled orders can be found in [1], [2]. We use the notation of [1], [2].

Suppose $\Lambda = \{\mathcal{O}, \mathcal{E}(\Lambda) = (\alpha_{ij})\}$ be tiled order in $M_n(D)$. For the tiled order Λ we construct graph $G(\Lambda)$ by the rule: vertices of a graph $G(\Lambda)$ are point $1, \ldots, n$ and vertices i and j are connected by an edge if and only if $\alpha_{ij} + \alpha_{ji} = 1$.

Let $d = \sum\limits_{i,j=1}^{n} \alpha_{ij}$.

Theorem 1. *Suppose* $\Lambda = \{\mathcal{O}, \mathcal{E} = (\alpha_{ij})\}$ *be tiled order with a connected graph* $G(\Lambda)$. *Then* $d \leq \dfrac{(n-1)n(n+1)}{6}$.

1. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 1, Series: Mathematics and Its Applications, **575**, Kluwer Acad. Publish., 2004. xii+380pp.
2. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 2, Series: Mathematics and Its Applications, **586**. Springer, Dordrecht, 2007. xii+400pp.

On the form of the solution of the simultaneous distributive equations

V. Zhuravlev, D. Zhuravlyov

Taras Shevchenko National University of Kyiv
vshur@univ.kiev.ua

Let $\mathcal{O}$ be a discrete valuation ring with unique maximal ideal $\mathfrak{m} = \pi\mathcal{O}$, where π is a prime element of the ring, let $\mathcal{O}$, $F = \mathcal{O}/\pi\mathcal{O}$ be a skew field, let $\Lambda = \{\mathcal{O}, \mathcal{E}(\Lambda) = (\alpha_{ij})\}$ be a reduced tiled order over $\mathcal{O}$ with exponent matrix $\mathcal{E}(\Lambda) = (\alpha_{ij}) \in M_n(Z)$. By a *tiled order* over a discrete valuation ring, we mean a Noetherian prime semiperfect semidistributive ring with nonzero Jacobson radical (see [1], [2]).

Definition 1. A right (resp. left) Λ-module M (resp. N) is called a right (resp. left) Λ-lattice if M (resp. N) is finitely generated free $\mathcal{O}$-module.

The equation of the form $\sum\limits_{i=1}^{s} a_i m_i = 0$, where $a_i \in F$ and m_i is the element of irreducible lattice M_i, which is the submodule of the distributive module M, we will call distributive.

The method of solving the system of distributive equations is described in [3].

The solution of the simultaneous distributive equations we will write as $\sum\limits_{i=1}^{r} M_i \bar{b}_i$, where M_i is irreducible lattice and $\bar{b}_i$ is a vector.

Remark. Form of solution of the simultaneous distributive equations depends on the order of the solutions of equations.

Let us consider the cases when the expression for the set of solutions can be simplyfied.

1) Let the module M_1 be the submodule of modules $M_{i_1}, \dots, M_{i_z}$ and vector $\bar{b}_1$ is linearly expressed over F through vectors $\bar{b}_{i_1}, \dots, \bar{b}_{i_z}$, i.e. $\bar{b}_1 = \alpha_1 \bar{b}_{i_1} + \dots + \alpha_z \bar{b}_{i_z}$, where $\alpha_j \in F$. Then $M_1\bar{b}_1 + M_{i_1}\bar{b}_{i_1} + \dots + M_{i_z}\bar{b}_{i_z} = M_{i_1}\bar{b}_{i_1} + M_{i_2}\bar{b}_{i_2} + \dots + M_{i_z}\bar{b}_{i_z}$.

2) Let the vectors $\bar{b}_i$ and $\bar{b}_j$ be collinear and not equal to zero. Then $M_i\bar{b}_i + M_j\bar{b}_j = (M_i + M_j)\bar{b}_i$.

