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ABSTRACTS

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This is the book of Abstracts of XII International Skorobohat'ko Mathematical Conference. Conference continues a series of previous meetings, initiated in 1987 by Professor Vitaliy Skorobohat'ko and Academician Anatoliy Dorodnitsyn. The new results in a few branches of mathematics relevant to interests of Prof. Vitaliy Skorobohatko (1927-1996) are presented. Problems in the fields of ordinary differential equations, integral equations, differential equations with partial derivatives are analyzed, tasks in function theory, rational approximations, functional analysis, numerical methods are considered. A number of applications to problems of theoretical physics and mechanics are developed.

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First page: portrait of V.Skorobohatko

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ON THE INTEGRAL EQUATION OF THE ANTENNA SYNTHESIS PROBLEM WITH RESTRICTION

The optimization problem with the restriction on the objective function is considered often in the antenna synthesis. This restriction has a physical sense and it consists of equality of the norms of the objective f and desired F functions. The variational statement of the optimization problems allows getting the nonlinear integral equation, which is solved effectively by the method of successive approximations.

The Euler equation for the proposed functional belongs to the Hammerstein nonlinear integral equations. It contains an unknown function of the third degree in the integrant.

The minimazation of functional

$$\sigma_\alpha(u) = ||F^2 - |f|^2||_f^2 + \alpha||u||_u^2, \tag{1}$$

is used to derive the respective nonlinear integral equation

$$\alpha f - 2AA^*[(F^2 - |f|^2)f] - \mu AA^* f = 0, \tag{2}$$

where A is a linear operator connecting the function f and u as $f = Au$, A^* is an operator joint with operator A. If the function f and parameter μ are determined, the function u, which has the practical interest in the antenna theory, can be determined by known relation [1].

The equation (2) is nonlinear; therefore the iterative procedure is applied for its solving. The numerical results for solving this equation will be presented for the cases of isometric and compact operators A.

The advantage of approach is that one can choose from the set of obtained solution more attractive from the point of view of engineering implementation, namely choose more simple distribution of current u in antenna.

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DETERMINATION OF MECHANICAL PARAMETERS OF A DISC-SIZED NON-CONTRAST INCLUSION OF VARIABLE THICKNESS IN AN ELASTIC MATRIX FROM THE AMPLITUDES OF SCATTERED TORSION WAVES

In this paper, an algorithm for determining the shear modulus and density of a thin-walled disk-shaped non-contrast inclusion of variable thickness from the amplitudes of scattered torsion waves into the Fraunhofer zone (far scattering zone) is proposed. The case of both elastic and piezoelectric non-contrast heterogeneity is considered, i.e. it is assumed that the elastic properties of the inclusion and the matrix differ slightly from each other compared to the small parameter ε which characterizes the relative thickness of the heterogeneity.

Let a thin disk piezoceramic inclusion of variable thickness be located in an elastic unbounded isotropic medium under conditions of axisymmetric torsion and loads that vary harmonically with time, occupying the region $W = \{(r, z) : 0 \leq r \leq a, 2|z| \leq a\varepsilon \mathrm{g}(r)\}$, where $Or\theta z$ is the cylindrical coordinate system; a and $h(r)$ is the radius and thickness of the inclusion, $g(r)$ is a sufficiently smooth positive definite function. The matrix is in ideal mechanical contact with the inclusion, and the perturbed displacements in it satisfy the condition of radiation at infinity.

Using asymptotic methods, the solution of the axisymmetric problem of torsion wave scattering by a thin disk-shaped inclusion of low contrast, contained in elastic space, was obtained in analytical form. Based on the results of the study of the direct wave scattering problem, an effective algorithm for determining the mechanical parameters of such inclusions from the data of shear wave scattering into the far zone was proposed.

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EXISTENCE AND UNIQUENESS FOR p-ADIC POROUS MEDIUM EQUATION

We study generalized solutions of the nonlinear evolution equations for complex-valued functions of a real positive time variable and p-adic spatial variable, which can be seen as non-Archimedean counterparts of the fractional porous medium equation. We construct the appropriate fractional Sobolev type spaces using the algebraic structure of the field of p-adic numbers and applying the Pontryagin duality theory.

We prove the existence and uniqueness results for the corresponding nonlinear equation and define an associated nonlinear semigroup.

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SOBOLEV SPACE ASPECTS OF DIFFERENTIAL EQUATIONS

We consider the most general (generic) class of linear inhomogeneous boundary-value problems for systems of ordinary differential equations of arbitrary order, the solutions and right-hand sides of which belong to the corresponding Sobolev spaces.

Let $(a,b) \subset \mathbb{R}$ be a finite interval and the parameters

$$\{n,k\} \subset \mathbb{N} \cup \{0\}, \quad \{m,r\} \subset \mathbb{N}, \quad 1 \leq p \leq \infty,$$

are arbitrarily chosen. $W_p^{n+r}\big([a,b];\mathbb{C}\big)$ be a complex Sobolev space.

We consider a sequence of linear boundary-value problems

$$(L_k y_k)(t) := y_k^{(r)}(t) + \sum_{j=1}^{r} A_{r-j}(t,k) y_k^{(r-j)}(t) = f(t), \quad t \in (a,b), \qquad (1)$$

$$B_k y_k = c, \qquad (2)$$

where the matrix-valued functions $A_{r-j}(\cdot,k) \in (W_p^n)^{m\times m}$, the vector-valued function $f(\cdot) \in (W_p^n)^m$, the vector $c \in \mathbb{C}^{rm}$, and the linear continuous operators

$$B_k \colon (W_p^{n+r})^m \to \mathbb{C}^{rm}.$$

As proven in the article [1], each of the operators $\big(L_k, B_k\big)$ is a bounded Fredholm operator with index zero.

We find constructive sufficient conditions on the left-hand sides of boundary-value problems under which these problems are well-posed for arbitrary right-hand sides of the problems (1), (2)

$$y_k \to y_0 \quad \text{in} \quad (W_p^{n+r})^m \quad \text{as} \quad k \to \infty.$$

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ON LOCAL ASYMPTOTIC STABILIZATION OF A CERTAIN CLASS OF INHERENTLY NONLINEAR SYSTEMS WITH UNKNOWN PARAMETERS

Let $a_1, \dots, a_n$ be positive numbers that are not known in advance. We consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = a_1 x_2^{2m+1}, & \\ \dot{x}_i = a_i x_{i+1}, & i = 2, \dots, n-1, \\ \dot{x}_n = a_n u, & \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control input, $m > 0$ is a given integer number.

Our objective is to find a control law $u = u(x)$ that does not depend on a_i and ensures the asymptotic stability of the equilibrium $x = 0$ for the closed-loop system obtained by substituting $u = u(x)$ into (1).

We emphasize that the stabilization problem for system (1) represents a so-called critical case, since the linearization of the system at the origin is uncontrollable and unstabilizable. A stabilizing control for the case of known a_i was designed in [1] using the Lyapunov function method. The linear stabilization for three-dimensional case was studied in [2]. The case of unknown a_i for linear systems ($m = 0$) was addressed in [3].

We shown that the nested control of the form

$$u = -(\dots(((k_1 x_1)^{p_1} + k_2 x_2)^{p_2} + k_3 x_3)^{p_3} + \dots + k_n x_n)$$

stabilizes system (1).

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ZEROS OF SUBSYMMETRIC POLYNOMIALS ON ℓ_p

A polynomial P on ℓ_p is *subsymmetric* if it is invariant with respect to the following operators $\mathcal{C}_j$ on ℓ_p:

$$\mathcal{C}_j\colon (x_1,\ldots,x_n,\ldots)\mapsto(x_1,\ldots,x_{j-1},0,x_j,x_{j+1}\ldots),\quad j\in\mathbb{N}.$$

It is known [1] that polynomials

$$P_{\alpha_1,\ldots,\alpha_n}(x)=\sum_{i_1<\cdots<i_n} x_{i_1}^{\alpha_1}\cdots x_{i_n}^{\alpha_n},\quad \alpha_j\geq\lceil p\rceil,$$

form a linear basis in the linear space of polynomials on ℓ_p. We denote by $\mathfrak{S}$ the minimal semigroup generated by operators $\mathcal{C}_j$ and by $\mathcal{P}_{\mathfrak{S}}(\ell_p)$ the algebra of all subsymmetric polynomials on ℓ_p. Linear subspaces in zero-sets of complex polynomials on Banach spaces were investigated first in [2]. Zeros of symmetric and block-symmetric polynomials on ℓ_p were considered in [3]. This talk is devoted to linear subspaces in zero-sets of complex subsymmetric polynomials on ℓ_p.

Theorem. *For every subsymmetric homogeneous polynomial P on ℓ_p and a natural number d there is a d-dimensional subspace V_d in ℓ_p and a linear basis (g_i) in V_d such that P is symmetric on V_d with respect to the basis. Also, there exists a d-dimensional subspace W_d in ℓ_p such that* $W_d\subset\ker P$.

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SOME CONVERGENCE THEOREMS FOR BRANCHED CONTINUED FRACTIONS OF THE SPECIAL FORM

Branched continued fractions with independent variables, when the variables are fixed numbers, are called branched continued fractions of the special form, and these fractions have the following form

$$b_0 + \mathop{\mathrm{D}}_{k=1}^{\infty} \sum_{i_k=1}^{i_{k-1}} \frac{a_{i(k)}}{b_{i(k)}},$$

where $b_0,\ a_{i(k)},\ b_{i(k)} \in \mathbb{C},\ i(k) \in \mathcal{I}$,

$$\mathcal{I} = \left\{ i(k) = (i_1, i_2, \ldots, i_k) : 1 \le i_k \le i_{k-1} \le \ldots \le i_0;\ k \ge 1;\ i_0 = N \right\}.$$

In the case when the elements of this fraction are positive numbers, the convergence criterion and the effective sufficient convergence condition are established.

For branched continued fractions of the special form, researchers often consider parabolic, angular, or circular element regions, along with their corresponding value regions. If the inclusion of the elements in such regions guarantees the convergence of the fraction, the region is called a convergence region.

Numerous convergence criteria for continued fractions and their multidimensional generalizations, such as branched continued fractions, are expressed in terms of convergence regions. When these regions are unbounded, additional conditions on the elements of the fractions are required to ensure convergence. This gives rise to the notion of conditional convergence regions.

By analyzing the classical results obtained for continued fractions, new convergence conditions have been established for branched continued fractions of the special form.

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ESTABLISHMENT AND DEVELOPMENT OF THE ANALYTICAL THEORY OF BRANCHED CONTINUED FRACTIONS

The first generalizations of continued fractions appeared in number theory and algebra. The primary issue that prompted this was the expansion of arbitrary algebraic irrationalities into regular multidimensional continued fractions when the partial numerators are equal to 1 and the partial denominators are natural numbers. To date, this problem remains unsolved. V. Ya. Skorobohatko founded a new scientific direction, the analytical theory and application of branched continued fractions (BCF). Unlike other multidimensional generalizations of continued fractions, BCFs are used to construct rational approximations of analytic functions of many variables. At the initial stage, we considered BCFs with N branches of branching

$$b_0 + \mathop{\mathrm{D}}_{n=1}^{\infty} \sum_{i_n=1}^{N} \frac{a_{i_1,i_2,\ldots,i_n}}{b_{i_1,i_2,\ldots,i_n}}$$

with complex elements or functions. V. Ya. Skorobohatko was convinced of the correctness of his choice after M. O. Nedashkovskyi presented the solution of a system of linear algebraic equations in the form of a finite BCF, the elements of which were the coefficients of this system. The main issues of the analytical theory of BCFs are the conditions of convergence, the classification of functional BCFs, the expansion of different classes of analytic functions into such fractions, and the estimation of approximation errors. From the point of view of possible applications, the stability to perturbations of the BCF is very important. The works of V. R. Hladun are devoted to such studies.

One of the main algorithms for expanding functions into C-fractions is the principle of correspondence between power series and C-fractions. However, in the class of general BCFs, this problem did not have an unambiguous solution. This led to the construction and study of two-dimensional corresponding continued fractions in the works of J. Murphy and M. O'Donoghue, Kh. Kuchminska, A. Cuyt, W. Siemaszko, and O. M. Sus'. Somewhat different, sometimes even very original, constructions of the BCF arise when constructing expansions of relations of various hypergeometric functions of many variables. Such works were first started by O. S. Manzij and N. P. Hoyenko. Today, this direction is being actively developed in the works of R. I. Dmytryshyn and T. M. Antonova. The use of the principle of correspondence for multiple power series has led to the emergence of a very important class of BCFs, the BCFs with independent variables, which, at fixed values of the variables, are called BCFs of the special form

$$b_0 + \mathop{\mathrm{D}}_{n=1}^{\infty} \sum_{i_n=1}^{i_{n-1}} \frac{a_{i_1,i_2,\ldots,i_n}}{b_{i_1,i_2,\ldots,i_n}},$$

where $i_0 = N$. The first results in the construction of an analytical theory of such fractions, in particular, are the convergence theorems, which were obtained by O. E. Baran. This direction is being developed very intensively in the works of R. I. Dmytryshyn, T. M. Antonova, and I. B. Bilanyk.

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WELL-POSEDNESS OF PROBLEMS FOR NONLINEAR ELLIPTIC-PARABOLIC EQUATIONS IN UNBOUNDED DOMAINS

As is well known, in order to guarantee the uniqueness of the solutions of the initial-boundary value problems for the linear and the some nonlinear parabolic equations in unbounded domains, we need to impose certain restrictions on their growth to infinity, for example, to require limitations of solutions or their belonging to some functional spaces. However, there are nonlinear parabolic equations, for which initial-boundary value problems is uniquely solvable without any conditions at infinity.

In this topic we consider weak solutions (from the *generalized Lebesgue and Sobolev spaces*) of problems for nonlinear elliptic-parabolic equations with the variable exponents of the nonlinearity in unbounded domains, and we prove uniquely solvability of the problems either with conditions at infinity or without it.

A typical example of the equations, that are considering here, is

$$(b(x)u)_t - \sum_{i=1}^{n}\Big(\widehat{a}_i(x,t)|u_{x_i}|^{p_i(x)-2}u_{x_i}\Big)_{x_i} + \widehat{a}_0(x,t)|u|^{p_0(x)-2}u = f(x,t),$$

$(x,t) \in \Omega \times (0,T)$, where $n \in \mathbb{N}$ and $T > 0$ be arbitrary fixed numbers; Ω be an unbounded domain in $\mathbb{R}^n$; $b : \Omega \to \mathbb{R}$ is a measurable bounded function, and there exists an open set $\Omega_0 \subset \Omega$ such that $b(x) > 0$ for a.e. $x \in \Omega_0$, and $b(x) = 0$ for a.e. $x \in \Omega \setminus \Omega_0$; $\widehat{a}_i$, $i = \overline{0,n}$, are measurable, bounded, positive and separated from zero functions; the *variable exponents of the nonlinearity* $p_i > 1$, $i = \overline{0,n}$, are functions such that for all $i \in \{0, 1, \ldots, n\}$ the function $p_i : \Omega \to \mathbb{R}$ is measurable, $1 < \operatorname*{ess\,inf}_{x\in\Omega'} p_i(x) \le \operatorname*{ess\,sup}_{x\in\Omega'} p_i(x) < +\infty$ for every bounded subdomain Ω' of domain Ω. It is obvious that on the set $\Omega_0 \times (0,T)$ this equation is parabolic, and on the set $(\Omega \setminus \Omega_0) \times (0,T)$ it is elliptic, and therefore it is called *elliptic-parabolic*.

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WEAKLY PERTURBED BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF INTEGRO-DIFFERENTIAL EQUATIONS IN THE GENERAL CASE. AN EQUIVALENT SYSTEM OF ALGEBRAIC EQUATIONS

Consider a boundary value problem for a linear system of integrodifferential equations with a small parameter ε:

$$\dot{x}(t) - \Phi(t)\int_a^b \Big[A(s)x(s) + B(s)\dot{x}(s)\Big]ds =$$

$$= f(t) + \varepsilon\int_a^b \Big[K(t,s)x(s) + K_1(t,s)\dot{x}(s)\Big]ds, \tag{1}$$

$$\ell x(\cdot,\varepsilon) = \alpha + \varepsilon\ell_1 x(\cdot,\varepsilon) \ \in \ \mathbb{R}^q. \tag{2}$$

Using perturbation theory methods, the coefficient conditions for the existence of a solution to the boundary value problem (1), (2) are found in the general case. The structure of the solution set for this problem is constructed in the space $D_2[a,b]$ n-dimensional absolutely continuous differentiable vector-functions $x = x(t,\varepsilon) : x(\cdot,\varepsilon) \in D_2[a,b]$, $\dot{x}(\cdot,\varepsilon) \in L_2[a,b]$, $x(t,\cdot) \in C(0,\varepsilon_0]$. It is proven that for arbitrary inhomogeneities on the right sides of equations (1), (2), there exists a unique solution in the form of a convergent series $x(t,\varepsilon) = \sum\limits_{i=-(k+1)}^{\infty} \varepsilon^i x_i(t,c_i)$ for $\varepsilon \in (0,\varepsilon_*] \subseteq (0,\varepsilon_0]$. Here, $A(t), B(t),\ \Phi(t)$ are $m\times n,\ m\times n,\ n\times m$-dimensional matrices whose components belong to the space $L_2[a,b]$; the column vectors of the matrix $\Phi(t)$ are linearly independent on $[a,b]$, $f(t)$ is an n-dimensional vector-function from $L_2[a,b]$; ℓ,ℓ_1 are linear bounded q-dimensional vector functionals, $\alpha = col(\alpha_1,\alpha_2,...,\alpha_q) \in R^q$. $K(t,s), K_1(t,s)$ are $(n\times n)$-dimensional matrices whose components are defined in the space of functions integrable on the interval $L_2[a,b]$. An equivalent algebraic system for the given problem is constructed.

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MFS-BASED ITERATIVE METHOD FOR A NON-LINEAR INVERSE HEAT CONDUCTION PROBLEM

Let D be a doubly-connected domain in $\mathbb{R}^d$, $d = 2, 3$, with an inner boundary Γ_1 and an outer boundary Γ_2. We consider a non-linear ill-posed inverse problem of determining the boundary Γ_1 from Cauchy data of an unknown function u that satisfies the heat equation

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u & \text{in } D \times [0, T], \\ u = f_2, \ \dfrac{\partial u}{\partial \boldsymbol{\nu}} = g_2 & \text{on } \Gamma_2 \times [0, T], \\ u = 0 & \text{on } \Gamma_1 \times [0, T], \\ u(\cdot, 0) = 0 & \text{in } D, \end{cases}$$

where f_2 and g_2 are given smooth functions, $\boldsymbol{\nu}$ is the outward unit normal to Γ_2 and $T > 0$ is the final time.

The problem is formulated as a nonlinear operator equation, which is solved by the Newton's method. At each step of the iterative procedure, it is necessary to solve a set of Dirichlet initial-boundary value problems for the heat equation (direct problems), which we perform using a two-step method. First, Rothe's method is applied to reduce every direct problem to a recurrent sequence of Dirichlet problems for the modified Helmholtz equation. Then, this sequence is fully discretized using the method of fundamental solutions. The initial guess for Newton iterations is obtained with the help of a genetic algorithm. The effectiveness of the method is confirmed by results of numerical experiments.