Theorem 1. *Form of solution of the simultaneous distributive equations after simplification independs on the order of the solution of equations.*

1. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 1, Series: Mathematics and Its Applications, **575**, Kluwer Acad. Publish., 2004. xii+380pp.
2. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 2, Series: Mathematics and Its Applications, **586**. Springer, Dordrecht, 2007. xii+400pp.
3. V. Zhuravlev, D. Zhuravlyov. Projective resolution of irreducible modules over tiled order. / V. Zhuravlev, D. Zhuravlyov. // Algebra and discrete mathematics. – Volume 14 (2012). Number 2. pp. 323 – 336.

Admissible quivers implemented by a finite number of non-equivalent exponent matrices

T. Zhuravleva

Taras Shevchenko National University of Kyiv
tatizhuravleva@gmail.com

Denote by $M_n(\mathbb{Z})$ the ring of all square $n \times n$-matrices over the ring of integers $\mathbb{Z}$. Let $\mathcal{E} \in M_n(\mathbb{Z})$.

Definition 1. An integer-valued matrix $\mathcal{E} = (\alpha_{ij}) \in M_n(\mathbb{Z})$ is called exponent matrix, if

(1) $\alpha_{ii} = 0$ for all $i = 1, \ldots, n$;

(2) $\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}$ for $1 \leq i, j, k \leq n$.

An exponent matrix $\mathcal{E}$ is said to be reduced, if $\alpha_{ij} + \alpha_{ji} > 0$, $i, j = 1, \ldots, n$; $i \neq j$.

Let $\mathcal{E} = (\alpha_{ij})$ be a reduced exponent matrix. Let $\mathcal{E}^{(1)} = (\beta_{ij})$, where $\beta_{ij} = \alpha_{ij}$ for $i \neq j$ and $\beta_{ii} = 1$ for $i = 1, \ldots, n$, and $\mathcal{E}^{(2)} = (\gamma_{ij})$, where $\gamma_{ij} = \min\limits_{1 \leq k \leq n} (\beta_{ik} + \beta_{kj})$. Obviously, $[Q] = \mathcal{E}^{(2)} - \mathcal{E}^{(1)}$ is an $(0,1)$-matrix.

Theorem 1. *[2] The matrix $[Q] = \mathcal{E}^{(2)} - \mathcal{E}^{(1)}$ is the adjacency matrix of a strongly connected simply laced quiver $Q = Q(\mathcal{E})$.*

Definition 2. The quiver $Q(\mathcal{E})$ is called the quiver of a reduced exponent matrix $\mathcal{E}$.

Definition 3. A strongly connected simply laced quiver is said to be admissible, if it is the quiver of a reduced exponent matrix.

Definition 4. Two exponent matrices $\mathcal{E} = (\alpha_{ij})$ and $\Theta = (\theta_{ij})$ are said to be equivalent if one can be obtained from the other by transformations of the following two types:

(1) subtracting an integer α from all the entries of i-th row with simultaneous adding α to the entries of i-th column;

(2) simultaneous interchanging of two rows and the same numbered columns.

Definition 5. An admissible quiver Q is said to be rigid, if, up to equivalence, there exists a unique exponent matrix $\mathcal{E}$ such that $Q = Q(\mathcal{E})$.

We describe all admissible quivers Q on n vertices with $n \leq 5$, such that for ever quiver there is only a finite number of non-equivalent matrices of $\mathcal{E}$ such that $Q = Q(\mathcal{E})$. In particular, we describe all rigid quivers on n vertices with $n \leq 5$.

1. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 1, Series: Mathematics and Its Applications, **575**, Kluwer Acad. Publish., 2004. xii+380pp.
2. M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules.* Vol. 2, Series: Mathematics and Its Applications, **586**. Springer, Dordrecht, 2007. xii+400pp.

On matrix representations of semigroups generated by two potents

O. V. Zubaruk

Taras Shevchenko National University of Kyiv
sambrinka@ukr.net

An element a of a semigroup S is said to be *potent* (more precisely, m-*potent*) if $a^m = a$ for some $m > 1$ (see, e.g., [1]). Potent elements of the semigroup of $n \times n$-matrices are called *potent matrices*; they were studied by many authors (see, e.g., [2]–[5]).