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ON INITIAL INVERSE PROBLEMS WITH INTEGRAL CONDITIONS

We find sufficient conditions for a unique solvability of the inverse problem on reconstruction the initial values u_0 (from the entire scale of spaces of Bessel potentials $H^{s+\alpha+\gamma,p}(\mathbb{R}^n)$) of the solution for a time-space fractional diffusion equation under a time-integral additional condition, that is the problem of determining the pair (u, u_0):

$$
\begin{gathered}
{}^cD_t^\beta u + (1-\Delta)^{\gamma/2}(-\Delta)^{\alpha/2}u = F(x,t), \quad (x,t) \in \mathbb{R}^n \times (0,T], \\
u(x,0) = u_0(x), \quad x \in \mathbb{R}^n, \\
\frac{1}{T}\int_0^T u(x,t)dt = \Phi(x), \quad x \in \mathbb{R}^n,
\end{gathered}
$$

where ${}^cD_t^\beta u$ is the Caputo fractional derivative of order $\beta \in (0,1)$ of u, $\alpha > \beta$, $\gamma \geq 0$, $s \in \mathbb{R}$, $p > 1$ and $(1-\Delta)^{\gamma/2}(-\Delta)^{\alpha/2}$ is determined using the Fourier transform: $\mathcal{F}[(-\Delta)^{\alpha/2}(1-\Delta)^{\gamma/2}\psi(x)] = |\xi|^\alpha(1+|\xi|^2)^{\gamma/2}\mathcal{F}[\psi(x)]$.

We also find sufficient conditions for the unique solvability of the inverse problem

$$
\begin{gathered}
{}^cD_t^\beta u - \Delta u = F(x,t), \quad (x,t) \in \mathbb{R}^n \times (0,T], \quad \beta \in (1,2), \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}^n, \\
\frac{1}{T}\int_0^T u(x,t)\eta_1(t)dt = \Phi_1(x), \quad \frac{1}{T}\int_0^T u(x,t)\eta_2(t)dt = \Phi_2(x), \quad x \in \mathbb{R}^n,
\end{gathered}
$$

of determining the triple (u, u_0, u_1) with values in Schwartz-type spaces of smooth functions rapidly decreasing to zero at infinity.

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APPLICATION OF THE PYRAMIDS METHOD FOR SOLVING THE PROBLEM OF DIGITAL DATA FILTERING

The relevance of the work is determined by the quick access to undistorted information, which is extremely important in real life.

The objective of the research is to apply the pyramids method for loops parallelization to solve a one-dimensional digital filtering problem (DFP) and obtaining real speedup for the constructed parallel algorithms using the modern high-level programming language Python.

One-dimensional DFP consists in performing k smoothing recalculations of a certain array of n variable values according to the formula:

$$x_i = \sum_{s=-m}^{m} x_{i+s} \cdot f_s, \quad i = \overline{1, n}.$$

Here $f_s \quad (s = \overline{-m, m})$ are specified weight coefficients.

Almost always, DFP has to be solved in real time for preliminary processing of signals or restoration of large arrays of damaged or distorted data. Therefore, parallel algorithms must be used for this purpose. The pyramids method allows constructing such algorithms with autonomous branches.

We propose efficient parallel algorithms with autonomous branches and limited parallelism [1] to solve the formulated DFP.

The software implementation of the constructed parallel algorithms has been carried out with the ability to generate an input distorted signal, graphically display the filtration results and obtain real speedup based on the execution time of the sequential and corresponding parallel algorithms.

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INITIAL-BOUNDARY VALUE PROBLEM FOR SYSTEM OF EQUATIONS WITH SHIFT

Let $n \in \mathbb{N}$, $T > 0$ and $\alpha > 0$ be fixed numbers, $n \geq 2$, $\Omega \subset \mathbb{R}^n$ be a bounded domain with the Lipschitz boundary $\partial\Omega$, $Q_{0,T} = \Omega \times (0,T)$.

We consider the problem of finding function $u = (u_1, \ldots, u_n) : Q_{0,T} \to \mathbb{R}^n$ that satisfies the following relations:

$$u_{k,t} + \alpha\, \Delta^2\big(u_k + b_k\big) + \Delta\Big(c_k(x,t)\Big|u + b(x,t)\Big|^{\gamma(x,t)-2}\Big(u_k + b_k(x,t)\Big)\Big) -$$

$$-\sum_{i=1}^{n}\Big(a_{ik}(x,t)\Big|u_{x_i} + b_{x_i}(x,t)\Big|^{p(x,t)-2}\Big(u_k + b_k(x,t)\Big)_{x_i}\Big)_{x_i} + \qquad (1)$$

$$+\Big(\mathsf{N}\Big(u+b\Big)\Big)_k(x,t) + \phi_k\Big(\Big(\mathsf{E}(u+b)\Big)_k(x,t)\Big) = F_k(x,t), \quad (x,t) \in Q_{0,T},$$

$$u(x,t) = \Delta u(x,t) = 0, \quad x \in \partial\Omega, \quad t \in [0,T], \qquad (2)$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \qquad (3)$$

where Δ denotes the Laplacian, $\Delta^2 := \Delta(\Delta)$, $a_{ik}, b_k, c_k, \phi_k, F_k, u_0, \mathfrak{Z}, \gamma, p$ are some functions,

$$(\mathsf{N}u)_k(x,t) := g_k(x,t)|u(x,t)|^{q(x,t)-2}u_k(x,t),$$

$$(\mathsf{E}u)_k(x,t) := \int\limits_{\Omega} \mathfrak{Z}_k(x,t,y)u_k(y,t)\, dy, \quad (x,t) \in Q_{0,T}, \quad k = \overline{1,n}.$$

Under additional conditions for data-in, we prove the solvability of problem (1)–(3).

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APPROXIMATION SCHEMES FOR DIFFERENTIAL-DIFFERENCE EQUATIONS AND ALGORITHMS OF THEIR APPLICATIONS

An important task in the study of differential-difference equations is the development of algorithms for constructing approximate solutions, since no universal methods for obtaining exact solutions are currently available. Of particular interest are studies that enable the application of methods from the theory of ordinary differential equations to the analysis of delay differential equations.

Let us consider the initial value problem for a differential equation with delay:

$$x'(t) = f\big(t, x(t), x(t-\tau)\big), \quad t \in [t_0, T], \tag{1}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - \tau, t_0]. \tag{2}$$

To the problem (1)–(2), we assign the Cauchy problem for a system of ordinary differential equations:

$$z_0'(t) = f\big(t, z_0(t), z_m(t)\big), \tag{3}$$

$$z_j'(t) = \frac{m}{\tau}\left(z_{j-1}(t) - z_j(t)\right), \quad j = \overline{0,m}, \tag{4}$$

$$z_j(t_0) = \varphi\left(t_0 - \frac{j\tau}{m}\right), \quad j = \overline{0,m}. \tag{5}$$

The theoretical justification of approximation schemes for differential-difference equations within various functional spaces were provided in [1–4].

The study of the relationships between differential-difference equations and their corresponding approximating systems of ordinary differential equations has made it possible to develop algorithms for solving a range of applied problems. Approximation schemes for non-asymptotic roots of quasipolynomials of linear differential-difference equations were proposed in [5], and a method for studying the stability of solutions of such equations was presented in [6]. Constructive algorithms for building stability regions of linear systems with multiple delays were obtained in [7].

Numerical experiments conducted on model test examples confirm the effectiveness of the proposed approximation schemes for delay differential equations.

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SUPERSYMMETRIC ANALYTIC FUNCTIONS OF INFINITELY MANY VARIABLES

We consider the ring of multisets consisting of elements in a given multiplicative semigroup of a Banach algebra and endowed with some natural "supersymmetric" operations of addition and multiplication. We construct a complete metrizable topology of the ring of multisets generated by a ring norm. In addition, we investigat homomorphisms of the ring and their relations with supersymmetric polynomials.

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CONDITIONAL SYMMETRIES AND EXACT SOLUTIONS OF A SIMPLIFIED SHIGESADA–KAWASAKI–TERAMOTO MODEL

We study a simplified form of the Shigesada–Kawasaki–Teramoto model [1], which generalizes the diffusive Lotka–Volterra system by incorporating nonlinear cross-diffusion terms. The model is given by the system:

$$\begin{aligned} u_t &= \left[(d_1 + d_{12}v)u\right]_{xx} + u(a_1 - b_1 u - c_1 v), \\ v_t &= \left[(d_2 + d_{21}u)v\right]_{xx} + v(a_2 - b_2 u - c_2 v), \end{aligned} \tag{1}$$

where $u(t,x)$ and $v(t,x)$ denote the densities of two interacting species, and the constants represent diffusion and reaction processes.

We construct a complete set of Q-conditional (nonclassical) symmetries for system (1), where the symmetry operator Q is given by

$$Q = \partial_t + \xi(t,x,u,v)\partial_x + \eta^1(t,x,u,v)\partial_u + \eta^2(t,x,u,v)\partial_v.$$

We apply the obtained Q-conditional symmetry operators to reduce the original system to systems of ordinary differential equations. Using these reductions, exact solutions are constructed explicitly. Their properties are examined in detail, and the results are interpreted from a biological perspective, emphasizing their impact on population growth and species interactions within the model.

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ON SOME MATRIX PARABOLIC PROBLEMS IN SOBOLEV SPACES OF GENERALIZED SMOOTHNESS

We consider a linear initial-boundary-value problem for a Petrovskii parabolic system of second order differential equations with the Dirichlet or a general first-order boundary conditions. All coefficients of the corresponding PDOs are complex-valued C^{∞}-functions. We study the solvability character of this problem and the regularity of its solutions in Hilbert anisotropic Sobolev spaces $H^{s,s/2;\varphi}$ over a finite multidimensional cylinder. Here, s is a positive number and $\varphi : [1,\infty) \to (0,\infty)$ is a Borel measurable function that varies slowly at infinity in the sense of Karamata, i.e. $\varphi(\lambda r)/\varphi(r) \to 1$, as $r \to \infty$, whenever $\lambda > 0$. This function sets a generalized smoothness subject to the main anisotropic smoothness $(s, s/2)$.

The operator of this problem sets isomorphisms on appropriate pairs of the indicated spaces [1].

We obtain new sharp conditions for generalized or classical regularity of the problem solutions [2,3].

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ON FUNDAMENTAL SOLUTION AND CORRECT SOLVABILITY OF THE CAUCHY PROBLEM FOR DEGENERATED PARABOLIC EQUATIONS OF KOLMOGOROV TYPE OF ARBITRARY ORDER WITH BLOCK STRUCTURE

We consider a class of the equations $\Big(\partial_t - \sum\limits_{j=1}^{n_2}\Big(\sum\limits_{s=1}^{n_1} b^1_{sj}x_{1s}\Big)\partial_{x_{2j}} - \sum\limits_{j=1}^{n_3}\Big(\sum\limits_{s=1}^{n_2} b^2_{sj}x_{2s}\Big)\partial_{x_{3j}} - \sum\limits_{|k_1|\le 2b} a_{k_1}(t,x)\partial^{k_1}_{x_1}\Big)u(t,x) = 0$, $(t,x)\in\Pi_{(0,T]}$, where n, n_1, n_2 and n_3 be given positive integers such that $n_1 \ge n_2 \ge n_3 \ge 1$ and $n = n_1+n_2+n_3$; spatial variable $x := (x_1,x_2,x_3)\in\mathbb{R}^n$ with components $x_j := (x_{j1},\dots,x_{jn_j})\in\mathbb{R}^{n_j}$, $j\in\{1,2,3\}$; multi-index $k := (k_1,k_2,k_3)\in\mathbb{Z}^{n_j}_+$ with $k_j := (k_{j1},\dots,k_{jn_j})\in\mathbb{Z}^{n_j}_+$, $|k_j| := |k_{j1}| + \dots + |k_{jn_j}|$, $j\in\{1,2,3\}$; $\Pi_H := \{(t,x)|t\in H, x\in\mathbb{R}^n\}$, if $H\subset\mathbb{R}$.

This class under some conditions for the coefficients of the equations is a generalization of the class of degenerate parabolic equations of Kolmogorov type $\mathbf{E}_{21}$ from the monograph [1]. Using the results of the paper [2] we built and studied the classic fundamental solutions of the Cauchy problem for the equations. Also we proved the correct solvability of the Cauchy problem in special weighed spaces, receive integral presentation of classic solutions of the Cauchy problem for the equations. Classes of well-posedness of the Cauchy problem were described.

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POLYNOMIAL MATRICES, THEIR FACTORIZATIONS AND DIFFERENTIAL EQUATIONS IN INVESTIGATIONS OF LOPATYNS'KII, KAZIMIRS'KII AND MODERN SCIENTISTS

Ya.B. Lopatyns'kii in [1, 2] established a connection between the factorization of a matrix polynomial into regular factors and the solvability of systems of differential equations with constant coefficients. In particular, he established a criterion for separating a regular factor from a regular polynomial matrix over the field of complex numbers in the case when the factors do not have common characteristic roots. According to his remark, the proof of this algebraic result is based on the theory of systems of differential equations with constant coefficients.

P.S. Kazimirs'kii in [3] proposed a new algebraic approach to the factorization of polynomial matrices. He introduced the concepts of the value of a polynomial matrix on the root system of a polynomial, the accompanying matrix, and the semiscalar equivalence of polynomial matrices. This approach allowed him to solve, in general, the problem of separation of a regular factor from a polynomial matrix over an algebraically closed field of characteristic zero [4].

P.S. Kazimirs'kii's students considered the problem of factorization of matrices in more general cases.

In [5], the method of factorization of polynomial matrices over an arbitrary field is suggested. For matrices over adequate rings, the concept of generalized equivalence of a pair of matrices is introduced. Based on the constructed standard form of a pair of matrices with respect to this equivalence, their factorizations are described.

In [6], using a special normal form of matrices with respect to unilateral equivalence, the method of factorization of matrices over rings of elementary divisors is developed.

It should be noted that the problem of using the factorability of polynomial matrices to solve differential equations is still relevant. In [7], from the prime decomposition of a quadratic matrix polynomial, a formula of the general solution to the corresponding second-order differential equation is obtained. The authors called a prime decomposition the factorization of a polynomial matrix of second degree into linear factors for which the condition of existence of a solution to a certain matrix polynomial equation of the Sylvester-type with two variables is satisfied. In [8], we proposed the methods for solving such matrix equations and investigated the structure of their solutions.

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CONTROLLABILITY AND APPROXIMATE CONTROLLABILITY OF THE HEAT EQUATION ON A HALF-PLANE

Let $T > 0$. Consider the heat equation on a half-plane $w_t = \Delta w$ controlled by the Dirichlet boundary condition $w(0, x_2, t) = \delta_{[2]} u(t)$ or by the Neumann boundary condition $w_{x_1}(0, x_2, t) = \delta_{[2]} u(t)$ under the initial condition $w(x_1, x_2, 0) = w^0(x_1, x_2)$, $x_1 > 0$, $x_2 \in \mathbb{R}$, $t \in (0, T)$, where $u \in L^\infty(0, T)$ is a control; $\delta_{[2]}$ is the Dirac distribution in x_2.

We investigate two control systems in subspaces of Sobolev spaces $H^s(\mathbb{R}^2)$ with $s = \overline{-3, -1}$ for the Dirichlet boundary condition and with $s = \overline{-2, 0}$ for the Neumann boundary condition. We consider the following (approximate) controllability problems: For given $T > 0$ and w^0, we aim to find a control $u \in L^\infty(0, T)$ that steers w^0 to a given end state exactly or approximately.

In both cases, the Dirichlet and Neumann boundary controls, we obtain necessary and sufficient conditions for approximate controllability in a given time; necessary and sufficient conditions for controllability in a given time under the control bounded by a given constant; sufficient conditions for approximate controllability in a given time under the control bounded by a given constant; the lack of controllability to the origin. These results have been published in [1] and [2].

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ON SYMMETRY REDUCTION OF THE (1 + 3)-DIMENSIONAL MONGE-AMPÈRE EQUATION TO ALGEBRAIC EQUATIONS

It is well known, that the symmetry reduction is one of the most powerful tools for investigation of partial differential equations.

We consider the $(1+3)$-dimensional inhomogeneous Monge-Ampère equation of the form:

$$\det\left(u_{\mu\nu}\right) = \lambda\left(1 - u_\nu u^\nu\right)^3, \quad \lambda \neq 0,$$

where $u = u(x)$, $x = (x_0, x_1, x_2, x_3) \in M(1,3)$, $u_\mu \equiv \dfrac{\partial u}{\partial x^\mu}$, $u_{\mu\nu} \equiv \dfrac{\partial u_\mu}{\partial x^\nu}$, $u^\mu = g^{\mu\nu} u_\nu, g_{\mu\nu} = (1,-1,-1,-1)\delta_{\mu\nu},\ \ \mu,\nu = 0,1,2,3.$

Here $M(1,3)$ is a $(1+3)$-dimensional Minkowski space.

By now, we have performed symmetry reduction of the equation under study to algebraic equations, using three-dimensional nonconjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$. Some invariant solutions are constructed. I plan to present some of the results obtained.

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WEAKLY NONLINEAR BOUNDARY-VALUE PROBLEM FOR THE SYSTEM OF FRACTIONAL DIFFERENTIAL EQUATIONS WITH TEMPERED CAPUTO DERIVATIVE

We consider weakly nonlinear boundary-value problem

$$ {}^{C}\mathrm{D}_{a+}^{\alpha,\lambda} x(t) = A(t)x(t) + f(t) + \varepsilon Z(x(t,\varepsilon),t,\varepsilon), \tag{1}$$

$$ l\left(e^{\lambda\cdot} x(\cdot)\right) = q + \varepsilon J(e^{\lambda\cdot} x(\cdot,\varepsilon),\varepsilon), \tag{2}$$

where ${}^{C}\mathrm{D}_{a+}^{\alpha,\lambda}$ is the left tempered Caputo fractional derivative of order α $(0 < \alpha < 1)$, $A(t)$ is a continuous $(n \times n)$-matrix on $[a,b]$, $f(t)$ is a continuous n-vector on $[a,b]$, $l = \operatorname{col}\begin{pmatrix} l_1, & l_2, & \ldots, & l_p \end{pmatrix} : C[a,b] \to \mathbb{R}^p$ is a bounded linear vector functional, $l_\nu : C[a,b] \to \mathbb{R}$, $\nu = \overline{1,p}$, $q = \operatorname{col}\begin{pmatrix} q_1, & q_2, & \ldots, & q_p \end{pmatrix} \in \mathbb{R}^p$, $Z(x(t,\varepsilon),t,\varepsilon)$ is the function nonlinear with respect to the first component and such that

$$ Z(\cdot,t,\varepsilon) \in C^1[\|x - x_0\| \leq \mu], $$

$$ Z(x(\cdot,\varepsilon),\cdot,\varepsilon) \in C[a,b], \quad Z(x(t,\cdot),t,\cdot) \in C[0,\varepsilon_0], $$

$J(e^{\lambda\cdot} x(\cdot,\varepsilon),\varepsilon)$ is the nonlinear bounded p-dimensional vector functional, continuously Fréchet differentiable with respect to $z(t,\varepsilon) = e^{\lambda t}x(t,\varepsilon)$ and continuous with respect to ε in the neighborhood of the generating solution, μ and ε_0 are sufficiently small constants, and ε is a small parameter.

We establish necessary and sufficient conditions for the solvability of the generating boundary-value problem $(\varepsilon = 0)$ and find the general form of the solution to this problem under the assumption that the number of boundary conditions (2) do not coincide with the number of unknowns of the system of fractional differential equations (1). The necessary and sufficient conditions for the existence of a solutions of the boundary-value problem (1) (2) have been obtained. An iterative algorithm for building such a solutions has been proposed.

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COPOLYNOMIALS OVER A COMMUTATIVE RING AND THEIR APPLICATIONS TO NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Let K be an arbitrary commutative integral domain with identity of characteristic 0. We study the copolynomials of n variables, i.e., K-linear mappings from the ring of polynomials $K[x_1, \dots, x_n]$ into K. We consider copolynomials as algebraic analogues of distributions. With the help of the Cauchy-Stieltjes transform of a copolynomial we introduce and study a multiplication of copolynomials. We prove the existence and uniqueness theorem of the Cauchy problem for some nonlinear partial differential equations in the ring of formal power series with copolynomial coefficients. We study a connection between some classical nonlinear partial differential equations and integer sequences. In particular, for the Cauchy problem for the Burgers equation

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + bu\frac{\partial u}{\partial x}, \quad u(0,x) = \delta(x), \quad (a, b \in \mathbb{Z})$$

we obtain the representation of the unique solution to this problem in the form of the series in powers of the δ-function with integer coefficients.

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CERTAIN COMBINATORIAL IDENTITIES INVOLVING MERSENNE NUMBERS AND MULTINOMIAL COEFFICIENTS

We study certain families of Toeplitz–Hessenberg determinants whose entries are the Mersenne numbers $M_n = 2^n - 1$, where $n \in \mathbb{N}_0$. Recall that a Toeplitz–Hessenberg matrix is an $n \times n$ matrix $A_n = (a_{ij})$, where $a_{ij} = 0$ for all $j > i+1$, $a_{ij} = a_{i-j+1}$, and $a_{i,i+1} = a_0$ with $a_0 \neq 0$ and at least one $a_k \neq 0$. We focus on matrices A_n with $a_0 = \pm 2$.

Using Trudi's formula, we can express our determinant formulas as identities involving sums of products of Mersenne numbers and multinomial coefficients (see [1, 2] for further details and related references).

Theorem. *If $n \in \mathbb{N}$, then*

$$\sum_{\sigma_n = n} \tfrac{m_n(s)}{(-2)^{|s|}} M_0^{s_1} M_1^{s_2} \cdots M_{n-1}^{s_n} = \tfrac{i}{2}\Big(\big(\tfrac{3+i}{2}\big)^{n-1} - \big(\tfrac{3-i}{2}\big)^{n-1}\Big),$$

$$\sum_{\sigma_n = n} \tfrac{m_n(s)}{2^{|s|}} M_0^{s_1} M_1^{s_2} \cdots M_{n-1}^{s_n} = \tfrac{\sqrt{3}}{6}\Big(\big(\tfrac{3+\sqrt{3}}{2}\big)^{n-1} - \big(\tfrac{3-\sqrt{3}}{2}\big)^{n-1}\Big),$$

$$\sum_{\sigma_n = n} \tfrac{m_n(s)}{2^{|s|}} M_1^{s_1} M_2^{s_2} \cdots M_{n}^{s_n} = \tfrac{\sqrt{17}}{17}\Big(\big(\tfrac{7+\sqrt{17}}{4}\big)^{n} - \big(\tfrac{7-\sqrt{17}}{4}\big)^{n}\Big),$$

$$\sum_{\sigma_n = n} \tfrac{m_n(s)}{(-2)^{|s|}} M_0^{s_1} M_2^{s_2} \cdots M_{2(n-1)}^{s_n} = \tfrac{\sqrt{3}}{2}\Big(\big(\tfrac{5-\sqrt{3}}{2}\big)^{n-1} - \big(\tfrac{5+\sqrt{3}}{2}\big)^{n-1}\Big),$$

$$\sum_{\sigma_n = n} \tfrac{m_n(s)}{2^{|s|}} M_0^{s_1} M_2^{s_2} \cdots M_{2(n-1)}^{s_n} = \tfrac{\sqrt{15}}{10}\Big(\big(\tfrac{5+\sqrt{15}}{2}\big)^{n-1} - \big(\tfrac{5-\sqrt{15}}{2}\big)^{n-1}\Big),$$

where $i^2 = -1$, $|s| = s_1 + \cdots + s_n$, $\sigma_n = s_1 + 2s_2 + \cdots + ns_n$ with $s_i \geq 0$, and $m_n(s) = \binom{s_1+\cdots+s_n}{s_1,\ldots,s_n}$.

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IMPLICIT LINEAR DIFFERENCE EQUATIONS OVER ONE FINITE COMMUTATIVE RING OF ORDER p^2

Let us denote $\mathbb{Z}_m$ the residue class ring modulo m, and let p be a prime number. Consider the following ring of order p^2: $\mathcal{S}_p = \mathbb{Z}_p[t]/(t^2)$. Let $A, B, F_n, Y_0 \in \mathcal{S}_p$ ($n \in \mathbb{Z}_+ = \{0, 1, 2, \ldots\}$) be given. In the ring $\mathcal{S}_p$ consider the following initial problem

$$BX_{n+1} = AX_n + F_n, \quad n \in \mathbb{Z}_+, \quad X_0 = Y_0 \tag{1}$$

for the first-order implicit linear difference equation. Recently this initial problem was investigated over the ring $\mathbb{Z}_m$ for an arbitrary natural $m \geq 2$ [1]. The following theorem establishes a solvability criterion for the initial problem (1) over the ring $\mathcal{S}_p$.

Theorem. *Let $A = A_0 + A_1 t$, $B = B_0 + B_1 t \neq 0$, $F_n = F_{0,n} + F_{1,n} t$, where $A_0, A_1, B_0, B_1, F_{0,n}, F_{1,n}$ ($n \in \mathbb{Z}_+$) are the elements of the ring $\mathbb{Z}_p$. The following assertions hold.*

1. *The initial problem* (1) *has a unique solution if and only if one of the following conditions holds:*
   - *(a)* $B_0 \neq 0$.
   - *(b)* $B_0 = 0$, $A_0 \neq 0$ *and* $Y_0 = -A^{-1}F_0 - A^{-2}BF_1$.
2. *The initial problem* (1) *has infinitely many solutions if and only if* $A_0 = B_0 = 0$ *and* $F_{0,n} = 0$ *for all* $n \in \mathbb{Z}_+$.
3. *The initial problem* (1) *has no solutions if and only if* $B_0 = 0$ *and one of the following conditions holds:*
   - *(a)* $A_0 = 0$ *and* $F_{0,n} \neq 0$ *for some* $n \in \mathbb{Z}_+$.
   - *(b)* $A_0 \neq 0$ *and* $Y_0 \neq -A^{-1}F_0 - A^{-2}BF_1$.

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RATIONALLY FACTORIZED LAX TYPE FLOWS RELATED WITH THE CENTRALLY EXTENDED LIE ALGEBRA OF FRACTIONAL INTEGRO-DIFFERENTIAL OPERATORS AND INTEGRABLE BENNEY TYPE HYDRODYNAMIC SYSTEMS

In a recently published article by O. Hentosh and A. Prykarpatski the rational factorization method for the Lax type Hamiltonian flows in the space dual to the Lie algebra of fractional integro-differential operators has been proposed for constructing new hierarchies of integrable nonlinear dynamical systems with fractional derivatives by one spatial variable and hydrodynamic Benney type systems as their quasi-classical approximations. In this report the rational factorization method is developed for the Lax type flows obtained as Hamiltonian ones in the space dual to the central extension by the Maurer-Cartan two-cocycle of the parameterized Lie algebra of fractional integro-differential operators within the general Lie-algebraic Adler-Kostant-Symes scheme.

By means of the Backlund mapping given by the rational factorization, one shows that the system of Hamiltonian flows for a pair of different elements of the parameterized operator Lie algebra, connected by the generalized gauge transformation, is equivalent to the system of two evolution equations for the fractional differential operators which rationally factorize these elements and proves the Hamiltonicity of the latter. Its quasi-classical approximation is established to lead to the system of two evolution equations for the polynomials by the fractional power of some complex variable as elements of the parameterized Lie algebra of fractional symbols.

The rational factorization method developed for the central extension is applied to construct a new integrable hierarchy of two-dimensional nonlinear dynamical systems with fractional derivatives by one spatial variable and a corresponding integrable hierarchy of two-dimensional hydrodynamic Benney type systems.

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DEGENERATE PROBLEMS FOR COUNTABLE 1D HYPERBOLIC SYSTEMS

For a countable hyperbolic system of first-order equations of the form

$$\frac{\partial u_i}{\partial t} + \lambda_i\,(x, t, u_1, u_2, \ldots)\,\frac{\partial u_i}{\partial x} = f_i(x, t, u_1, u_2, \ldots), \quad i \in \mathbb{N},$$

consider the correct solvability of various problems with certain degeneracies. First of all, the characteristics of the system may be degenerate (horizontal and vertical with respect to the coordinate axes); the domain of the solution may also be degenerate (the line of initial conditions degenerates into a point, from which a certain number of the system's characteristics enter the domain).

Theorems are proved on the closeness of solutions of the countable and the truncated finite system, and local and global generalized solutions are constructed for mixed problems of countable hyperbolic systems of quasilinear equations.

The idea of proving the main theorems is based on the reduction of mixed problems for such systems to systems of countable integro-functional equations, for which the method of contracting mappings is applied. The investigation is carried out on the basis of the results of works [1–2].

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PARABOLIC CONVERGENCE REGION OF SOME EXPANSION INTO BRANCHED CONTINUED FRACTION FOR APPELL HYPERGEOMETRIC FUNCTION $F_4(1,2;2,2,z_1,z_2)$

We consider Appell hypergeometric function F_4 defined by double power series:

$$F_4(a,b;c,c';z_1,z_2)=\sum_{m,n=0}^{\infty}\frac{(a)_{m+n}(b)_{m+n}}{(c)_m(c')_n}\frac{z_1^m z_2^n}{m!n!},$$

where parameters a, b, c, c' are complex numbers, z_1, z_2 are complex variables, c, c' are not equal to $0, -1, -2, \ldots$, $(d)_k = d(d+1)\ldots(d+k-1)$ is the Pochhammer symbol.

The expansion into branched continued fraction for Appell function $F_4(1,2;2,2,z_1,z_2)$ was built

$$\cfrac{1}{1-\cfrac{z_1}{1-\cfrac{1/2z_2}{1-\ddots}}-\cfrac{z_2}{1-\cfrac{z_1}{1-\ddots}}}.$$

We investigate the convergence region of this branched continued fraction.

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ASYMPTOTICS OF THE SPECTRUM OF SCHRÖDINGER OPERATORS WITH SINGULAR POTENTIALS AND SINGULAR WEIGHT FUNCTIONS

The asymptotics of the spectrum of the Schrödinger operator on $\mathbb{R}$ with singular perturbations in a neighborhood of the origin of a potential $W(x)$ and a weight function $m(x)$ are investigated. Namely, the goal is to study the asymptotics of the eigenvalues of the problem

$$\Big(-\frac{d^2}{dx^2}+W(x)+\varepsilon^{-2}V(\varepsilon^{-1}x)+\varepsilon^{-1}U(\varepsilon^{-1}x)\Big)y_\varepsilon = \\ = \lambda_\varepsilon\left(m(x)+\varepsilon^{-1}M(\varepsilon^{-1}x)\right)y_\varepsilon. \quad (1)$$

These perturbations can be considered as a regularization of the pseudopotential $W(x)+a\delta'(x)+b\delta(x)$ and the singular weight $m(x)+c\delta'(x)$, where δ is Dirac's delta function. It is known that (1) can have at most a finite number of eigenvalues convergent to $-\infty$ as $\varepsilon\to 0$; the rest have finite limits and present a finite part of the spectrum.

The δ'-like potential is sensitive to its regularization way. Therefore, the spectral properties of (1) significantly depend on the "profile" $V(\varepsilon^{-1}x)$ of the δ'-like potential's regularization [1]. We focused on the more complicated case of the so-called resonant potential V, when equation $-\frac{d^2v}{d\xi^2}+V(\xi)=0$ has non-trivial solutions bounded on $\mathbb{R}$. For this case, a limiting problem and problems for the main terms of the asymptotics in the small zone $|x|<\varepsilon$ are established as $\varepsilon\to 0$. The spectral properties of the limiting problem are studied. The convergence of the eigenvalues with fixed numbers belonging to the finite part of the problem (1) spectrum to the corresponding eigenvalues of the limiting problem with the accuracy $c\varepsilon$ is proved.

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COEFFICIENT INVERSE PROBLEMS FOR DEGENERATE PARABOLIC EQUATION

In a domain $Q_T = \{(x,t) : 0 < x < h, 0 < t < T\}$ we consider an inverse problems of identification of the time-dependent functions $b_1 = b_1(t), \quad b_2 = b_2(t)$ in a coefficient at the minor derivative in the degenerate parabolic equation

$$u_t = a(t)t^{\beta}u_{xx} + (b_1(t)x + b_2(t))u_x + c(x,t)u + f(x,t) \tag{1}$$

with initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

boundary conditions

$$u(0,t) = \mu_1(t),\ u(h,t) = \mu_2(t),\ t \in [0,T] \tag{3}$$

and integral overdetermination conditions

$$\int_0^h u(x,t)dx = \mu_3(t), \quad t \in [0,T], \tag{4}$$

$$\int_0^h xu(x,t)dx = \mu_4(t), \quad t \in [0,T]. \tag{5}$$

It is known that $a(t) > 0$, $t \in [0,T]$ and a degeneration of the equation (1) is caused by the power function t^{β}. The conditions of existence and uniqueness of the classical solution to the named problems are established for both cases of weak ($0 < \beta < 1$) and strong ($\beta \geq 1$) degenerations. For this aim we use the Schauder fixed point theorem and the properties of the solutions to the homogeneous integral equations with integrable kernels.

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SOLVABILITY OF BOUNDARY VALUE TWOPOINT PROBLEM FOR A DIFFERENTIAL EQUATION WITH PARAMETER IN A REFINED SOBOLEV SCALE

In the cylinder $G = [0,T] \times \Omega$, where $\Omega = \mathbb{R}/(2\pi\mathbb{Z})$, we investigate a problem with nonlocal conditions for a homogeneous partial differential equation of order n

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u(t,x) \equiv \sum_{s_0+s_1\le n} a_{s_0,s_1} \frac{\partial^{s_0+s_1} u(t,x)}{\partial t^{s_0} \partial x^{s_1}} = 0, \qquad a_{n,0} = 1,$$

$$L_m u = \mu \frac{\partial^{m-1} u(t,x)}{\partial t^{m-1}}\Big|_{t=0} - \frac{\partial^{m-1} u(t,x)}{\partial t^{m-1}}\Big|_{t=T} = \varphi_m(x), \quad m = 1,\dots,n,$$

where $n \in \mathbb{N}$, $\mu \in \mathbb{C}\setminus\{0\}$ is a parameter in the nonlocal conditions, a_{s_0,s_1} are complex coefficients of the equation, $\varphi_1 = \varphi_1(x),\dots,\varphi_n = \varphi_n(x)$ are given functions and $u = u(t,x)$ is the unknown solution.

A distinctive feature of this paper is the investigation of conditions for the unique solvability of a nonlocal boundary value problem for the partial differential equation with constant coefficients in a scale of Hormander spaces, which form a refined Sobolev scale. In Sobolev spaces, the smoothness of functions is characterized by a single numerical parameter, whereas in the refined Sobolev scale it is determined by two parameters, a numerical one and a functional one (a slowly varying function at infinity). The paper establishes necessary and sufficient conditions for the uniqueness of the solution to the problem under consideration within the refined scale, as well as sufficient conditions for the existence of a solution in this scale. The Hadamard well-posedness of the problem is established, which distinguishes it from the Hadamard ill-posed multidimensional problem whose solvability is associated with the small divisors issue. It is proved that the small divisors problem does not arise, since the corresponding expressions are bounded below by positive constants.

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INTEGRAL PROBLEM FOR A SYSTEM OF PARTIAL DIFFERENTIAL EQUATION IN AN UNBOUNDED LAYER

We consider the next problem with integral conditions for a system of partial differential equations:

$$L\left(\frac{\partial}{\partial t},\frac{\partial}{\partial x}\right)\vec{u}(t,x)\equiv\frac{\partial^n\vec{u}}{\partial t^n}+\sum_{j=0}^{n-1}A_j\frac{\partial^n\vec{u}}{\partial t^j\partial x^{n-j}}=\vec{0},\ t\in(0;T),\ x\in\mathbb{R}, \quad (1)$$

$$\int_0^T t^{j-1}\vec{u}(t,x)dt=\vec{\varphi}_j(x),\quad j=1,\ldots,n,\quad x\in\mathbb{R}, \quad (2)$$

where $A_j=\|a_{q,r}^j\|_{q,r=1}^m$, $j=0,1,\ldots,n-1$, are $m\times m$ matrices with complex entries, $\vec{u}(t,x)=\mathrm{col}(u^1(t,x),\ldots,u^m(t,x))$, $\vec{0}=\mathrm{col}\,(\underbrace{0,\ldots,0}_{m})$, $\vec{\varphi}_j(x)=\mathrm{col}(\varphi_j^1(x),\ldots,\varphi_j^m(x))$, $j=1,\ldots,n$.

The conditions for the well-posedness of problem (1), (2) in the corresponding functional spaces are obtained, provided that the μ-roots of equation $\det\|L(\mu,i)\|=0$ have distinct nonzero real parts. To establish this result, we used the methods developed in [1].

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PERIODIC PROBLEM FOR THE CAMASSA-HOLM EQUATION

This work addresses the development of the Riemann-Hilbert problem (RHP) formalism for the Camassa-Holm equation under periodic boundary conditions. Particularly, we present a representation of the solution to this problem in terms of the solution of the associated Riemann-Hilbert problem, the data for which are determined by the intial data for the problem in terms of the associated spectral functions.

Recently [1, 2], it was shown that the initial boundary value problem on a finite interval with x-periodic boundary conditions for the nonlinear Schrödinger (NLS) equation $iq_t+q_{xx}+2|q|^2q=0$ belong to the class of so-called linearizable problems: the solution $q(x,t)$ can be expressed in terms of the solution of a RH problem, the data for which – the jump matrix across a certain contour and the residue conditions – can be expressed in terms of the entries of the scattering matrix associated with the initial data.

In our talk, we develop this formalism for the Camassa-Holm equation

$$\left(\sqrt{m+1}\right)_t = -\left(u\sqrt{m+1}\right)_x, \quad m = u - u_{xx}$$

which models the unidirectional propagation of shallow water waves over a flat bottom.

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TWO-TERM ASYMPTOTICS OF THE LOGARITHMIC DERIVATIVE OF ENTIRE FUNCTIONS WITH IMPROVED DISTRIBUTION OF ZEROS

Let f be an entire function of finite order, $f(0) = 1$, $(\lambda_n)_{n\in\mathbb{N}}$ be a sequence of its positive zeros and $n(t) := \sum_{\lambda_n\leq t} 1$ be the counting function of the sequence (λ_n).

We investigate the connection between the regularity of the growth of the logarithmic derivative f'/f of the function f and the improved distribution of its zeros on a positive ray in terms of the two-term asymptotics [1–3].