We consider matrix representations of finite and infinite semigroups generated by two potents, and focuses on the representation type of the semigroups.

These studies were carried out together with V. M. Bondarenko.

1. J. B. Kim. On r-potents in a full transformation semigroup // Kyungpook Math. J., 1975, vol. 15, P. 109–110.
2. J. P. McCloskey. Characterizations of r-potent matrices // Math. Proc. Cambridge Philos. Soc., 1984, vol. 96, N2, P. 213–222.
3. J. Stuart. Reducible sign k-potent sign pattern matrices // Linear Algebra Appl., 1999, vol. 294, P. 197–211.
4. Y. Tian, G. Styan. Rank equalities for idempotent and involutory matrices // Linear Algebra Appl., 2001, vol. 335, P. 101–117.
5. R. Huang, J. Liu, L. Zhu. A structural characterization of real k-potent matrices // Linear Multilinear Algebra, 2011, vol. 59 N4, P. 433–439.

О нелокальных классах Локетта, удовлетворяющих гипотезе Локетта

Е.Н.Залесская

УО "Витебский государственный университет имени П.М. Машерова"
alenushka0404@mail.ru

В настоящей работе рассматриваются только конечные группы.

Решение многих задач описания строения классов Фиттинга и их классификации связано с применением операторов Локетта "$*$" и "$_*$" [1]. Напомним, что каждому непустому классу Фиттинга $\mathfrak{F}$ Локетт [1] сопоставляет класс $\mathfrak{F}^*$, который определяется как наименьший из классов Фиттинга, содержащий $\mathfrak{F}$, такой, что для все групп G и H справедливо равенство $(G \times H)_{\mathfrak{F}^*} = G_{\mathfrak{F}^*} \times H_{\mathfrak{F}^*}$, и класс $\mathfrak{F}_*$ как пересечение всех таких классов Фиттинга $\mathfrak{X}$, для которых $\mathfrak{X}^* = \mathfrak{F}^*$. Класс Фиттинга называют классом Локетта [1], если $\mathfrak{F} = \mathfrak{F}^*$.

Заметим, что семейство классов Локетта обширно: оно содержит наследственные и обобщенно наследственные классы Фиттинга (классы Фишера), а так же классы Фиттинга, замкнутые относительно гомоморфных образов или конечных подпрямых произведений (в частности, формации Фиттинга).

Напомним, что непустой класс Фиттинга $\mathfrak{F}$ называется нормальным, если $\mathfrak{F}$-радикал $G_{\mathfrak{F}}$ является $\mathfrak{F}$-максимальной подгруппой G для любой группы G.

Как установлено [1], для любого класса Фиттинга $\mathfrak{F}$ справедливы включения: $\mathfrak{F}_* \subseteq \mathfrak{F} \subseteq \mathfrak{F}^*$ и $\mathfrak{F}_* \subseteq \mathfrak{F} \cap \mathfrak{X} \subseteq \mathfrak{F}^*$, где $\mathfrak{X}$ – некоторый нормальный класс Фиттинга. В связи с этим Локеттом была сформулирована следующая проблема, которая в настоящее время известна как

Гипотеза Локетта ([1]). Каждый класс Фиттинга $\mathfrak{F}$ совпадает с пересечением некоторого нормального класса Фиттинга $\mathfrak{X}$ и $\mathfrak{F}^*$.

Первоначально гипотеза Локетта была подтверждена Брайсом и Косси [2] для разрешимых локальных наследственных классов Фиттинга. В [2] также установлено, что класс Фиттинга $\mathfrak{F}$ удовлетворяет гипотезе Локетта, если справедливо равенство $\mathfrak{F}_* = \mathfrak{F}^* \cap \mathfrak{S}_*$, где $\mathfrak{S}_*$ – минимальный нормальный класс Фиттинга. В последующем гипотеза нашла подтверждение для следующих семейств нормальных классов Фиттинга: разрешимых локальных вида $\mathfrak{X}\mathfrak{Y}$, $\mathfrak{X}\mathfrak{S}_\pi\mathfrak{S}_{\pi'}$ (Бейдлеман и Хаук [3]), произвольных разрешимых локальных (Н.Т. Воробьев [4]). Кроме того, позднее Галледжи [5] было установлено, что локальные классы Фиттинга произвольных групп также удовлетворяют гипотезе Локетта.