Theorem 1. *Let* $\Delta \in (0; +\infty)$, $\Delta_1 \in \mathbb{R}$, $\rho \in (0; +\infty) \setminus \mathbb{N}$, $[\rho] < \rho_2 < \rho_1 < \rho < [\rho] + 1$ *and*

$$n(t) = \Delta t^{\rho} + \Delta_1 t^{\rho_1} + o(t^{\rho_2}), \quad t \to +\infty.$$

Then for an entire function f *of order* ρ, *one has*

$$\frac{f'(re^{i\varphi})}{f(re^{i\varphi})} = \frac{\pi\Delta\rho r^{\rho-1}}{\sin\pi\rho} e^{-i\pi\rho} e^{i(\rho-1)\varphi} + \frac{\pi\Delta_1\rho_1 r^{\rho_1-1}}{\sin\pi\rho_1} e^{-i\pi\rho_1} e^{i(\rho_1-1)\varphi} + \frac{o(r^{\rho_2-1})}{\sin(\varphi/2)}$$

as $r \to +\infty$ *uniformly with respect to* $\varphi \in (0; 2\pi)$.

An analogue of Theorem 1 is also true for $\rho \in \mathbb{N}$.

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COMPLETENESS OF THE SYSTEMS OF BESSEL FUNCTIONS OF INDEX $-m-1/2$ WITH $m \in \mathbb{N}$

Let $m \in \mathbb{N}$ and $L^2((0;1);x^{2m}dx)$ be the weighted Lebesgue space of all measurable functions $f : (0;1) \to \mathbb{C}$, satisfying $\int_0^1 x^{2m}|f(x)|^2\,dx < +\infty$, let $J_{-m-1/2}$ be the Bessel function of the first kind of index $-m-1/2$ (see [1]) and $(\rho_k)_{k\in\mathbb{N}}$ be a sequence of distinct nonzero complex numbers.

We establish necessary and sufficient conditions for the completeness of the system $\left\{\rho_k^m\sqrt{x\rho_k}J_{-m-1/2}(x\rho_k) : k \in \mathbb{N}\right\}$ in $L^2((0;1);x^{2m}dx)$ in terms of sequences of zeros of functions from certain classes of even entire functions of exponential type $\sigma \le 1$ (see [2–4]). We also obtain an analog of the Paley-Wiener theorem related to this system (see [2, 5–7]).

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GLOBAL BOUNDED SOLUTIONS TO QUASILINEAR FIRST ORDER HYPERBOLIC SYSTEMS

In the strip $\Pi = \{(x,t) \in \mathbb{R}^2 : 0 \le x \le 1\}$ we consider first order quasilinear hyperbolic systems of the type

$$\partial_t u + A(x,t,u)\partial_x u = F(x,t,u), \quad x \in (0,1) \tag{1}$$

where $u = (u_1, \dots, u_n)$, $F = (F_1, \dots, F_n)$, $A = diag(A_1, \dots, A_n)$, and $n \ge 2$. Suppose that the matrix A fulfills the condition

$$A_1(x,t,u) > \dots > A_m(x,t,u) > 0 > A_{m+1}(x,t,u) > \dots > A_n(x,t,u)$$

for all $(x,t) \in \Pi$, an integer m in the range $0 \le m \le n$, and small u. The system (1) is supplemented with the boundary conditions

$$\begin{aligned} u_j(0,t) &= R_j u(\cdot,t) + H_j(t, Q_j(t)u(\cdot,t)), \quad 1 \le j \le m, \\ u_j(1,t) &= R_j u(\cdot,t) + H_j(t, Q_j(t)u(\cdot,t)), \quad m < j \le n, \end{aligned} \tag{2}$$

where R_j are linear bounded operators from $C([0,1],\mathbb{R}^n)$ to $\mathbb{R}$ defined by

$$R_j v = \sum_{k=m+1}^{n} r_{jk} v_k(0) + \sum_{k=1}^{m} r_{jk} v_k(1), \quad j \le n,$$

r_{jk} are real constants, and $H_j(t,v)$ are real valued functions from $\mathbb{R} \times \mathbb{R}$ to $\mathbb{R}$. Moreover, for any $t \in \mathbb{R}$, $Q_j(t)$ are linear bounded operators from $C([0,1];\mathbb{R}^n)$ to $\mathbb{R}$. Our main result consists in establishing conditions on A, F, R, and H_j ensuring that the problem (1)–(2) has a unique small global (in particular, periodic and almost periodic) classical C^2-solution, for any sufficiently small $F(x,t,0)$ and $H_j(t,0)$.

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SOME EXPLICIT FORMS OF ALTERNATING q-MULTIPLE t-VALUES

Recently, various alternating multiple zeta values have been proposed and studied as variations of ordinary classical multiple zeta values and their general or modified forms. We consider alternating multiple zeta values as an application of alternating Stirling numbers based on our study of Stirling numbers. In this paper, we give the values of a certain kind of alternative q-multiple t-function at roots of unity.

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WARING-GIRARD FORMULAS FOR BLOCK-SYMMETRIC POLYNOMIALS

We denote by $\ell_p(\mathbb{C}^s) = \ell_p(\mathbb{N}, \mathbb{C}^s)$, $1 \le p < \infty$ the linear space of all sequences $x = (x_1, x_2, \ldots, x_j, \ldots)$, such that $x_j = (x_j^{(1)}, \ldots, x_j^{(s)}) \in \mathbb{C}^s$ for $j \in \mathbb{N}$, and the series $\sum\limits_{j=1}^{\infty} \sum\limits_{r=1}^{s} \left| x_j^{(r)} \right|^p$ converges. The linear space $\ell_p(\mathbb{C}^s)$ endowed with the norm $\|x\| = \left(\sum\limits_{j=1}^{\infty} \sum\limits_{r=1}^{s} \left| x_j^{(r)} \right|^p \right)^{1/p}$ is a Banach space. A polynomial P on the space $\ell_p(\mathbb{C}^s)$ is called *block-symmetric (or vector-symmetric)* if: $P(x_1, x_2, \ldots, x_m, \ldots) = P(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(m)}, \ldots)$ for every permutation $\sigma \in S_{\mathbb{N}}$ and $x_m \in \mathbb{C}^s$. We denote by $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$ the algebra of all block-symmetric polynomials on $\ell_p(\mathbb{C}^s)$.

Polynomials

$$H^{\mathbf{k}}(x) = H^{k_1, k_2, \ldots, k_s}(x) = \sum_{j=1}^{\infty} \prod_{\substack{r=1 \\ |\mathbf{k}| \ge \lceil p \rceil}}^{s} (x_j^{(r)})^{k_r}$$

form an algebraic basis in $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$, $1 \le p < \infty$, where

$$x = (x_1, \ldots, x_m, \ldots) \in \ell_p(\mathbb{C}^s), \quad x_j = (x_j^{(1)}, \ldots, x_j^{(s)}) \in \mathbb{C}^s$$

and $\lceil p \rceil$ is a ceiling of p. In the case of the space $\ell_1(\mathbb{C}^s)$ there exist another important algebraic basis:

$$R^{\mathbf{k}}(x) = R^{k_1, k_2, \ldots, k_s}(x) = \sum_{\substack{i_1^j < \cdots < i_{k_j}^j, \\ 1 \le j \le s}}^{\infty} \prod_{j=1}^{s} x_{i_1^j}^{(j)} \ldots x_{i_{k_j}^j}^{(j)}.$$

The talk will present analogues of the Waring–Girard formulas for block-symmetric polynomials on the space $\ell_p(\mathbb{C}^s), 1 \le p < \infty$, and their potential applications in combinatorics.

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A SIMPLE ALGORITHM FOR EXPANDING A FORMAL TRIPLE POWER SERIES INTO THE THREE-DIMENSIONAL CORRESPONDING ASSOCIATED CONTINUED FRACTION

The formal triple power series

$$P(z) = \sum_{|k|=0}^{\infty} c_k z^k, \quad z = (z_1, z_2, z_3) \in \mathbb{C}^3, \quad k = (k_1, k_2, k_3) \in \mathbb{Z}^3_{\geq 0},$$

$$|k| = k_1 + k_2 + k_3, \quad z^k = z_1^{k_1} z_2^{k_2} z_3^{k_3},$$

can be represented as the sum:

$$c_0 + \sum_{k_1=1}^{\infty} c_{k_1,0,0} z_1^{k_1} + \sum_{k_2=1}^{\infty} c_{0,k_2,0} z_2^{k_2} + \sum_{k_3=1}^{\infty} c_{0,0,k_3} z_3^{k_3} + z_1 z_2 \times$$
$$\times \sum_{k_1+k_2 \geq 0} c_{k_1+1,k_2+1,0} z_1^{k_1} z_2^{k_2} + z_1 z_3 \sum_{k_1+k_3 \geq 0} c_{k_1+1,0,k_3+1} z_1^{k_1} z_3^{k_3} + z_2 z_3 \times$$
$$\times \sum_{k_2+k_3 \geq 0} c_{0,k_2+1,k_3+1} z_2^{k_2} z_3^{k_3} + z_1 z_2 z_3 \sum_{|k| \geq 0} c_{k_1+1,k_2+1,k_3+1} z_1^{k_1} z_2^{k_2} z_3^{k_3}. \quad (1)$$

The first six series in (1) one can expand into A-fractions and two-dimensional corresponding associated continued fractions (A_2) [1], [2] and for the seventh term in (1) we use the representation for

$$\left(\sum_{k_1+k_2+k_3 \geq 0} c_{k_1+1,k_2+1,k_3+1} \, z_1^{k_1} z_2^{k_2} z_3^{k_3} \right)^{-1}.$$

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PROBLEM WITH INTEGRAL CONDITION FOR NONHOMOGENEOUS PARTIAL DIFFERENTIAL EQUATIONS OF THIRD ORDER

Let $K_{\mathbb{C},M}$, $M \subseteq \mathbb{C}$, be a class of quasipolynomials of the form

$$f(t,x) = \sum_{j=1}^{m} P_j(t,x) \exp[\alpha_j x + \beta_j t], \quad \alpha_j \in M, \quad \beta_j \in \mathbb{C}, \quad (t,x) \in \mathbb{R}^2,$$

where $\alpha_j \neq \alpha_k$, $\beta_j \neq \beta_k$, for $k \neq j$, $x \in \mathbb{R}$, $P_j(t,x)$, $j = 1, \ldots, n$, are polynomials with complex coeffients. Each quasipolynomial $f(t,x)$ defines a pseudodifferential operator $f\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right)$, whose action on a function $\Phi(\nu, \lambda)$ is defined by the equality

$$f\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right) \Phi(\nu, \lambda) = \sum_{j=1}^{m} P_j\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right) \Phi(\lambda, \nu)\Big|_{\nu=0, \lambda=\alpha_j}.$$

In the strip $\Omega = \{(t,x) \in \mathbb{R}^2 : t \in (T_1, T_2) \cup (T_3, T_4), x \in \mathbb{R}\}$ we consider the problem with integral conditions in the form

$$\frac{\partial^3 U(t,x)}{\partial t^3} + \sum_{i=1}^{3} a_i\left(\frac{\partial}{\partial x}\right) \frac{\partial^{3-i} U(t,x)}{\partial t^{3-i}} = f(t,x), \tag{1}$$

$$\int_{T_1}^{T_2} t^k U(t,x) dt + \int_{T_3}^{T_4} t^k U(t,x) dt = 0, \quad k = 0, 1, 2, \tag{2}$$

where $f(t,x)$ is a given function, $a_i\left(\frac{\partial}{\partial x}\right)$ is a differential expresion with entire symbol $a_i(\lambda)$. Using the differential-symbol method [1], we construct a solution of the problem (1)–(2).

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METHODOLOGY FOR DETERMINING THE QUASI-STATIC THERMOELASTIC STATE OF A MULTILAYER PLATE UNDER NONLINEAR THERMAL CONDITIONS

A multilayer non-uniformly heated plate, free from mechanical loads and aligned with a cylindrical coordinate system, is considered. The cylindrical surface of the plate is thermally insulated. The plate is heated by a heat source whose intensity depends on the axial coordinate and time. Convective heat exchange occurs through the flat boundary surfaces with surrounding media whose temperatures are time-dependent, and thermal radiation is emitted according to the Stefan-Boltzmann law. Volumetric heat capacities and thermal conductivities are assumed to be constant, while the heat transfer coefficients and emissivities are temperature-dependent.

The determination of the one-dimensional unsteady temperature field involves an integral representation of the heat conduction problem solution using the corresponding Green's function for a multilayer plate, the use of generalized functions, linear splines, and exact series expansions in terms of eigenfunctions. The values of the integral dependencies on the temperatures of the boundary surfaces are obtained from recurrent linear relations. In the formulas for stresses, strains, and displacements, terms describing the thermoelastic state under linear thermal boundary conditions are isolated and expressed as a superposition of the integral dependencies on the specified thermal factors.

For a five-layer plate, comparisons are made with temperature values calculated based on a methodology that requires prior solution of a recurrent system of nonlinear algebraic equations, and for large time values, with the corresponding temperatures calculated from the solution of a steady-state nonlinear heat conduction problem. Numerical studies confirm the stability of the unsteady heat conduction problem solution with respect to changes in the spline time grid step.

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FACTORIZED EVOLUTION EQUATIONS WITH EXPONENTIAL INTEGRAL CONSTRAINTS

In the domain $D^p_{t_0} = \left\{(t,x) \in \mathbb{R}^{p+1} : t \in (t_0, T), x \in \mathbb{R}^p\right\}$ we consider the following problem

$$\prod_{j=1}^{n} \left(\frac{\partial}{\partial t} + A_j \left(\frac{\partial}{\partial x} \right) \right) u(t,x) = 0, \quad (t,x) \in D^p_{t_0}, \tag{1}$$

$$\int_{t_0}^{T} e^{\alpha_j t} u(t,x) dt = \varphi_j(x), \quad j = 1, \ldots, n, \quad x \in \mathbb{R}^p, \tag{2}$$

where α_j, $j = 1, \ldots, n$, are pairwise different nonzero complex numbers; $A_j(\eta)$, $j = 1, \ldots, n$, $\eta \in \mathbb{R}^p$, are polynomials of the form $A_j(\eta) = \sum_{|s|=0}^{N_j} a_{j,s}\eta^s$, $a_{j,s} \in \mathbb{C}$, $N_j \in \mathbb{N}$. We assume that the operators $A_j(\partial/\partial x)$ are elliptic. The functions $\varphi_j(x)$ are considered as almost periodic with respect to x with given spectrum

$$\mathcal{M} := \{\mu_k \in \mathbb{R}^p : d_1|k|^{\sigma_1} \leqslant |\mu_k| \leqslant d_2|k|^{\sigma_2}, k \in \mathbb{Z}^p, d_1, d_2, \sigma_1, \sigma_2 > 0\}.$$

The aim of this work is to establish sufficient conditions for the existence and uniqueness of a solution to problem (1), (2) in the corresponding Sobolev-type spaces of almost periodic functions in x.

In general, such problems are ill-posed, and their correct solvability is related to the small denominators problem [1, 2] to solve which we use the metric approach.

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SIMULATING TURBULENCE WITH SYNTHETIC NOISE

Procedural noise functions are well known in computer graphics where they play a key role in the generation of realistic structures such as clouds, waves, terrain, and other natural textures. Thy key features of a synthetic noise are its unstructured pattern and limited frequency bandwidth, so combining multiple noise layers with different amplitudes yields random structures with controlled spectral properties. It means that one can efficiently generate turbulent fields that statistically reproducemain properties of turbulence using a noise function.

The developed approach captures the multi-scale nature of the turbulent cascade while maintaining high computational efficiency. By appropriately scaling noise layers with frequency-dependent amplitudes, the resulting synthetic structures demonstrate statistical properties consistent with real turbulent data following Kolmogorov's -5/3 law.

This approach can be applied to turbulent processes in astrophysical plasmas where turbulence plays a key role in energy dissipation, particle acceleration, and magnetic field amplification across various cosmic environments.

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QUATERNION GENERALIZED INVERSE MATRICES

In 1920, Moore described the reciprocal of the general matrix $\mathbf{A} \in \mathbb{C}^{m\times n}$. Later, in 1955, Penrose uniquely characterized this matrix, now known as the *Moore–Penrose inverse* ($\mathbf{A}^{\dagger} \in \mathbb{C}^{n\times m}$), as the solution to the following four equations:

$$(1)\ \mathbf{AXA} = \mathbf{A},\ (2)\ \mathbf{XAX} = \mathbf{X},\ (3)\ (\mathbf{AX})^{*} = \mathbf{AX},\ (4)\ (\mathbf{XA})^{*} = \mathbf{XA}$$

Penrose's characterization was based on the singular value decomposition (SVD). A matrix $\mathbf{X}$ satisfying only condition (2) is called a *generalized inverse* of $\mathbf{A}$ and is denoted by $\mathbf{A}^{(2)}$. Other notable generalized inverses include the *Drazin (D-) inverse* and the *weighted MP-* and *D- inverses.* The subsequent definitions and development of the *core inverse* (2010), the *core-EP (CEP-) inverse* (2014), along with their weighted counterparts, sparked a surge in research on generalized inverses. This led to the creation of new compositions, such as the MPCEP and MPD inverses, etc., which combine the properties of the MP-, D-, and CEP-inverses, as well as their weighted counterparts. Using the SVD of matrices is often a unique method to compute generalized inverses, while the familiar determinantal ($\mathcal{D}$-)representation of the ordinary inverse uses a matrix of cofactors. Based on the theory of row-column noncommutative determinants [1], we provide $\mathcal{D}$-representations of various quaternion generalized inverses and examine their properties, as well as their applications in solving quaternion matrix equations [2]. Note that the novelty of this research applies to complex matrices via the use of ordinary determinants.

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ERROR ESTIMATES FOR TRUNCATION LEGENDRE METHOD IN NUMERICAL DIFFERENTIATION

Numerical differentiation is a fundamental ill-posed problem in computational mathematics, particularly challenging with noisy data. We investigate (see also [1]) the truncation Legendre method for recovering derivatives of arbitrary order from functions in weighted Wiener classes $W_s^\mu = \{f \in L_2 : \|f\|_{s,\mu}^s := \sum_{k=0}^{\infty}(\max\{1,k\})^{s\mu}|\langle f, \varphi_k\rangle|^s < \infty\}$, where $\mu > 0, 1 \le s < \infty$.

Let, instead of the exact function $f \in W_s^\mu$, its perturbation $f^\delta \in L_2$ be known. Then we will use the truncation Legendre method $\mathcal{D}_N^{(r)} f^\delta(t) = \sum_{k=r}^{N}\langle f^\delta, \varphi_k\rangle \varphi_k^{(r)}(t)$.

We establish error estimates across different metrics, extending previous results [2] to arbitrary derivative orders and various function spaces.

Theorem. *Let $f \in W_s^\mu$ with $1 \le s < \infty$, $\mu > 2r - 1/s + 3/2$, and assume the perturbation condition $\|f - f^\delta\|_{\ell_p} \le \delta$ for $1 \le p \le \infty$. Then for $N \asymp \delta^{-\frac{1}{\mu - 1/p + 1/s}}$, the truncation method achieves the error bound $\|f^{(r)} - \mathcal{D}_N^{(r)} f^\delta\|_C \le c\delta^\eta$, $\eta = \frac{\mu - 2r + 1/s - 3/2}{\mu - 1/p + 1/s}$.*

Similar results hold for L_q, $2 \le q \le \infty$ metrics. The convergence rate depends on the Wiener class parameters μ and s, derivative order r, metrics of input and output spaces, and noise characteristics.

The parameter $N \asymp \delta^{-\frac{1}{\mu - 1/p + 1/s}}$ balances approximation error against noise amplification. These results provide both theoretical insights into numerical differentiation limitations and practical guidelines for robust algorithm development.

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A SYLVESTER-TYPE MATRIX EQUATION OVER SOME QUADRATIC RINGS

Let $\mathbb{K} = \mathbb{Z}[\sqrt{k}]$ be a quadratic ring. We establish the conditions for the solvability of the matrix equation

$$AX + YA^{\nabla} = C, \tag{1}$$

where A, C are $n \times n$ given matrices and X, Y are $n \times n$ unknown matrices over the ring $\mathbb{K}$, and ∇ is a non-trivial involution in $\mathbb{K}$.

Theorem. *If* $k \equiv 2, 3 \pmod 4$*, and*

$$A = A_1 + A_2\sqrt{k}, \quad A^{\nabla} = A_1^T - A_2^T\sqrt{k}, \quad C = C_1 + C_2\sqrt{k},$$

where A_i, C_i *are* $n \times n$ *matrices over a ring* $\mathbb{Z}$*, and* A_i^T *denotes the transpose of a matrix* A_i*,* $i = 1, 2$*, then equation* (1) *is solvable if and only if the following matrices*

$$\left\| \begin{matrix} A_1 \otimes I_n & A_2 k \otimes I_n & I_n \otimes A_1 & -I_n \otimes A_2 k & \mathbf{c}_1 \\ A_2 \otimes I_n & A_1 \otimes I_n & -I_n \otimes A_2 & I_n \otimes A_1 & \mathbf{c}_2 \end{matrix} \right\|,$$

$$\left\| \begin{matrix} A_1 \otimes I_n & A_2 k \otimes I_n & I_n \otimes A_1 & -I_n \otimes A_2 k & 0 \\ A_2 \otimes I_n & A_1 \otimes I_n & -I_n \otimes A_2 & I_n \otimes A_1 & 0 \end{matrix} \right\|,$$

are equivalent over $\mathbb{Z}$*, where* $\mathbf{c}_i = \left\| r_1(C_i) \ \cdots \ r_n(C_i) \right\|^T, i = 1, 2,$ $r_i(C)$ *denotes the i-th row of the matrix* C *and* I_n *is the* $n \times n$ *identity matrix.*

The result of the theorem holds in the case when $k \equiv 1 \pmod 4$ and

$$A = \frac{1}{2}(A_1 + A_2\sqrt{k}), \quad A^{\nabla} = \frac{1}{2}(A_1^T - A_2^T\sqrt{k}), \quad C = \frac{1}{2}(C_1 + C_2\sqrt{k}).$$

The criterion for the existence and uniqueness of solutions over the ring $\mathbb{Z}$ to such equations is given in [1].

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ON THE INTEGRAL EQUATIONS APPROACH FOR THE NUMERICAL SOLUTION OF THE 3D PARABOLIC LATERAL CAUCHY PROBLEM

We consider the numerical solution of the Cauchy problem for the heat equation in bounded doubly connected 3D domains with smooth surfaces. This problem, known to be ill-posed, arises in various applications such as nondestructive testing and thermal tomography.

For the time variable semi-discretization for the nonstationary problems Laplace transform is used. Applying this approach, initial boundary value problem can be reduced to the set of boundary value problems for the Helmholtz equation. An approximate solution of the original problem can be obtained by approximately integrating the inverse Laplace transform, in particular using sinc quadratures with a certain choice of integration contour [2].

Boundary value problems are numerically solved using a boundary integral equation method [1, 2]. The solution is represented via the combination of single-layer potentials, and the resulting integral equations are discretized using the Nyström method based on quadrature rules for surface integrals.

Due to the ill-posedness of the original problem, the resulting linear systems are ill-conditioned. To address this, we employ Tikhonov regularization. The regularization parameter is selected using the L-curves method.

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ON THE SOLUTION OF THE SYSTEM OF PDE RELATED TO A PIEZOELECTRIC BIMATERIAL WITH AN INTERFACE ELECTRODE

Consider the system of partial differential equations

$$(C_{ijkl}u_k + e_{lij}\phi)_{,li} = 0, \quad (e_{ikl}u_k - \chi_{il}\phi)_{,li} = 0, \tag{1}$$

where $i, j, k, l = 1, 3$; comma means derivatives with respect to the corresponding coordinate variables; u_1, u_3, ϕ are unknown functions independent of coordinate x_2; $C_{ijkl}, e_{lij}, \varepsilon_{il}$ are known coefficients with certain values for each subdomain and summation on repeated indexes is implied.

According to the Stroh's formalism, we find the solution of the system (1) in the form $\mathbf{V} = \mathbf{a}f(z)$, where $z = x_1 + px_3$, $\mathbf{V} = [u_1, u_3, \phi]^T$, and f is arbitrary function. Substituting $\mathbf{V}$ into (1) we arrive to system for the determination of eigenvalues p and eigenvectors $\mathbf{a} = [a_1, a_2, a_3]^T$. Its solution permits to present the general solution of (1) in the form $\mathbf{V} = \mathbf{A}\mathbf{f}(z) + \bar{\mathbf{A}}\bar{\mathbf{f}}(\bar{z})$, where $\mathbf{A}$ is (3×3) matrix composed of eigenvectors; $\mathbf{f}(z) = [f_1(z_1), f_3(z_3), f_4(z_4)]^T$, $z_s = x_1 + p_s x_3$ and overbar means complex conjugation. Considering that (1) is the electrostatic system for piezoelectric material the generalized stress vector $\mathbf{t} = [\sigma_{13}, \sigma_{33}, D_3]^T$ can be presented in the form $\mathbf{t} = \mathbf{B}\mathbf{f}'(z) + \bar{\mathbf{B}}\bar{\mathbf{f}}'(\bar{z})$. In case of bimaterial domains $x_3 > 0$ and $x_3 < 0$ the obtained presentations are usually used for analysis of cracks at the interfaces. Here we apply them for the consideration of rigid or soft interface electrodes. For this purpose, the formulas for $\mathbf{V}$ for lower and upper materials are equated on the interface $x_3 = 0$. The following transformation of obtained equality permit to get the relation: $\mathbf{t}^+(x_1, 0) - \mathbf{t}^-(x_1, 0) = \mathbf{W}^+(x_1) - \mathbf{W}^-(x_1)$, $\quad \mathbf{V'}^{(1)}(x_1, 0) = \mathbf{G}\mathbf{W}^+(x_1) - \bar{\mathbf{G}}\mathbf{W}^-(x_1)$, with known matrix $\mathbf{G}$ and vector-function $\mathbf{W}(z)$ analytic in each subdomain. The application of the obtained relations was used for the analysis of piezoelectric bimaterials with one or several interface electrodes. Besides, the similar analysis was applied for the case of electrically insulated interface inclusions. For this purpose it is only necessary to rearrange the components of the matrixes $\mathbf{A}$ and $\mathbf{B}$.

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SYMMETRIES AND TRANSFORMATION PROPERTIES OF LINEAR SYSTEMS OF SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS

We review our recent results [1] on the transformation properties of normal linear systems of second-order ordinary differential equations with an arbitrary number of dependent variables under several appropriate gauges of the arbitrary elements parameterizing these systems. We also present principal properties of the Lie symmetries of the systems under consideration, outline ways for completely classifying these symmetries, and demonstrate how equivalence transformations and Lie symmetries can be used for reduction of order of such systems and their integration. Additionally, we discuss some open problems and possible directions for future research.

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WEYL OPERATOR CALCULUS FOR EXPONENTIAL TYPE DISTRIBUTIONS

Weyl operator calculus plays an important role in the study of quantum mechanics and the theory of distributions [1], and we consider it for exponential type distributions $E'(\mathbb{R}^2)$.

Let A and B be closed and densely defined operators acting in a complex Banach space X. We assume that each of the operators iA and iB generates a uniformly bounded C_0- group on X.

The operator $e^{\mathrm{i}(uA+vB)} \in L(X)$ generated by the Weyl pair A, B in a Banach space X [2] is defined by $e^{\mathrm{i}(uA+vB)}\varphi(t) = e^{\mathrm{i}uv/2}e^{\mathrm{i}uA}e^{\mathrm{i}vB}\varphi(t) = e^{\mathrm{i}(uv+vt)/2}\varphi(t+v)$ for all $\varphi \in L_1(\mathbb{R}^2)$. For $\varphi \in E(\mathbb{R}^2)$, $\varphi(A,B) := \frac{1}{2\pi}\iint \widehat{\varphi}(u,v)e^{\mathrm{i}(uA+vB)}dudv$, where $\widehat{\varphi}(u,v) = \frac{1}{2\pi}\iint \varphi(x,y)e^{\mathrm{i}(xu+yv)}dxdy$. We get $\varphi(A,B)\circ\psi(A,B) = (\varphi\#\psi)(A,B)$, where the Moyal product is defined as $(\varphi\#\psi)(x,y) = \frac{1}{\pi^2}\iint\iint \varphi(x+u, y+u')\psi(x+v, y+v')e^{-2\mathrm{i}(vu'-uv')}$ $dudu'dvdv'$.

Define by $\langle \widehat{f} \mid \widehat{\varphi}\rangle := \frac{1}{2\pi}\langle f \mid \varphi\rangle$ with $f \in E'(\mathbb{R}^2)$ the duality $\langle \widehat{E}'(\mathbb{R}^2) \mid \widehat{E}(\mathbb{R}^2)\rangle$.

Define $(f\star x)(u,v) := (I\otimes K_f)x(u,v)$, $\forall x \in E(\mathbb{R}^2; X)$ and the isometry $E(\mathbb{R}^2; X) \ni x(u,v) \mapsto \widehat{x} \in \widehat{E}(X) := \left\{\widehat{x} = \iint (e^{\mathrm{i}(uA+vB)} \otimes I)x(u,v)dudv\right\}$.

Theorem. $\widehat{f}(A,B)\colon \widehat{E}(X) \ni \widehat{x} \longmapsto \widehat{f}(A,B)\widehat{x} := \iint (e^{\mathrm{i}(uA+vB)} \otimes K_f)x(u,v)dudv$ is continuous homomorphism in $L(\widehat{E}(X))$ with $\widehat{f}(A,B)\circ \widehat{g}(A,B) = \widehat{(f\#g)}(A,B)$, where $\widehat{(f\#g)}(A,B)$ is defined by the duality $\langle \widehat{E}'(\mathbb{R}^2) \mid \widehat{E}(\mathbb{R}^2)\rangle$.

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STRESS FIELD SINGULARITY IN AN ANISOTROPIC COMPOSITE WEDGE SUBJECTED TO ANTIPLANE DEFORMATION

The problem of longitudinal shear in a composite anisotropic wedge composed of an arbitrary number of dissimilar elements with cylindrical anisotropy is considered [1]. To solve the problem, the method of the generalized conjugation problem [2] was employed, which made it possible to reduce the determination of the stress-strain state to a partially degenerate differential equation with boundary conditions of the first, second, or mixed type.

Using the Mellin transform, closed-form expressions were obtained to describe the stress-strain state in multi-wedge structures. For the case of a two-component wedge, characteristic equations were derived that allow the determination of the order of the stress field singularity near the tip under various boundary conditions (conditions of the first, second, and mixed elasticity problems).

The proposed method was extended to three-component systems. A study was conducted on the influence of geometric parameters and elastic properties of materials on the order of stress singularity at the tip of a two-component wedge. The obtained results enhance the understanding of the local behavior of composite structures and can be used in the assessment of their strength.

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INTERACTION OF A PERIODIC SYSTEM OF THIN NON-CONTRAST INCLUSIONS WITH A HALF-SPACE, THE MOTION OF WHICH IS DESCRIBED BY THE HELMHOLTZ EQUATION

Thin elastic inhomogeneities are widely used as fillers in composites, in particular when they are arranged periodically. In addition to solving traditional problems of assessing the strength and deformability of such structures, research aimed at studying the spectral characteristics of the fields scattered in them is important. For this purpose, the problem of plane wave propagation in an elastic half-space with a single-period array of deep collinear piezoelectric thin inclusions of constant thickness is considered. The motion of the half-space is described by the Helmholtz equation. All inclusions, the thickness of which is characterized by a small parameter ε, have the same geometric dimensions and electro-elastic properties. Using the singular perturbation method [1], the interaction of the half-space with such a set of scatterers is modeled by the jumps of displacements and stresses on the centerline of the inclusions. These jumps, proportional to a small parameter, are expressed in terms of a given incident wave and have the same form as in the case of a single inclusion [1], i.e. the interaction of scatterers in the resulting model is not taken into account. The solution to the problem of wave propagation in a half-space for given jumps of displacements and stresses on the inclusion line is determined by applying the Fourier transform in the spatial coordinate. As a result, we obtain a superposition of plane waves propagating from the inclusion line in the direction to infinity.

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WELL-POSEDNESS OF THE VARIATIONAL FORMULATION OF THE BOUNDARY VALUE PROBLEM OF CYLINDRICAL SHELL STATICS VIA THE H-ADAPTIVE FINITE ELEMENT METHOD

The aim of this work is to establish sufficient conditions for the well-posedness of variational problems arising in the Timoshenko shell model under various boundary and distributed load conditions, as well as to identify criteria for their singular perturbation. To clarify key aspects, we restrict ourselves to the analysis of a linear axisymmetric boundary value problem for a cylindrical shell.

In view of the potential singular perturbation of the equilibrium equations, we compute the eigenvalues, condition number, and determinant of the Hooke's law matrix, and provide its spectral decomposition, which is necessary for the analysis of the variational equation.

A dimensionless similarity criterion is derived for Timoshenko shells, which signals the possibility of singular perturbation for small values of shell thickness and/or radius relative to its length.

By invoking the Lax–Milgram–Vishik theorem and proving a Korn-type inequality, we derive sufficient regularity conditions on the problem data that ensure the well-posedness of the variational formulation.

Based on this, we construct an h-adaptive algorithm using piecewise linear finite element approximations capable of computing approximate solutions to the problem with guaranteed a priori accuracy.

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NUMERICAL MODELLING OF STEADY-STATE HARMONIC OSCILLATIONS IN THERMOVISCOELASTIC THIN-WALLED SHELLS

Thin-walled structures, especially shells of revolution, are widely used in modern engineering. When made of thermo-viscoelastic composite materials and exposed to combined thermal and mechanical loads, their behavior becomes complex. This work presents a mathematical model for analyzing steady-state harmonic vibrations in such shells.

The model is based on the Timoshenko–Mindlin hypothesis in a curvilinear orthogonal coordinate system. Displacements and temperatures are approximated by linear functions in the thickness direction. The Duhamel–Neumann hypothesis is used to describe constitutive relations with short-term memory effects.

The governing thermoelastic and heat equations are reformulated as a variational problem using Sobolev spaces. Bilinear and linear forms describe energy, dissipation, and external actions.

Assuming harmonic loading, the unknown fields are expanded in trigonometric functions. This leads to a symmetric frequency-dependent system solved numerically using the Galerkin method. Numerical experiments are performed for cylindrical shells made of isotropic thermo-viscoelastic materials, with the model expressed in cylindrical coordinates.

The approach provides an efficient tool for simulating harmonic vibrations of thermo-viscoelastic thin shells and can be applied in engineering design and analysis.

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RESOLVING DIFFERENTIAL EQUATION FOR FLUTTER OF A PLATE-STRIP COMPLIANT TO TRANSVERSE SHEAR

In modern aerospace engineering, thin-walled structural elements made of composite materials are widely used [3] due to their reduced weight and sufficient reliability. A key feature of such materials is their compliance to transverse shear, which significantly affects dynamic behavior.

A previously developed model based on refined plate theory [2] describes the deflection w of a plate-strip in aerodynamic flow [1]:

$$w^{(IV)} - \frac{2\rho h}{D}\ddot{w} - \frac{2\rho h}{\Lambda}\ddot{w}'' - \frac{M_a}{D}\left(\frac{D}{\Lambda}w''' + \frac{D}{\Lambda}\dot{w}'' - w' - \frac{1}{V}\dot{w}\right) = 0. \quad (1)$$

In the limit of vanishing shear compliance, equation (1) reduces to a (2), which is based on the Kirchhoff–Love plate theory:

$$w^{(IV)} - \frac{2\rho h}{D}\ddot{w} + \frac{M_a}{D}\left(w' + \frac{1}{V}\dot{w}\right) = 0. \quad (2)$$

The refined equation includes mixed and third-order derivatives, which complicate its solution. A corresponding fourth-order characteristic polynomial was derived, and a root-finding algorithm was implemented and tested for a simply supported plate-strip.

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VARIATIONAL PROLONGATION & VARIATIONAL CHECK IN TWO EXAMPLES

Recall that the Dixon's system describes the Relativistic Spherical Top. We incorporate the Lorentz–Dirac equation into the Dixon's system:

Proposition. *The system of equations*

$$\begin{cases} \dot{\wp} = \gamma \varkappa^2 \boldsymbol{u}\,, \\ \dfrac{1}{2}\,\dot{\boldsymbol{S}} = \wp \wedge \boldsymbol{u}\,, \\ \dot{\boldsymbol{S}}.\boldsymbol{u} = \gamma \left[\dfrac{(\boldsymbol{u}\cdot\dot{\boldsymbol{u}})}{\|\boldsymbol{u}\|^2}\,\boldsymbol{u} - \dot{\boldsymbol{u}} \right] \end{cases} \tag{1}$$

is equivalent to the Lorentz–Dirac equation.
Consider a *parameter-invariant* variation problem with the Action functional

$$\int (\varkappa^2 + A)\|\boldsymbol{u}\| d\zeta\,, \tag{2}$$

which gives rise to the following variational equation

$$\frac{d}{d\zeta}\left\{\left(\frac{2\,(\boldsymbol{u}\cdot\ddot{\boldsymbol{u}})}{\|\boldsymbol{u}\|^5} - \frac{(\dot{\boldsymbol{u}}\cdot\dot{\boldsymbol{u}})}{\|\boldsymbol{u}\|^5} - \frac{5\,(\boldsymbol{u}\cdot\dot{\boldsymbol{u}})^2}{\|\boldsymbol{u}\|^7} + \frac{A}{\|\boldsymbol{u}\|}\right)\boldsymbol{u} + \frac{6\,(\boldsymbol{u}\cdot\dot{\boldsymbol{u}})}{\|\boldsymbol{u}\|^5}\,\dot{\boldsymbol{u}} - \frac{2}{\|\boldsymbol{u}\|^3}\,\ddot{\boldsymbol{u}}\right\} = \boldsymbol{0}. \tag{3}$$

Definition. We call "a *(differential) variational extension* of some dynamical equation" such an equation (respectively, of an increased order), the set of integral curves of which includes the set of integral curves of the initial equation.

Proposition. *Equation (3) represents a variational extension of the relativistic Spherical Top equation.*

Proposition. *Equation (3) is a differential variational extension of the equation of the relativistic uniform acceleration motion.*

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CONVECTION-DOMINATED TRANSPORT PROBLEMS IN THIN GRAPH-LIKE NETWORKS

This talk presents recent joint work with Prof. C. Rohde and builds on results from [1–4], focusing on time-dependent convection-diffusion problems with a high Péclet number in thin three-dimensional networks consisting of cylinders that are interconnected by small domains (nodes) with diameters of order $\mathcal{O}(\varepsilon)$. Inhomogeneous flux boundary conditions with different intensity factors are considered on the lateral surfaces of the thin cylinders and the node boundaries.

The main goal is to study the asymptotic behavior of solutions of such problems as ε tends to zero, i.e. when a thin network shrinks to a graph and the diffusion operator disappears in the limit. A new asymptotic approach, which can take into account various factors, e.g., variable thickness of thin cylinders, inhomogeneous boundary conditions and different geometric characteristics of networks, is proposed. For linear problems, we construct asymptotic expansions and prove energetic and pointwise estimates [1,2]. For semilinear [3] and quasilinear [4] cases, we derive and rigorously justify asymptotic approximations.