Однако проблема описания нелокальных классов Фиттинга, удовлетворяющих гипотезе Локетта, остается по-прежнему актуальной.

Доказана следующая

Теорема. *Пусть E – простая неабелева группа, $\mathfrak{X} = FitE$ – класс Фиттинга, порожденный E, $\mathfrak{F} = \mathfrak{X}\mathfrak{N}_p$ и $\omega = \{p\}$, где p – простое число. Тогда $\mathfrak{F}$ – нелокальный ω-локальный класс Локетта, который ненормален и удовлетворяет гипотезе Локетта.*

1. 1. Lockett P. The Fitting class $\mathfrak{F}^*$ // Math. Z. - 1974. - Bd. 137, №2. - S.131-136.
2. 2. Bryce R.A., Cossey J. A problem in Theory of Normal Fitting classes // Math. Z. 1975. V. 141. № 2. P. 99-110.
3. 3. Beidleman J.C., Hauck P. Uber Fittingklassen und die Lockett-Vermutung // Math. Z. 1979. V. 167. № 2. P. 161-167.
4. 4. Воробьев Н.Т. О радикальных классах конечных групп с условием Локетта // Мат. заметки. - Т.43, №2. - 1988. - С. 161-168.
5. 5. Gallego M.P. Fitting pairs from direct limits and the Lockett conjecture // Comm. Algebra. - 1996. - Bd.24, №6. - S. 2011-2023.

Про приведення многочленних матриць над полем до блочно-діагонального вигляду

О.М. Мельник, Р.В. Коляда

Ivan Franko National University of Lviv, Lviv, Ukraine
melnykorest@gmail.com

Нехай $\mathbb{F}[x]$ – кільце многочленів над полем $\mathbb{F}$ характеристики нуль, $M_n(\mathbb{F}[x])$ – кільце $(n\times n)$-матриць над $\mathbb{F}[x]$. Надалі об'єктом нашого дослідження будуть неособливі матриці із $M_n(\mathbb{F}[x])$, характеристичний многочлен яких розкладається в добуток лінійних множників. Додаткові позначення використано із роботи [1].

Нехай характеристичний многочлен неособливої матриці $A(x) \in M_n(\mathbb{F}[x])$ зображений у вигляді добутку $\det A(x) = \varphi(x)\phi(x)$, де $\deg \varphi(x) = kr$, $\deg \phi(x) = (n-k)r$, $1 \le k < n$. Якщо $(\varphi(x), \phi(x)) = 1$, то легко довести, що для матриці $A(x)$ не завжди існують матриці $U(x), V(x) \in GL(n, \mathbb{F}[x])$ такі, що

$$U(x)A(x)V(x) = \operatorname{diag}\big(A_1(x)\,,\quad A_2(x)\big)$$

– блочно-діагональна матриця, де $A_1(x) \in M_k(\mathbb{F}[x]), A_2(x) \in M_{n-k}(\mathbb{F}[x])$ – унітальні многочленні матриці степеня r із визначниками $\det A_1(x) = \varphi(x)$ та $\det A_2(x) = \phi(x)$ відповідно. В даному повідомленні анонсовано наступне твердження.

Теорема. *Нехай для неособливої матриці* $A(x) \in M_n(\mathbb{F}[x])$ *виконується* $\deg\det A(x) = nr$, *де* $1 \le r \le \deg A(x)$. *Нехай, далі, форма Сміта матриці* $A(x)$ *допускає зображення у вигляді добутку* $\mathbf{d}$*-матриць, тобто*