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ON AN EXTENDED HILBERT SCALE AND ITS APPLICATIONS

We propose a new viewpoint on Hilbert scales extending them by means of all Hilbert spaces that are interpolation ones between spaces on the scale. Choosing a Hilbert space H and a positive definite self-adjoint unbounded operator A acting in H, one obtains the Hilbert scale $\{H^s_A : s \in \mathbb{R}\}$, where H^s_A is the completion of the domain of A^s in the norm $\|A^s u\|_H$ of a vector u. By definition, the extended Hilbert scale generated by A consists of all Hilbert spaces each of which is an interpolation space for a certain pair $[H^{s_0}_A, H^{s_1}_A]$ where $s_0 < s_1$ (the real numbers s_0 and s_1 may depend on the interpolation Hilbert space). This scale admits an explicit description with the help of OR-varying functions of A, is obtained by the quadratic interpolation (with function parameter) between the spaces $H^{s_0}_A$ and $H^{s_1}_A$, and is closed with respect to the quadratic interpolation between Hilbert spaces. We discuss applications of the extended Hilbert scale to interpolational inequalities, Sobolev spaces of generalized smoothness, and spectral expansions induced by abstract and elliptic operators. Specifically, this allows obtaining new results for multiple Fourier series. These results are published in [1, 2].

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ON SOLUTION OF INTEGRO-DIFFERENTIAL EQUATION DESCRIBING SURFACE TENSION IN DYNAMIC CRACK THEORY

An efficient method of analitycal-numerical solution of the 2D hypersingular integro-differential equation of a second kind and with the Helmholtz potential kernel, where free term models the crack surface tension [1-3] under time-harmonic loading conditions, is proposed. Corresponding algorithm is based on the transformation of initial equation to the weakly-singular form by using the Laplace-operator inversion technique and its subsequent regularization by the singularity subtraction approach. In addition, the the uniqueness of the solution is provided by the consideration of unknown function vanishing on the circular contour of the integration domain. A discrete analogue of the problem is constructed via satisfaction of resulting integral equation in the collocation points. The influence of the free-term of the equation on the solution is investigated for the different spectral parameters or frequencies

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THIELE'S INTERPOLATION. CASES OF DEGENERACY OF THE FRACTION

The theory of interpolating functions by continued fractions considers problems that are inherent only to this method of approximation.

A function is approximated by a continued fraction on the set of notes. The constructed fraction can be either a *Thiele interpolation continued fraction* or a *degenerate fraction.* New examples are given that refute the assumption that if a function takes the same values at different nodes, then the constructed continued fraction will always be degenerate. In the case when the function at the nodes takes on different values, the resulting continued fraction can also be either degenerate or an interpolation.

The equality of zero or infinity of the coefficients of a continued fraction does not always indicate the degeneracy of the fraction.

Theorem 1. *(A) The inverse difference* $\rho_{2k-1}[t_0, \ldots, t_{2k-1}; p]$ *of the polynomial* $p(z) = a_0 + \cdots + a_n z^k$*, where* $a_i \in \mathbb{C}$, $i = \overline{0, k}, k \geq 2$, *is identically equal to zero. (B) The inverse difference* 1*-th order of the binomial* $g(z) = a_0 + a_1 z$ *is equal to* $1/a_1$.

Theorem 2. *The inverse difference* $(2k)$*–th order of the elementary fraction* $g(z) = \dfrac{a}{(z-\alpha)^k}$, $k \in \mathbb{N}$, $a \in \mathbb{C}$, *where* α *is not a node, is equal to zero.*

Conditions have been found when the coefficients of a continued fraction are not equal to zero or infinity.

An algorithm is obtained that determines cases of interpolation of polynomials or rational functions, and also either constructs an interpolation continued fraction from given values of the function at the nodes, or indicates the impossibility of such construction.

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SOLVING A SYSTEM OF TWO POLYNOMIAL EQUATIONS VIA CONTINUED FRACTIONS

This work is dedicated to the development of an iterative method for solving systems of polynomial equations of the form:

$$\begin{cases} a_{n,0}x^n + a_{n-1,1}x^{n-1}y + \ldots + a_{0,n}y^n + \ldots + a_{1,0}x + a_{0,1}y + a_{0,0} = 0; \\ b_{n,0}x^n + b_{n-1,1}x^{n-1}y + \ldots + b_{0,n}y^n + \ldots + b_{1,0}x + b_{0,1}y + b_{0,0} = 0. \end{cases}$$

Here $x, y, a_{i,j}, b_{i,j} \in \mathbb{R}\ \ (i = 0, \ldots, n; j = 0, \ldots, n)$.

To find an approximate solution to the system, we will use an iterative formula of the form:

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} = \Big[\Big(E - \Big(-E - A_{2,0}^{1,1}\Big(\Big(E - \Big(E + A_{1,1}^{2,0}\Big(A_{n-1,1}^{n,0}x^{n-3} + \\ + \sum_{<k_1,k_2>\in S_i} A_{k_1-1,k_2+1}^{k_1,k_2} x^{k_1-3}y^{k_2}\Big)^{-1}\Big)^{-1}\Big)^{-1} A_{1,1}^{2,0}x\Big)^{-1}\Big)^{-1}\Big)^{-1} A_{1,1}^{2,0}x + \\ + \Big(E - \Big(-E - A_{2,0}^{1,1}\Big(\ \Big(E - \Big(E + A_{1,1}^{0,2}\Big(A_{0,n}^{1,n-1}y^{n-3} + \\ + \sum_{<k_1,k_2>\in S_i} A_{k_1,k_2}^{k_1-1,k_2+1} x^{k_1+1}y^{k_2-3}\Big)^{-1}\Big)^{-1}\Big)^{-1} A_{1,1}^{0,2}\Big)^{-1}\Big)^{-1}\Big)^{-1} A_{0,2}^{1,1}y + A_{0,1}^{1,0}\Big]^{-1} D, \end{aligned}$$

where $\quad S_i = \{(k_1, k_2) \mid \ 0 < k_1 < n, \ \ 0 < k_2 < n, \ \ k_1 + k_2 = i\}$,

$$A_{n,0}^{n-1,1} = \begin{pmatrix} a_{n,0} & ma_{n-1,1} \\ b_{n,0} & lb_{n-1,1} \end{pmatrix}, A_{k_1-1,k_2+1}^{k_1,k_2} = \begin{pmatrix} ma_{k_1,k_2} & ma_{k_1-1,k_2+1} \\ lb_{k_1,k_2} & lb_{k_1-1,k_2+1} \end{pmatrix},$$

$$A_{0,n}^{1,n-1} = \begin{pmatrix} (1-m)a_{1,n-1} & a_{0,n} \\ (1-l)b_{1,n-1} & b_{0,n} \end{pmatrix}, A_{0,1}^{1,0} = \begin{pmatrix} a_{1,0} & a_{0,1} \\ b_{1,0} & b_{0,1} \end{pmatrix},$$

$$A_{k_1,k_2}^{k_1-1,k_2+1} = \begin{pmatrix} (1-m)a_{k_1,k_2} & (1-m)a_{k_1-1,k_2+1} \\ (1-l)b_{k_1,k_2} & (1-l)b_{k_1-1,k_2+1} \end{pmatrix}, D = \begin{pmatrix} -a_{0,0} \\ -b_{0,0} \end{pmatrix},$$

The algorithm has also been implemented and tested in the Python environment for the case of systems with many unknowns.

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DYNAMICAL SYSTEMS ASSOCIATED WITH THE LOGISTIC MAP

We introduce the logistic map on the ring of multisets, demonstrating how it can be understood as an infinite-dimensional symmetric logistic equation. This approach provides a new framework for modeling dynamical systems where states are represented by multisets rather than single numbers.

Let us consider a mapping $L_{\mathbf{r}}$ of the form

$$L_{\mathbf{r}}(\mathbf{u}_n) = \mathbf{u}_{n+1} = \mathbf{r}\mathbf{u}_n(\mathbf{1} - \mathbf{u}_n),$$

where $\mathbf{u}_n = [u_n] = [(y_n|x_n)] = [(\ldots, y_{n2}, y_{n1}|x_{n1}, x_{n2}, \ldots)]$, $n \in \mathbb{N}$, and $\mathbf{r} = [(l|s)] = [(\ldots, l_2, l_1|s_1, s_2, \ldots)]$ are elements in commutative semirings $\mathcal{M}_p$. Polynomials T_k form an algebraic basis in the algebra of supersymmetric polynomials on $\ell_p \times \ell_p$, where

$$T_k(u) = F_k(x) - F_k(y) = \sum_{j=1}^{\infty} x_j^k - \sum_{j=1}^{\infty} y_j^k.$$

Suppose that $T_k(\mathbf{u}_1)$ and $T_k(\mathbf{r})$ are real positive numbers for every k in some subset $M = \{m_1, m_2, \ldots\} \subset \mathbb{N}$ satisfying the following inequalities: $0 < T^k(\mathbf{u}_0) < 1$, and $0 \leq T_k(\mathbf{r}) \leq 4$. Then we say that the sequence of multinumbers $(\mathbf{u}_n)$ satisfies logistic equation $L_{\mathbf{r},\mathbf{u}_0,M}$.

Theorem. *Let $(\mathbf{u}_n)$ satisfies logistic equation $L_{\mathbf{r},\mathbf{u}_0,M}$. Then for every $k \in M$, the sequence $T_k(\mathbf{u}_n)$ satisfies logistic equation L_{λ,t_0} for $\lambda = T_k(\mathbf{r})$ and $t_0 = T_k(\mathbf{u}_0)$, that is, $T_k(\mathbf{u}_{n+1}) = T_k(\mathbf{r})T_k(\mathbf{u}_n)\big(1 - T_k(\mathbf{u}_n)\big)$.*

We study the fixed points of this infinite-dimensional logistic mapping. Understanding the conditions for their existence and their properties is key to predicting the stable behavior of the dynamical systems modeled by this map.

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ON THE NUMERICAL SOLUTION OF THE PARABOLIC VOLTERRA PIDE WITH WEAK SINGULAR KERNELS

We consider a parabolic PIDE that models nonstationary diffusion processes with temporal memory effects

$$\frac{\partial u}{\partial t}(\mathbf{x},t)+\mathcal{L}_x u(\mathbf{x},t)+\int_0^t K(t-s)\,\mathcal{B}_x u(\mathbf{x},s)\,ds = f(\mathbf{x},t),\ \mathbf{x}\in D,\ t\in[0,T],$$

where $D\subset\mathbb{R}^d$ (for $d=2$ or 3) is a bounded domain, T is the final time, $\mathcal{L}$ is a second-order elliptic operator in divergence form with symmetric, bounded, and uniformly positive definite coefficients, $\mathcal{B}$ is a bounded linear operator, $K\colon[0,T]\to\mathbb{R}$ is a given weak singular kernel and $f\colon D\times[0,T]\to\mathbb{R}$ is a given smooth function. The equation is supplemented with the Dirichlet boundary value condition and an initial condition.

Existence and uniqueness of the solution for this time-dependent problem are established in the corresponding anisotropic spaces using the Galerkin method.

For numerical approximation, we use a two-step approach (see, for example, [1]). First, we apply semi-discretisation in time using Rothe's method, transforming the original problem into a sequence of elliptic boundary value problems. To calculate the time integral, we approximate the regular part of the kernel K on the introduced equidistant mesh using a piecewise constant. Each stationary problem is then solved using the radial basis function (RBF) collocation method, leading to linear systems with a fixed matrix and varying recurrent right-hand sides. The RBF shape parameter is optimised via genetic algorithms.

Numerical experiments confirm the accuracy and efficiency of the proposed approach.

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ON THE CLASSICAL FUNDAMENTAL SOLUTION OF THE CAUCHY PROBLEM FOR A DISSIPATIVE ULTRAPARABOLIC EQUATION WITH TWO GROUPS OF DEGENERATE VARIABLES AND WITH COEFFICIENTS INDEPENDENT OF THEM

In a layer $\{(t,x) : t \in (0,T],\ x \in \mathbb{R}^n\}$ of finite thickness $T > 0$ we consider the ultraparabolic equation

$$L\,(\partial_t, \partial_x)u(t,x) := \Bigg(\partial_t - \sum_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} - \sum_{j=1}^{n_3} x_{2j}\partial_{x_{3j}} - \\ - \sum_{j,s=1}^{n_1} a_{js}(t,x_1)\partial_{x_{1j}}\partial_{x_{1s}} - \sum_{j=1}^{n_1} a_j(t,x_1)\partial_{x_{1j}} - a_0(t,x_1)\Bigg) u(t,x) = 0, \quad (1)$$

in which n_1, n_2, n_3 are given natural numbers such that $n_3 \le n_2 \le n_1$; $n := n_1+n_2+n_3$; the variable $x \in \mathbb{R}^n$ consists of three groups of variables $x_l := (x_{l1}, \ldots, x_{ln_l}) \in \mathbb{R}^{n_l}$, $l \in \{1,2,3\}$, so that $x := (x_1, x_2, x_3)$.

The variables x_2 and x_3 are the degeneracy variables of the equation (1). It is assumed that the equation is dissipative ultraparabolic [1, 2], and the coefficients of the equation are differentiable and satisfy the local Hölder condition. A classical fundamental solution of the Cauchy problem for the equation (1) is constructed and estimates of its derivatives for all groups of variables are found.

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SEMI-ANALYTICAL SOLUTION FOR THE MOMENTUM SPECTRUM OF PARTICLES ACCELERATED AT A PARTIALLY-RADIATIVE SHOCK

We present a semi-analytical solution for the momentum distribution of particles accelerated at a shock transitioning from the adiabatic to the radiative phase of evolution. This regime arises for example when a supernova remnant encounters dense molecular material, leading to rapid shock deceleration and significant radiative cooling. The modified hydrodynamic structure, particularly in the immediate post-shock region, alters the transport equation governing particle acceleration. We model this by solving analytically the stationary diffusion-convection equation with spatially varying flow velocity and magnetic field upstream and downstream of the shock. Then the particle distributions are calculated numerically on the background of the flow structure obtained by numerically solving a system of MHD equations incorporating a radiative cooling term. The resulting momentum spectrum reveals explicit dependencies on length scales relevant for cooling and particle acceleration, on compression ratios and diffusion coefficients. These results demonstrate a tight link between MHD structures and spectra of accelerated particles, opening a possibility to explain some observational properties of interacting supernova remnants.

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SOLUTIONS OF THE MATHISSON-PAPAPETROU EQUATIONS IN SCHWARZSHILD'S METRIC FOR ULTRARELATIVISTIC PARTICLES

The Mathisson-Papapetrou (MP) equations [1] are a generalization of the geodesic equations to describe motions of rotating (spinning) particles in general relativity. There is a correspondence between the general relativistic Dirac equation for quantum particles and the MP equations in the linear spin approximation. It is important that solutions of the MP equations reveal new properties of the gravity interaction when the velocity of a spinning particle relative to the source of the gravitational field is close to the speed of light. The corresponding examples for Schwarzschild's gravitational field were considered, in particular, in [2].

Now we investigate the analytical solutions of the MP equations which describe the ultrarelativistic nonequatorial circular orbits of spinning particles in Schwarzschild's field using the standard spherical and time coordinates. The necessary and sufficient conditions for existence of these solutions are obtained. Illustrations of the comparison of motions of a spinning and a spinless particle with the same initial coordinates and velocity are presented. Estimates of the spin-gravity force which deviates its motion from the geodesic one are given. The properties of the ultrarelativistic spin-gravity coupling at the classical level are useful for studying the gravity interaction of quantum particles at very high energies [3].

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BOUNDARY VALUE PROBLEMS FOR LINEAR AND NONLINEAR DISCRETE SYSTEMS

This report investigates boundary value problems (BVPs) for both linear and nonlinear discrete systems with a focus on input-to-state stability (ISS). We begin by analyzing homogeneous and nonhomogeneous linear systems, deriving explicit solution formulas using mathematical induction. Solvability conditions for these systems are established, providing necessary and sufficient criteria for the existence of solutions. The study is then extended to nonlinear discrete systems, where perturbation terms are introduced, and equivalent transformed BVPs are derived. By applying solvability conditions, we analyze the transition from nonlinear to linear cases as perturbations approach zero. The results contribute to the understanding of ISS properties in discrete-time dynamical systems and serve as a foundation for stability analysis and numerical methods in solving BVPs in discrete settings.

In the presented report we try to investigate the following boundary-value problem for the following systems of differential and difference equations, which model the simple version of the neural networks

$$x_i^{'}(t,\varepsilon) = d_i I_i x_i(t,\varepsilon) - \varepsilon c_i d_i x_i^2(t,\varepsilon) +$$

$$x_i(n+1,\varepsilon) = d_i I_i x_i(n,\varepsilon) - \varepsilon c_i d_i x_i^2(n,\varepsilon) +$$

$$+\varepsilon d_i x_i(t,\varepsilon) \sum_{j=1}^{n} a_{ij} f_j(x_j(t,\varepsilon)), \tag{1}$$

with additional boundary conditions

$$x_i(0,\varepsilon) - x_i(T,\varepsilon) = \alpha_i, \tag{2}$$

where a_{ij} can be considered as a control parameters. In this model $x_i(t,\varepsilon)$ denotes the state of the i-th neuron at time t, $d_i x_i$ and $c_i x_i$ represent the amplification and behavior functions at time t, respectively, ε is a small parameter. Moreover, in further considerations, we suppose that $d_i I_i T \neq 0$. Condition (2) represents the two-point boundary condition, α_i are the set of parameters. We can consider such boundary-value problem as in the finite-dimensional case ($i = \overline{1,n}$), as in the countable case ($i = \overline{1,\infty}$) and in the case when we consider the given problem in the case of Hilbert or Banach spaces (in this case, quadratic nonlinearity can be interpreted in another way).

In the paper which we can see below the main results were proved. We are going also use the proposed technique for the investigating another type of neural networks such as considered below.

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ALGEBRAS OF SYMMETRIC POLYNOMIALS ON CARTESIAN PRODUCTS OF REAL AND COMPLEX BANACH SPACES OF LEBESGUE INTEGRABLE FUNCTIONS

Let $L_p^{(\mathbb{K})}$, where $p \in [1;+\infty)$, $\mathbb{K} \in \{\mathbb{C}, \mathbb{R}\}$, be the Banach space of measurable functions $x : [0;1] \to \mathbb{K}$ for which the pth power of the absolute value is Lebesgue integrable. Let $\Xi_{[0;1]}$ be the set of all bijections $\sigma : [0;1] \to [0;1]$ such that both σ and σ^{-1} are measurable and preserve Lebesgue measure, i.e., $\mu(\sigma(E)) = \mu(\sigma^{-1}(E)) = \mu(E)$ for every Lebesgue measurable set $E \subset [0;1]$, where μ is Lebesgue measure. For $\sigma \in \Xi_{[0;1]}$ and $x = (x_1; \ldots; x_n) \in L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})}$, where $p_i \in [1;+\infty)$ for $i \in \{1;\ldots;n\}$, let $x \circ \sigma = (x_1 \circ \sigma; \ldots; x_n \circ \sigma)$. A function $f : L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})} \to \mathbb{K}$ is called symmetric if $f(x) = f(x \circ \sigma)$ for every $x \in L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})}$ and $\sigma \in \Xi_{[0;1]}$. For every $\alpha = (k_1; k_2; \ldots; k_n) \in \mathbb{Z}_+^n \setminus \{(0;\ldots;0)\}$ let $S_\alpha^{(p_1,\ldots,p_n)} = \sum_{i=1}^n \frac{k_i}{p_i}$. Let

$$\aleph_{p_1,\ldots,p_n} = \{\alpha \in \mathbb{Z}_+^n \setminus \{(0;\ldots;0)\} : S_\alpha^{(p_1,\ldots,p_n)} \le 1\}.$$

For every $\alpha = (k_1; k_2; \ldots; k_n) \in \aleph_{p_1,\ldots,p_n}$ let $R_\alpha^{(L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})})} : L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})} \to \mathbb{K}$ be defined by

$$R_\alpha^{(L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})})}(x_1; \ldots; x_n) = \int_0^1 x_1^{k_1}(t) \cdot \ldots \cdot x_n^{k_n}(t)\, dt,$$

where $x_i \in L_{p_i}^{(\mathbb{K})}$ for $i \in \{1;\ldots;n\}$. In the case $k_i = 0$ and $x_i \equiv 0$ we will consider $x_i^{k_i}$ to be equal 1.