$$S_A(x) = \operatorname{diag}(a_1(x)a_2(x), \dots, a_n(x) = \Phi(x)\Psi(x),$$

де $\Phi(x) = \operatorname{diag}(\varphi_1(x), \varphi_2(x), \dots, \varphi_n(x)) \in M_n(\mathbb{F}[x])$, $\varphi_i(x)|\varphi_{i+1}(x)$; $\deg\det\Phi(x) = kr$ і $\Psi(x) = \operatorname{diag}(\phi_1(x), \phi_2(x), \dots, \phi_n(x)) \in M_n(\mathbb{F}[x])$, $\phi_i(x)|\phi_{i+1}(x)$; $\deg\det\Psi(x) = (n-k)r$. *Якщо* $(\det\Phi(x), \det\Psi(x)) = 1$, *то для матриці* $A(x)$ *існують матриці* $P \in GL(n, \mathbb{F})$ *та* $Q(x) \in GL(n, \mathbb{F}[x])$ *такі, що*

$$PA(x)Q(x) = \begin{Vmatrix} A_1(x) & 0 \\ 0 & A_2(x) \end{Vmatrix},$$

де $A_1(x) \in M_k(\mathbb{F}[x])$, $A_2(x) \in M_{n-k}(\mathbb{F}[x])$ – *унітальні многочленні матриці степеня* r *із визначниками* $\det A_1(x) = \det\Phi(x)$ *та* $\det A_2(x) = \det\Psi(x)$ *відповідно, тоді і тільки тоді, коли виконуються наступні умови:*

1. $\operatorname{rank} M_{A_*^r(x)}[a_n(x)] = nr$;

2. $\operatorname{rank} M_{A_*^r(x)}[\varphi_n(x)] = \operatorname{rank} M_{(A_*^T(x))^r}[\varphi_n(x)] = kr$.

1. П.С. Казімірський. Розклад матричних многочленів на множники. К.:Наук. думка. – 1981.

Локально-нільпотентні групи з централізатором елемента скінченого рангу

В.О. Оніщук

ЛНТУ, Луцьк, Україна

Нагадаємо означення спеціального рангу групи, введеного О.І.Мальцевим [1].

Група G має скінчений спеціальний ранг r, якщо r є найменшим числом з такою властивістю, що довільна скінченно породжена підгрупа групи G може бути породжена не більше ніж r елементами.

Якщо такого натурального числа не існує, то спеціальний ранг групи вважається нескінченним. Спеціальний ранг групи G будемо позначати через $r(G)$ і називати просто рангом групи.

Теорема 1. Нехай G — локально нільпотентна група без кручення і f — деякий її елемент. Якщо $r(C_G(f)) = r$, то $r(C_G(fZ)) \leq r$, де $Z = Z(G)$ — центр групи.

З цієї теореми випливає, що ранги всіх факторів верхнього центрального ряду групи з натуральними номерами не перевищують числа r.

Теорема 2. Якщо в локально-нільпотентній групі G без кручення централізатор $C_G(f)$ деякого елементу f в групі G має скінчений ранг, то вона гіперцентральна.

Якщо — довільна локально нільпотентна група, то питання про гіперцентральність групи вирішується при додаткових умовах.

Теорема 3. Нехай G — локально нільпотентна група, $\langle f\rangle$— деяка її циклічна підгрупа і $P \neq 1$ — силовська p-підгрупа в періодичній частині $t(G)$. Якщо $C_P(f)$ є скінченною підгрупою, то $P \cap Z \neq 1$.

Теорема 4. Нехай G — локально нільпотентна група і f — деякий її елемент. Якщо $C_G(f)$ має скінчений ранг і всі силовські підгрупи періодичної частини $t(C_G(f))$ скінченні, то G — гіперцентральна група.

Теорема 5. Якщо G — локально нільпотентна група з періодичною частиною $T = t(G)$ і F — деяка її скінченно породжена підгрупа. Якщо ранг централізатора $C_G(F)$ підгрупи F в групі G скінченний, то ранг централізатора $C_{G/T}(FT/T)$ образу підгрупи F в фактор-групі G/T також скінченний.

1. Мальцев А.И. О группах конечного ранга. // Мат.сб. — 1948. — 22, №2. — С.351—352.

Мінімальні системи твірних метазнакозмінних груп нескінченного рангу

В.С. Сікора

Чернівецький національний університет імені Юрія Федьковича
sikoravira@rambler.ru

Нехай $\bar{n} =< n_1, n_2, ... >$ — нескінченна послідовність натуральних чисел,

$$A(\bar{n}) = A_{n_1} \wr A_{n_2} \wr ...$$

— метазнакозмінна група нескінченного рангу та метастепеня $(n_1, n_2, ...)$ (для означень див. [1]).

У роботі [2] встановлено, що при $n_i \geq 5$, $i \in N$, група $A(\bar{n})$ є топологічно 2-породженою. У повідомленні досліджується мінімальне число твірних цієї групи за умови $n_i \geq 3$, $i \in N$.

Доведено такі твердження.

Теорема 1. Якщо $\bar{n} =< 3, 3, ... >$ або $\bar{n} =< 4, 4, ... >$, то метазнакозмінна група $A(\bar{n})$ є нескінченно породженою в топологічному сенсі.

Символом $k(\bar{n})$ позначимо кількість членів послідовності $\bar{n} =< n_1, n_2, ... >$, які дорівнюють 3 або 4.

Теорема 2. Група $A(\bar{n})$ буде скінченно породженою в топологічному сенсі тоді і тільки тоді, коли $k(\bar{n}) < \infty$.

Якщо виконується умова $k(\bar{n}) < \infty$, то мінімальна кількість топологічних твірних групи $A(\bar{n}) = k(\bar{n}) + 2$.

1. Олійник Б.В., Сущанський В.І., Сікора В.С. Метасиметричні та метазнакозмінні групи нескінченного рангу // Математичні студії.— **29**, N 2.— С.139-150.
2. Bhattacharjee M. The probability of generating certain profinite groups by two elements // Israel Journal of Mathematics.— 1994.— **86**.— P. 311—329.

Абелева група модуля

О.В. Щенсневич

National Taras Shevchenko University of Kiev, Kiev, Ukraine
schensnevichov@mail.ru

Означення. Нехай R, S –– кільця. $R{-}S$-бімодуль, буде збіжним, якщо $J(R)M - MJ(S) = 0$.

Теорема. *Нехай R –– напівдосконале кільце. Група R^* є абелевою, тоді і тільки тоді, коли R є пряма сума власних ідеалів $S \oplus T$, де S або дорівнює нулю, або ізоморфне до прямої суми локальних комутативних кілець L_i, а T або дорівнює нулю, або ізоморфне кільцю $[R_{ij}, M_{ij}]$, де R_i – локальне комутативне кільце таке, що будь-яке поле $R_i/J(R_i)$ складається з двох елементів, а кожний M_{ij} є збіжним $R_i - R_j$-бімодулем.*

1. W. K. Nicholson, Semiperfect rings with abelian group of units, Pacific J. Math., 49. №1 (1973), 191—198.
2. R. W. Gilmer, Finite rings with a cyclic multiplicative group of units, Amer. J. Math., 85 (1963), 447-452.

Index

Гомоморфизмы матричных групп над ассоциативными кольцами

В.М.Петечук

Закарпатский институт последипломного педагогического образования, Ужгород, Украина
vasil-petechuk@rambler.ru

Пусть R – ассоциативное кольцо с 1, $\mathrm{R_n}$ – кольцо всех $n \times n$ матриц над R, $GL(n, R)$ – группа обратимых матриц кольца $\mathrm{R_n}$, $E(n, R)$ – подгруппа $GL(n, R)$, порожденная трансвекциями $t_{ij}(r) = E + re_{ij}$, $r \in R$, $1 \leqslant i \neq j \leqslant n$, G – произвольная группа такая, что $E(n, R) \subseteq G \subseteq GL(n, R)$.

Пусть K – ассоциативное кольцо с 1, W – левый K-модуль, а $GL(W)$ – группа обратимых элементов кольца $End(W)$.