Theorem *Let $p_1; \ldots; p_n \in [1;+\infty)$. Let $L^{(\mathbb{K})} = L_{p_1}^{(\mathbb{K})} \times \ldots \times L_{p_n}^{(\mathbb{K})}$. The set of polynomials $\{R_\alpha^{(L^{(\mathbb{K})})} : \alpha \in \aleph_{p_1,\ldots,p_n}\}$ is an algebraic basis of the algebra of all symmetric continuous polynomials on $L^{(\mathbb{K})}$ with the values in $\mathbb{K}$.*

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FRACTIONAL NONLOCAL ELASTICITY

In nonlocal elasticity, the stress constitutive equation has the integral form which takes into account interatomic long-range forces and reduces to the classical theory of elasticity in the long-wave-length limit and to the atomic lattice theory in the short-wave-length limit.

We propose the new nonlocal elasticity theory with the weight function in the stress constitutive equation being the Green function of the corresponding Cauchy problem for the space-time fractional diffusion equation [1, 2]. The weight function (the nonlocality kernel) is expressed in terms of the Mainardi function and Wright function being the generalizations of the exponential function [3, 4].

The considered theory can be used for better matching the theory of elasticity and the atomic lattice theory. Several examples are solved in the framework of the proposed approach.

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ON DETERMINATION OF RIGHT-HAND SIDE FUNCTION OF THIRD ORDER WAVE EQUATION

In the domain $Q_T = \Omega\times(0,T)$, where $0 < T < \infty$, and $\Omega \subset \mathbb{R}^n, n \in \mathbb{N}$, is a bounded domain with the smooth boundary $\partial\Omega$, we consider the following inverse problem: find the sufficient conditions for the existence of a pair of functions $(u(x,t), g(t))$ that satisfies the equation with the strong damping

$$u_{tt} - a^2\Delta u - b^2\Delta u_t + \varphi(x,u,u_t) = f(x)g(t),\ x \in \Omega,\ t \in [0,T], \tag{1}$$

and the initial, boundary and overdetermination conditions

$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \qquad x \in \Omega, \tag{2}$$

$$u|_{\partial\Omega\times(0,T)} = 0, \tag{3}$$

$$\int\limits_{\Omega} K(x)u(x,t)dx = E(t), \qquad t \in [0,T]. \tag{4}$$

Here $a \neq 0,\ b \neq 0$, function $\varphi(x,\xi,\eta)$ is measurable with respect to $x \in \Omega$ for all $\xi,\eta \in \mathbb{R}$ and continuously differentiable with respect to $\xi,\eta \in \mathbb{R}$. Moreover, $|\varphi(x,\xi,\eta) - \varphi(x,\tilde{\xi},\tilde{\eta})| \leq L_0\left(|\xi-\tilde{\xi}| + |\eta-\tilde{\eta}|\right)$, $|\varphi(x,\xi,\eta)| \leq L_1\left(|\xi| + |\eta|\right)$, for almost all $x \in \Omega$ and $\xi,\eta,\tilde{\xi},\tilde{\eta} \in \mathbb{R}$, where L_0, L_1 are positive constants.

Functions $f \in L^2(\Omega)$, $u_0 \in H_0^1(\Omega)$, $u_1 \in H_0^1(\Omega)$, $K \in H^2(\Omega)\cap H_0^1(\Omega)$ $E \in C^2([0,T])$.

Theorem. *Let* $\int\limits_{\Omega} K(x)f(x)\,dx \neq 0$. *Then there exists a unique weak solution to the problem (1)–(4), and* $u, u_t \in C([0,T]; H_0^1(\Omega))$, $u_{tt} \in L^2(Q_T)$, $g \in C([0,T])$.

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UNSTEADY HEAT CONDUCTION PROBLEM FOR A PIECEWISE-HOMOGENEOUS THERMOSENSITIVE MEDIUM

This work is a continuation of the studies [2] concerning the method for solving unsteady heat conduction problems in a two-component medium with temperature-dependent thermophysical properties under the action of plane impulsive heat sources. The approach involves the use of the Kirchhoff transformation, generalized functions, Green's functions [1] for a piecewise homogeneous medium, and linear splines.

An approximate solution to the corresponding problem in terms of the Kirchhoff variable has been obtained. This solution contains unknown values of the Kirchhoff variable in the second half-space at the interface. To find them, a nonlinear relation was derived: $\Theta_2(0, Fo) = \Theta_2^{(w)}(0, Fo) + \frac{\delta_1}{\delta_1+\delta_2} F_2\left(\Theta_2(0, Fo)\right)$, where the first term describe the temperature field with constant thermophysical properties, and δ_i are the square roots of the product of the corresponding thermal conductivity and volumetric heat capacity coefficients. The form of the function F_2 depends on the type of temperature dependence of the thermal conductivity coefficients. As examples, linear and exponential dependencies were considered.

The temperature distribution was studied under a linear dependence of the thermal conductivity coefficients and different values of the specific power of the heat source in the first half-space.

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APPLICATION OF CLASSICAL ORTHOGONAL POLYNOMIALS IN FRACTIONAL CALCULUS

This paper offers a brief overview of the existing fractional order derivatives, their positive aspects and existing unsolved problems.

Differential equations in partial differentials of fractional order, being a generalization of partial differentials of integer order, are of great practical importance in addition to their theoretical interest. Fractional calculus has found its application in physics, mechanics, viscoelasticity, biomedical engineering, civil engineering, electrochemistry, and electromagnetism. Other interesting applications include signal processing: noise, fractional chaos, fractional Gaussian noise, and fractional Brownian motion.

The positive aspects include: taking into account the history of the process and the frontality of the environment in which the process takes place; the possibility of obtaining a qualitative picture of the process.

The unresolved problems include: the question of choosing the type of fractional derivative for modeling the processes under study remains open; as practice shows, mathematical models are sensitive to the order of the fractional derivative, but the criterion for its selection is unknown; currently there are no sufficiently effective methods for solving boundary value problems using fractional order derivatives; existing application packages for solving boundary value problems do not always give satisfactory results.

The fractional calculus has been tested in the study of mass transfer processes in pipeline systems and complex porous media. The following conclusions can be drawn from the results obtained.

Modeling the transfer processes in pipeline systems using fractional order derivatives with the same input data does not lead to improved results compared to the classical approach, but computational difficulties increase significantly.

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ON THE CONSTRUCTION OF LYAPUNOV FUNCTIONS USING GENETIC PROGRAMMING

We developed a method using an evolutionary approach to the automated construction of polynomial Lyapunov functions for certifying exponential stability in nonlinear dynamical systems. Building on the theoretical guarantee that such functions exist under sufficient smoothness and stability assumptions, we propose a genetic programming framework that represents candidate Lyapunov functions as symbolic binary trees. The core of our method is a carefully crafted fitness metric that encodes Lyapunov conditions—positivity, quadratic bounds, and orbital dissipation—as a minimax optimization problem over the state space. The GP algorithm iteratively refines candidate functions by applying mutation, crossover, and elitist selection, driving the population toward valid stability certificates. We validate the approach on several benchmark systems, including linear, polynomial, and nonlinear models such as the damped pendulum and Van der Pol oscillator. The experiments demonstrate that our method can reliably discover Lyapunov functions that satisfy the required stability conditions. Our research [1] highlights the potential of symbolic evolutionary computation in control theory and opens a path toward interpretable, data-driven stability analysis tools. Future research directions include the synthesis of control Lyapunov functions, adaptive parameter tuning, and improving scalability to high-dimensional systems.

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ASYMPTOTIC MODEL OF DYNAMIC PROCESSES IN PIEZOELECTROMAGNETIC MATRIX WITH THIN METAL LAYER

Piezoelectric materials are widely used as sensors, actuators and transducers for managing vibrations in various devices. Addressing the problems related to the complex interactions of wave propagation and coupled physicomechanical fields in piezoelectric composites will improve modeling and defect detection methods in advanced electromechanical systems.

We consider shear wave propagation in a composite consisting of metallic layer and piezoelectromagnetic matrix. The layer height is significantly less than the wavelength of incident waves and described by a small parameter. Ideal mechanical contact and closed circuit boundary conditions are prescribed on the contact line between composite elements. The asymptotic expansions method [1] was adapted to derive effective models for the interaction between the components of the given structure. Appropriate interaction model for the given composite is obtained and can be utilized in boundary element formulations to investigate properties of SH-wave propagation [2].

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DIRICHLET-NEUMANN PROBLEM FOR A HIGH-ORDER PARTIAL DIFFERENTIAL EQUATION IN AN UNBOUNDED LAYER

In a multidimensional unbounded layer $Q_T^p = \{(t,x) \in \mathbb{R}^{p+1} : t \in (0,T), x \in \mathbb{R}^p\}$, $T>0$, we consider such boundary value problem :

$$\frac{\partial^{2n} u(t,x)}{\partial t^{2n}} + \sum_{j=0}^{n-1} A_{2n-2j}(D_x) \frac{\partial^{2j} u(t,x)}{\partial t^{2j}} = 0, \quad (t,x) \in Q_T^p, \tag{1}$$

$$\begin{cases} \left. \dfrac{\partial^{2j-2} u(t,x)}{\partial t^{2j-2}} \right|_{t=0} = \varphi_j(x), \quad j=1,\dots,n, \quad x \in \mathbb{R}^p, \\ \left. \dfrac{\partial^{2j-1} u(t,x)}{\partial t^{2j-1}} \right|_{t=T} = \varphi_{n+j}(x), \quad j=1,\dots,n, \quad x \in \mathbb{R}^p, \end{cases} \tag{2}$$

where $x=(x_1,\dots,x_p)$, $D_x = (-i\partial/\partial x_1, \dots, -i\partial/\partial x_p)$,

$$A_{2j}(D_x) = \sum_{\substack{s=(s_1,\dots,s_p)\in\mathbb{Z}_+^p, \\ |s|\le 2j}} A_{2j}^s \left(-i\frac{\partial}{\partial x_1}\right)^{s_1} \cdots \left(-i\frac{\partial}{\partial x_p}\right)^{s_p}, \quad j=1,\dots,n.$$

Given suitable assumptions on the parameters of problem (1), (2), the well-posedness of this problem in the Sobolev space is established.

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PHYSICALLY GROUNDED PARTIAL SOLUTIONS OF THE SYSTEM OF NAVIER EQUATIONS

The construction of partial solutions of the equations of mathematical physics, if they do not contain an unknown homogeneous solution, is a difficult task. In [1], when the volume force potential $F = 0$, the physical features of the thermoelastic state are accurately taken into account and the following partial solution of the system of Navier equations is found:

$$u_j^\tau = \frac{\partial \vartheta}{\partial x_j} + \beta_1 \Omega_j,\ j = \overline{1,\,3},\ e^\tau = 3\alpha T,\ \Theta^\tau = \sigma_1^\tau + \sigma_2^\tau + \sigma_3^\tau = 0, \tag{1}$$

where T, $\Omega_j = \int \mathrm{T} dx_j$, $j = \overline{1,\,3}$ are three-dimensional harmonic functions, $\vartheta = \beta(x\Omega_1 + y\Omega_2 + z\Omega_3)$, $\nabla^2\psi = -\alpha T$, $\beta = -\frac{1}{6}\alpha$, $\beta_1 = \frac{4}{3}\alpha$. It was proved that the displacements and stresses (1) do not contain elastic components.

Consider an elastic body that is freely deformed under the action of volume forces with the potential $F \neq 0$, $T = 0$. For the first time, a partial solution to the system of Navier equations has been found that does not contain elastic displacements (homogeneous solutions):

$$u_j^f = \frac{\partial \vartheta}{\partial x_j} + \chi_1 \Omega_j,\ j = \overline{1,\,3},\ e^f = -\frac{(1-2\nu)}{E} F,\ \Theta^f = -F, \tag{2}$$

where $\Omega_j = \int F dx_j$, $j = \overline{1,\,3}$, F are three-dimensional harmonic functions, $\vartheta = \chi(x\Omega_1 + y\Omega_2 + z\Omega_3)$, $\chi = -\frac{(1+2\nu)}{6E}$, $\chi_1 = \frac{4\nu}{3E}$. If we put $(1-2\nu)F = -2\alpha G\,(1+\nu)\,T$, then the right-hand sides of the Navier equations that determine the influence of volume forces and temperature will be the same.

Conclusion. *It has been established that when finding a partial solution of the system of Navier equations, it is necessary to take into account the physical content of the right-hand side.*

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ON BÉZOUT RINGS OF STABLE RANGE 1

Let R be a Bézout ring. Recall that a ring R is a right (left) Bézout ring if every finitely generated right (left) ideal is principal. The concept of stable range plays an important role in the study of many matrix problems over different rings [1]. This concept was introduced by Bass [2].

Definition 1. *The stable range of a ring R is called the least natural number n such that for any nonzero left relatively prime elements $a_1, \ldots, a_n, a_{n+1} \in R$, there exist elements $r_1, \ldots, r_n \in R$ such that the elements*

$$a_1 + a_{n+1}r_1, a_2 + a_{n+1}r_2, \ldots, a_n + a_{n+1}r_n$$

are left relatively prime.

A ring R is called a ring of stable range 1 if for any elements $a, b \in R$ the equality $aR + bR = R$ implies that there is some $x \in R$ such that $(a + bx)R = R$.

Definition 2. *A ring R is said to be a ring of almost right (left) stable range 1 if for any elements $a, b, c \in R$ where $a \neq 0$ such that $aR + bR + cR = R$ $(Ra + Rb + Rc = R)$ there exist an element $r \in R$ such that*

$$aR + (b + cr)R = R \quad (Ra + R(b + rc) = R).$$

Theorem. *Let R be a right (left) Bézout ring of stable range 1 then R is of almost right (left) stable range 1.*

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THE CAYLEY TRANSFORM METHOD FOR AN ABSTRACT EVOLUTION EQUATION WITH A FRACTIONAL DERIVATIVE

We consider an initial-value problem for an abstract evolution equation:

$$u'(t) + \partial_t^{-\alpha} Au = f(t), \quad 0 < t \leqslant T, \quad u(0) = u_0,$$

where A is a self-adjoint positive definite operator in a Hilbert space H, $f : [0;T] \to H$, $u : [0;T] \to H$ is an unknown solution, $u_0 \in H$, and

$$(\partial_t^{-\alpha} u)(t) = \begin{cases} \dfrac{d}{dt} \int_0^t \dfrac{(t-s)^\alpha}{\Gamma(1+\alpha)} u(s)\, ds & \text{if} \quad -1 < \alpha \leqslant 0; \\ \int_0^t \dfrac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} u(s)\, ds & \text{if} \quad 0 < \alpha < 1. \end{cases}$$

The solution $u(t)$ can be formally presented in the form [1]:

$$u(t) = U(t)u_0 + \int_0^t U(t-s) f(s)\, ds,$$

where $U(t) = \sum\limits_{n=0}^{\infty} p_n^{(\alpha)}(t^{1+\alpha}) Q^n, \quad Q = A(I+A)^{-1},$

$$p_n^{(\alpha)}(t^{1+\alpha}) = \sum_{r=0}^{n-1} \frac{(-1)^{r+1} C_{n-1}^r t^{(r+1)(\alpha+1)}}{\Gamma\big(1 + (r+1)(\alpha+1)\big)}, \quad n \in \mathbb{N}, \quad p_0^{(\alpha)}(t^{\alpha+1}) \equiv 1.$$

We approximate the solution $u(t)$ as follows:

$$u_N(t) = U_N(t)u_0 + \int_0^t U_N(s) f(t-s)\, ds, \quad U_N(t) = \sum_{n=0}^{N} p_n^{(\alpha)}(t^{1+\alpha}) Q^n.$$

We study the accuracy of $u_N(t)$ under various assumptions regarding the input data u_0 and $f(t)$: we obtain a power-type convergence rate or an exponential convergence rate depending on N if u_0 and $f(t)$ are finitely or infinitely smooth in some sense.

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GLOBAL WELL-POSEDNESS OF THE TWO-COMPONENT NOVIKOV SYSTEM

My talk deals with the global semigroup of conservative weak solutions of the following two-component Novikov system [2]:

$$\begin{aligned} & m_t + (uvm)_x + vu_x m = 0, && m = m(t,x),\, u = u(t,x),\, v = v(t,x), \\ & n_t + (uvn)_x + uv_x n = 0, && n = n(t,x), \\ & m = u - u_{xx},\; n = v - v_{xx}, && u, v \in \mathbb{R}, \quad t, x \in \mathbb{R}. \end{aligned} \tag{1}$$

Taking $u = v$, system (1) reduces to the Novikov equation, which reads

$$m_t + (u^2 m)_x + u u_x m = 0.$$

Applying the Bressan-Constantin approach, the work [1] constructs the global conservative weak solutions $(u,v)(t)$ of (1) for the initial data $(u_0, v_0) \in \left(H^1 \cap W^{1,4}\right)^2$, such that $(u,v)(t) \in \Sigma$, where

$$\Sigma = \left\{(f,g) : f, g \in H^1(\mathbb{R}) \text{ and } \|f_x g_x\|_{L^2(\mathbb{R})} < \infty\right\},\ \Sigma \subsetneq \left(H^1 \cap W^{1,4}\right)^2.$$

Since these solutions do not preserve the regularity, there is no hope to have a semigroup property for such solutions.

In our work [3], we address this question and obtain a global semigroup of conservative solutions of (1) for $(u_0, v_0) \in \Sigma$. We also show the possible energy concentration of either $u_x^2\,dx$, $v_x^2\,dx$, or $\left(u_x^2 v_x^2\right) dx$.

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TIME-PERIODIC HEAT CONDUCTION PROBLEM WITH ROBIN BOUNDARY CONDITIONS IN A MULTILAYERED DOMAIN

In the classical formulation of multilayer heat conduction problems in composite structures, the initial temperature distribution is typically prescribed in each layer [1,2]. However, in the case of a time-periodic steady-state thermal regime, the initial conditions are naturally replaced by time-periodic conditions.

Let $\Omega = \mathbb{R}/2\pi\mathbb{Z}$ be a unit circle and let $\mathcal{D} = (x_0, x_n) \times \Omega$, $x_0, x_n \in \mathbb{R}$, $\mathcal{D}_j = (x_{j-1}, x_j) \subset \mathcal{D}$, $x_{j-1} < x_j$, $u_j = u_j(x,t)$, $j = 1, \ldots, n$.