Гомоморфизм $\Lambda : G \to GL(W)$ удовлетворяет условие (*), если для произвольного ненулевого нильпотентного элемента $m \in \mathrm{End(W)}$, $m^2 = 0$ существуют обратимые в K натуральные числа s_1 и s_2 и $A \in G$ такие что $\Lambda A = 1 + s_1 m$ и из равенства $\Lambda A \Lambda B = \Lambda B \Lambda A$, $B \in G$ следует что $A^{s_2} B = B A^{s_2}$.

В частности, условие (*) удовлетворяет произвольный изоморфизм группы G на группу $GL(W)$.

Теорема. *Пусть R и K - ассоциативные кольца с 1,* $E(n, R) \subseteq G \subseteq GL(n, R)$, $n \geqslant 4$, *W- левый K-модуль, гомоморфизм* $\Lambda : G \to GL(W)$ *удовлетворяет условие (*). Тогда существуют подмодули L и P модуля W и изоморфизм* $\mathrm{g : W} \to \underbrace{\mathrm{L} \oplus \ldots \oplus \mathrm{L}}_{n} \oplus \mathrm{P}$, *такие что*

$$\Lambda(E) = \mathrm{g}^{-1} \left[\bar{\delta}(x) e + \bar{\nu}(x)^{-1}(1 - e) + e_1 \right] g,$$

где $x \in E(n, R)$, $\bar{\delta}$ *и* $\vec{\nu}$*- кольцевые гомоморфизм и антигомоморфизм* $\mathrm{R_n}$, *индуцированные кольцевыми гомоморфизмом* $\delta\colon \mathrm{R} \to \mathrm{End(L)}$ *и антигомоморфизмом* $\nu\colon \mathrm{R} \to \mathrm{End(L)}$ *соответственно, e - центральный идемпотент кольца* $\mathrm{End(L)}$, 1 *– единица* $\mathrm{End}(\underbrace{\mathrm{L} \oplus \ldots \oplus \mathrm{L}}_{n})$, *а* e_1*- единица кольца* $\mathrm{End(P)}$.

Замечание. Если дополнительно предположить, что $2 \in \mathrm{R}^*$, то теорема верна при $n \geqslant 3$. Если $\mathrm{n} = 3$ и $2 \notin \mathrm{R}^*$, то существует нестандартный гомоморфизм [6].

1. Golubchik I.Z. Isomorphism of the General Linear Group $\mathrm{GL_n}(R)$, $n \geqslant 4$ over on associative Ring // Contemporary Mathematics. – 1992. – Vol. 131. – Part 1. – P.123-136.
2. Голубчик И.З., Михалев А.В. Изоморфизмы полной линейной группы над ассоциативными кольцами // Вест. МГУ. Сер. 1. Математика., механика. – 1983. –Т.3. – С.61-72.
3. Зельманов Е.И. Изоморфизмы линейных групп над ассоциативными кольцами // Сиб. мат. журн. – 1985. – Т.4. – С.49-67.
4. Петечук В.М. Автоморфизмы матричных групп над коммутативными кольцами // Мат. сб.. – 1982. – №.4. – С.539-547.
5. Петечук В.М. Гомоморфизмы линейных групп над кольцами // Математические заметки. – 1989. – Т.45. – В.2. – С.83-94.
6. Петечук В.М. Гомоморфизмы линейных групп над коммутативными кольцами // Математические заметки. – 1989. – Т.46. – В.5. – С.50-61.

Розклади матриць скінченного порядку над комутативними кільцями

Ю.В.Петечук

Ужгородський національний університет
vasil-petechuk@rambler.ru

Автором вводиться поняття повного добутку многочленів з елементами в комутативних кільцях. Більш точно, многочлен $p(x)$ кільця R[x] над комутативним кільцем R з $1 \neq 0$ називається повним добутком многочленів $p_1(x), ..., p_t(x)$, $t \geqslant 1$ кільця R[x] з елементом $p \in R$, якщо $p(x) = p_1(x)...p_t(x)$ і при $t > 1$ $p \in \langle p_i(x), p_j(x) \rangle_{R[x]}$ для всіх $1 \leqslant i \neq j \leqslant t$. Якщо $p \in R^*$, то $p(x)$ називається повним добутком многочленів $p_1(x), ..., p_t(x)$.

Теорема. Нехай R – комутативне кільце з $1 \neq 0$, $n = p_1^{n_1} \dots p_k^{n_k} > 1$, $p = n^{(1+n_1)\dots(1+n_k)}$. Тоді $x^n - 1$ є повним добутком поліномів ділення круга $\Phi_d(x), d\,|n$ з елементом p. Якщо поліноми ділення круга $\Phi_d(x), d\,|n$ є повними добутками многочленів $p_{dl}(x)$ з елементами $p_d \in R$, то $x^n - 1$ є повним добутком многочленів $p_{dl}(x)$, $d\,|n$ з елементом $p \prod_{d|n} p_d$.

Теорема. Нехай R – комутативне кільце з $1 \neq 0$, V – лівий R-модуль, $a \in EndV$, $p\,(x)$– повний добуток степенів многочленів $p_1\,(x)\,, ..., p_t\,(x)$, $t \geqslant 1$ кільця $R\,[x]$ з елементом$p \in$ R, таким що $pV = V$ і $Ann_V p = 0$, $p\,(a) = 0$. Тоді існує розклад $V = V_1 \oplus \cdots \oplus V_t$, де $aV_i \subset V_i$, $A_i = a|_{V_i}$, $a = diag\,(A_1, ..., A_t)$ і $p_i\,(A_i)$ – нільпотентні елементи для всіх $1 \leqslant i \leqslant t$.Якщо $b \in EndV$ і $ab = ba$, то $bV_i = V_i$, $1 \leqslant i \leqslant t$.

1. Винберг Э.Б. Курс алгебры. – 2-е изд. испр. и доп. – М.: Факториал Пресс, 2001. – 544 с.
2. Гудивок П.М. Представления конечных групп над коммутативными локальными кольцами. – Ужгород: Ужгородский национальный університет, 2003. – 119 с.
3. Дрозд Ю.А., Кириченко В.В. Конечномерные алгебры. – Киев: Изд. объед. «Вища школа», 1980. – 192 с.
4. Каргаполов М.И., Мерзляков Ю.И. Основы теории групп. – 3-е изд. перероб. и доп. – Москва: Наука, 1982. – 288 с.
5. Петечук В.М. Стабильность колец // Наук. вісник Ужгород. ун-ту. Сер. матем. і інформ. – 2009. Вип. 19. – с. 87 – 111.

Right MPI morphic rings and their applications

O.S. Sorokin

Ivan Franko National University of Lviv, Lviv, Ukraine
neverhalluet@gmail.com

Let R be a associative ring with $1 \neq 0$. We say that ring R is *reversible* if for any $x, y \in R$ that $xy = 0$ we also have $yx = 0$ [1]. A ring R is called *left (right) morphic ring* if for any $a \in R$ we have that $R/Ra \cong l(a)$ $(R/aR \cong r(a))$, where $l(a), r(a)$ are left and right annihilators of element a respectively [2]. We say that ring R is *right MPI ring* if every maximal right ideal is left pure ideal (ideal I is said to be *pure* if for any $a \in R$ one can find some $b \in R$ that $a = ba$).

Theorem 1. *Let R be left and right morphic duo ring. Then R is reversible.*

Theorem 2. *Left and right morphic right MPI duo ring is reduced.*

As a corollary we have next result.

Theorem 3. *For a left and right morphic right MPI ring the following properties are equivalent:*
1).R is a right duo ring;
2).R is a reversible ring;
3).R is a reduced ring;
4).R is a Jacobson semisimple ring;
5).R is a semiprimary;
6).R is a von Neumann regular ring;
7).R is an unit-regular ring;
8).R is a strongly regular ring;
8).R is a left duo ring;
10).R is a duo ring.

1. Cohn P. M. Right principal Bezout domains // J. London Math. Soc. – 1987. – 35, № 2. – P. 251–262.
2. W. K. Nicholson and E.Sanchez Campos, Rings with the dual of the isomorphism theorem, J. Algebra 271 (2004), 391–406.