In domain $\mathcal{D} \times \Omega$, we consider the problem for $u = (u_1, \ldots, u_n)$:

$$\tfrac{\partial u_j}{\partial t} = \alpha_j \tfrac{\partial^2 u_j}{\partial x^2}, \quad (x,t) \in \mathcal{D}_j \times \Omega, \quad j = 1, \ldots, n,$$

$$\left(\nu_1 u_1 + \nu_2 \tfrac{\partial u_1}{\partial x}\right)|_{x=x_0} = g_1(t), \ \left(\nu_3 u_n + \nu_4 \tfrac{\partial u_n}{\partial x}\right)|_{x=x_n} = g_2(t), \ t \in \Omega,$$

$$u_j\big|_{x=x_{j\,-}} = u_{j+1}\big|_{x=x_{j\,+}}, \ \kappa_j \frac{\partial u_j}{\partial x}\Big|_{x=x_{j\,-}} = \kappa_{j+1} \frac{\partial u_{j+1}}{\partial x}\Big|_{x=x_{j\,+}}, \ j = 1, \ldots, n-1.$$

where $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{R}_+$ that are pairwise distinct, $\nu_1, \nu_2, \nu_3, \nu_4 \in \mathbb{R}$, $|\nu_1| + |\nu_2| \neq 0$, $|\nu_3| + |\nu_4| \neq 0$, $\kappa_j > 0$ is the thermal conductivity of the material in the j-th layer, the functions g_1 and g_2 are 2π-periodic from the space $\mathbf{H}_q(\Omega)$.

The well-posedness of the problem is investigated in Sobolev spaces, and sufficient conditions are obtained for the existence of a unique solution u with components $u_j \in \mathbf{C}^2(\mathcal{D}_j; \mathbf{H}_q(\Omega))$, for $j = 1, \ldots, n$. The analysis is based on the method of separation of variables, Fourier series expansion, and estimates of the determinants related to the problem.

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FIXED POINT THEOREM IN AN ARBITRARY TOPOLOGICAL VECTOR SPACE

Classical fixed point theorem says that if X is an arbitrary complete metric space and $f : X \to X$ a contractive mapping then f has a unique fixed point $x_0 \in X$, i.e., $f(x_0) = x_0$.

In our work we use the technique of filters for studying more general situation. Recall that for nonempty set Ω filter $\mathfrak{F} \subset 2^\Omega$ is a family of subsets in Ω satisfuying the following axioms: $\Omega \in \mathfrak{F}$, $\emptyset \notin \mathfrak{F}$, if $A, B \in \mathfrak{F}$ then $A \cap B \in \mathfrak{F}$, if $A \in \mathfrak{F}$, $D \supset A$ then $D \in \mathfrak{F}$.

In [1] we introduced the following generalisation of completeness for general topological vector spaces. Let $\mathfrak{F}$ be a filter on $\mathbb{N}$. A topological vector space X is called to be complete over $\mathfrak{F}$, if every Cauchy sequence over $\mathfrak{F}$ in X has a limit over $\mathfrak{F}$.

Let us introduce the following

Definition. Let X be an arbitrary topological vector spaces. We say that the mapping $f : X \to X$ is *tvs-contractive* if for all $x \in X$ and for all neighbourhood U of $f(x)$ there exists a neighbourhood V of x such that $f(U) \subset V$.

In our work we study the next question: is it possible to obtain the fixed point theorem in the case of an arbitrary topological vector spaces.

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FUNDAMENTAL SOLUTION FOR A NEW CLASS OF NON-ARCHIMEDEAN PSEUDO-DIFFERENTIAL EQUATIONS

In the analysis of real- and complex-valued functions over a non-Archimedean field of p-adic numbers $\mathbb{Q}_p$ and the spaces $\mathbb{Q}_p^n$, a typical linear operator is the Vladimirov-Taibleson operator $D^{\alpha,n}$, which can be seen as a kind of an elliptic operator.

Let $\alpha > 0$, $\beta > 0$, $n \geq 1$. We consider the Cauchy problem

$$D^{\alpha}_{|t|_p} u(|t|_p, x) - D^{\beta,n}_x u(|t|_p, x) = 0,\ (t,x) \in \mathbb{Q}_p \times \mathbb{Q}_p^n, \tag{1}$$

$$u(0,x) = u_0(x),\ x \in \mathbb{Q}_p^n, \tag{2}$$

where $u : \mathbb{Q}_p \times \mathbb{Q}_p^n \to \mathbb{C}$ is a radial function with respect to t.

Theorem. *Let $\alpha > 0$, $\beta > 0$ such that the condition*

$$\beta = K\alpha \text{ for some } K \in \mathbb{N}$$

holds. Suppose that the function u_0 is in $\Phi(\mathbb{Q}_p^n)$. Then the Cauchy problem (1)–(2) has a solution $u = u(|t|_p, x)$, radial in t, that belongs to the space $\Phi(\mathbb{Q}_p)$ for each $x \in \mathbb{Q}_p^n$, and belongs to $\Phi(\mathbb{Q}_p^n)$ for each $t \in \mathbb{Q}_p$.

If the condition $\beta \neq K\alpha$ for any $K \in \mathbb{N}$, then the equation (1) has only a zero solution $u(t,x) \equiv 0$, $(t,x) \in \mathbb{Q}_p \times \mathbb{Q}_p^n$.

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ON GENERALIZATIONS OF THEOREMS OF FUCHS AND KAHANE

Let a function $\gamma : [0; +\infty) \to \mathbb{R}$ satisfy the following conditions: $i_1)$ γ is a measurable and $|\gamma(t)| \le c_0(t+1)$ for all $t \in [0; +\infty)$ and some constant $c_0 > 0$; $i_2)$ $|\gamma(t) - t\eta(t)| \le c_1$ for all $t \in [0; +\infty)$, some constant $c_1 > 0$ and some function $\eta : [0; +\infty) \to \mathbb{R}$ of bounded variation on $[0; +\infty)$.

Let $H_\gamma(\mathbb{C}_+)$ be the class of holomorphic functions $f \not\equiv 0$ in the half-plane $\mathbb{C}_+ = \{z \in : \mathrm{Re} z > 0\}$ such that

$$\sup\{|f(z)| \exp(-\gamma(|z|) - c\,\mathrm{Re} z) : z \in \mathbb{C}_+\} < +\infty$$

for some constant $c = c_f \in \mathbb{R}$.

We generalize some results of W. Fuchs [1] and J.-P. Kahane [2] on sequences of zero of functions of exponential type in the half-plane.

Theorem 1. *Let (λ_k) be an arbitrary sequence in $\mathbb{C}_+$ and $h : \mathbb{R} \to \mathbb{R}$ be a non-increasing function, for which $h'(t) = 0$ almost everywhere. Then a function $f \in H_\gamma(\mathbb{C}_+)$, such that its sequence of zeros and singular boundary function coinciding with (λ_k) and h respectively, exist if and only if* $\inf\{\psi(r) : r \in [1; +\infty)\} > -\infty$, $\sum_{|\lambda_k| \le 1} \mathrm{Re}\lambda_k < +\infty$ *and*

$$\sup\left\{ \int\limits_{1 \le |t| \le r} \left(\frac{1}{t^2} - \frac{1}{r^2}\right) (ds(t) - dh(t)) - 2\psi(r) : r \in [1; +\infty) \right\} < +\infty,$$

where $s(t) = 2\pi \sum_{1 < |\lambda_k| \le t} \mathrm{Re}\lambda_k$, $\psi(r) = \int_1^r \frac{\gamma(t)}{t^2}\, dt$, *if* $r \in [1; +\infty)$ *and* $\psi(r) = 0$, *if* $r \notin [1; +\infty)$.

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ON FUNDAMENTAL SYSTEMS OF SOLUTIONS OF THE EQUATION $f'' + Af = 0$ ACCORDING TO GIVEN SEQUENCES

We investigate the question about construction two linearly independent solutions f_1 and f_2 of the equation

$$f'' + Af = 0$$

according to given sequences. Similar result is considered in [1].

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REDUCTION OF A MATRIX ORIENTED BY CHARACTERISTIC ROOTS TO A FORM WITH GIVEN CHARACTERISTICS

We consider polynomial matrices over the field of complex numbers. By a polynomial matrix of a simple structure we mean one whose elementary divisors are all linear [1]. Polynomial matrices are called semiscalar equivalent if one of them can be obtained by multiplying the other on the left and right by a numerical nonsingular and polynomial invertible matrix, respectively [1]. In the author's work [2], the definition of matrices of a simple structure oriented by characteristic roots is introduced.

In the proposed report, the possibility of reducing one type of matrices oriented by characteristic roots to a form with previously fixed characteristics using semiscalar equivalent transformations is proven. The proof of this fact is constructed as a sequence of transformations of the original matrix, which ultimately lead to a matrix with the desired properties. At each step of the reduction, the left and right transformation matrices are specified, from which, after the process is completed, the left and right matrices of the resulting transformation can be constructed. The resulting matrix with fixed values of its certain elements in the class of semiscalar equivalent matrices is uniquely defined [3]. This makes it possible to apply the resulting matrix to the problem of classifying a selected type of polynomial matrices with respect to semiscalar equivalence and to classifying sets of numerical matrices with accuracy up to similarity.

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NONLINEAR FOURIER SPECTRUM CHARACTERIZATION OF TIME-LIMITED SIGNALS

We address some problems in the theory of optical communication systems in the framework of the nonlinear Fourier transform (NFT) for the nonlinear Schrödinger equation

$$iq_z + q_{tt} + 2|q|^2 q = 0,$$

which is an integrable nonlinear PDE involving the slow-varying complex electromagnetic field envelope function $q(t, z)$ propagating along an optical fiber, where z is the distance along the fiber and t is the retarded time. Namely, we are interested in the properties of the signals emerging in the so-called "b-modulation" method, the nonlinear signal modulation technique that provides explicit control over the signal extent.

We analyze the case where the time-domain profile of a signal corresponding to the b-modulated data has a limited duration:

$$b(k) = \int_{-L}^{L} \beta(\tau) e^{ik\tau} d\tau \quad \text{with some } \beta(\tau) \in L^1(-L, L)$$

(which corresponds to a signal q such that $q(t) = 0$ for $|t| > \frac{L}{2}$) and when the bound states corresponding to specifically chosen discrete solitonic eigenvalues and norming constants, are also present. We prove that the number of solitary modes that we can embed without violating the exact localization of the time-domain profile, is actually infinite, and present the characterization of the set of all possible solitonic eigenvalues using the Riemann-Hilbert problem formalism.

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DIFFERENTIAL EQUATION $ay' = by^m$ IN THE RING OF POWER SERIES WITH INTEGER COEFFICIENTS

Let $m \in \mathbb{N}$ and $a, b, c_0 \in \mathbb{Z}$, with $a, b, c_0 \neq 0$. Consider the Cauchy problem

$$ay' = by^m, \quad y(0) = c_0$$

in the ring $\mathbb{Z}[[x]]$ of formal power series with integer coefficients. That is, we seek a solution of the form

$$y(x) = \sum_{k=0}^{\infty} c_k x^k, \quad c_k \in \mathbb{Z},$$

satisfying the given differential equation.

Theorem. *The existence of a solution depends on the value of m as follows:*

- *If $m = 1$, the equation has no solution in $\mathbb{Z}[[x]]$.*
- *If $m = 2$, the equation has a solution in $\mathbb{Z}[[x]]$ if and only if $b \mid ac_0$.*
- *If $m \geq 3$, the equation has a solution in $\mathbb{Z}[[x]]$ if and only if $db \mid ac_0^{m-1}$, where d is the product of the distinct prime divisors of $m - 1$.*

Corollary. *The equation $y' = y^m$ admits a formal power series solution with integer coefficients for every initial value $y(0) = c_0 \in \mathbb{Z}$ if and only if $m = 2$.*

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DIMENSIONLESS VARIABLES IN FINITE ELEMENT ANALYSIS OF LORD-SHULMAN THERMOPIEZOELECTRICITY TIME-DEPENDENT PROBLEM

The problem of transient analysis in Lord–Shulman thermopiezoelectricity is frequently solved using finite element method complemented with some time integration scheme. In paper [1] such a hybrid time integration scheme is considered. However, as mentioned in that paper, the resulting matrix of the numerical scheme is ill-conditioned to the extent that it is mandatory to perform variables scaling to be able to obtain an acceptable numerical solution.

To resolve the aforementioned issue with ill-conditioned matrix we propose to introduce dimensionless variables that correspond to the original unknowns as it was done in articles [2, 3] (where the authors use Laplace transform method). The nondimensionalization is performed at the level of variational problem statement. The hybrid time integration scheme depicted in [1] is then applied.

The proposed numerical scheme is implemented in Python programming language and shows better practical numerical stability comparing to the one from [1]. The obtained numerical results are in accordance with the ones presented in [3].

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MULTI-POINT PROBLEM FOR DIFFERENTIAL-OPERATOR EQUATIONS

We consider the following multipoint problem with multiple interpolation nodes for partial differential equations

$$\frac{\partial^n u(t,x)}{\partial t^n} + \sum_{j=0}^{n-1} A_j(D_x) \frac{\partial^j u(t,x)}{\partial t^j} = 0, \quad (t,x) \in (0,T) \times \Omega^p, \tag{1}$$

$$\begin{cases} U_{j,q_j}[u] \equiv \left. \dfrac{\partial^{q_j-1} u(t,x)}{\partial t^{q_j-1}} \right|_{t=t_j} = \varphi_{j,q_j}(x), \ q_j = 1, \ldots, r_j, \\ \\ j = 1, \ldots, l, \ 2 \le l \le n, \ r_1 + \ldots + r_l = n, \ 1 \le r_j \le n, \ x \in \Omega^p, \end{cases} \tag{2}$$

where $x = (x_1, \ldots, x_p) \in \Omega^p = (\mathbb{R}/2\pi\mathbb{Z})^p$, $D_x = (-i\partial/\partial x_1, \ldots, -i\partial/\partial x_p)$, $A_j(D_x)$ is a pseudodifferential operator, whose action on a periodic function $\varphi(x) = \sum_{k \in \mathbb{Z}^p} \varphi_k \exp(ik, x)$ is defined by the formula

$$A_j(D_x)\varphi(x) = \sum_{k \in \mathbb{Z}^p} A_j(k) \varphi_k \exp(ik, x), \quad j = 1, \ldots, n,$$

and the sequences $\{A_j(k) : k \in \mathbb{Z}^p\}$ satisfy the following condition

$$\sup_{k \in \mathbb{Z}^p} \left\{ |A_j(k)| / G^{n-j}(k) : j = 0, 1, \ldots, n-1 \right\} < \infty,$$

where $G : \mathbb{Z}^p \to \mathbb{R}_+$ is a positive function such that i) $G(k) \ge 1$ for all $k \in \mathbb{Z}^p$, and ii) $\lim\limits_{|k| \to +\infty} G(k) = +\infty$. Using a metric approach [1], it is shown that the well-posedness conditions of problem (1), (2) (in the function spaces characterized by the growth of their Fourier coefficients via the function G) are satisfied for almost all (with respect to the Lebesgue measure in $\mathbb{R}^l$) vectors $\vec{t} = (t_1, \ldots, t_l) \in [0,T]^l$, formed by the values of the interpolation nodes.

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COMPATIBILITY OF EQUATIONS WHICH DESCRIBE EINSTEIN-MAXWELL PETROV TYPE II SPACE-TIMES IN THE CASE OF ONE-WAY NULL ELECTROMAGNETIC FIELD

We consider Einstein-Maxwell spacetimes of Petrov type II with outgoing one-way null electromagnetic field

$$\begin{aligned}\psi_{ABCD} = \psi_4 o_A o_B o_C o_D - \psi_3 \left(o_A o_B o_C \iota_D + o_A o_B \iota_C o_D + \right.\\ \left. o_A \iota_B o_C o_D + \iota_A o_B o_C o_D\right) + \psi_2 \left(o_A o_B \iota_C \iota_D + o_A \iota_B o_C \iota_D + \right.\\ \left. + o_A \iota_B \iota_C o_D + \iota_A o_B o_C \iota_D \iota_A o_B \iota_C o_D + \iota_A \iota_B o_C o_D\right), \quad (1)\end{aligned}$$

$$\varphi_{AB} = \varphi_2 o_A o_B, \qquad (2)$$

ψ_{ABCD} is the Weyl spinor, φ_{AB} is the Maxwell spinor, ψ_4, ψ_3, ψ_2 are components of the Weyl spinor, φ_2 is a component of the Maxwell spinor, which describe outgoing radiation, o, ι — spin dyad.

We write down the Bianchi identities in the spinor formalism and investigate their compatibility. Second order integrabillity equations are obtained. We established that the Weyl spinor of such Einstein-Maxwell fields is not null.

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ANALYSIS AND PATCH TEST VERIFICATION OF A NONCONFORMING QUADRATIC BASIS FOR THE OCTAHEDRAL FINITE ELEMENT

In [1], the development of a quadratic basis for a finite element (FE) is presented in the form of an octahedron. Our study shows that this basis is C^0 discontinuous (nonconforming), despite its shape functions satisfying the Kronecker-delta property and the partition of unity condition. This occurs because each shape function, while zero on one of two faces non-incident to its node, is non-zero on the other.

To quantitatively assess this deviation, the square of the norm L_2 of each shape function was determined on the corresponding non-incident face. The value obtained is approximately 0.0032 for an element with a characteristic size of $h = 2$.

To verify the element's ability to reproduce fields and ensure FEM convergence when using it, two types of analytical checks were conducted. The first demonstrated the ability of the basis functions to accurately reproduce linear coordinate fields. Through symbolic computations of the corresponding sums, their complete agreement with the prescribed coordinate values was analytically confirmed.

The second check, which is essentially a patch test, was designed to verify the accurate recovery of the gradient for an arbitrary linear field. Analytical calculations of the approximate solution's gradient showed its complete correspondence with the exact solution's gradient.

The successful passing of both tests indicates the basis's first-order polynomial completeness and its suitability for accurately reproducing linear fields. This is a key prerequisite for the successful application of this FE in solving second-order elliptic differential equations, such as the Laplace or Poisson equations, thereby ensuring the convergence of FEM.

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ON TOPOLOGICAL ISOMORPHISM OF SOME COUNTABLY GENERATED ALGEBRAS OF ENTIRE FUNCTIONS ON BANACH SPACES

Let $L_\infty[0,1]$ be the complex Banach space of all complex-valued Lebesgue measurable essentially bounded functions on $[0,1]$. Let $L^2_\infty[0,1]$ be the Cartesian square of $L_\infty[0,1]$.

We consider the Fréchet algebra $H_{bs}(L^2_\infty[0,1])$ of all entire symmetric functions of bounded type on $L^2_\infty[0,1]$. We construct a countably generated Fréchet algebra of entire functions of bounded type on ℓ_∞ which is topologically isomorphic to $H_{bs}(L^2_\infty[0,1])$, where ℓ_∞ is a complex Banach space of all bounded sequences of complex numbers. Let

$$\mathcal{I} = \big(I_{11}, I_{12}, I_{21}, I_{22}, I_{23}, \ldots, I_{n1}, I_{n2}, \ldots, I_{n,n+1}, \ldots\big),$$

where

$$I_{11}(x) = x_1, I_{12}(x) = x_2, I_{21}(x) = x_3^2, I_{22}(x) = x_4^2, I_{23}(x) = x_5^2,$$

$$I_{31}(x) = x_6^3, I_{32}(x) = x_7^3, I_{33}(x) = x_8^3, I_{34}(x) = x_9^3, \ldots$$

for $x = (x_1, x_2, \ldots) \in \ell_\infty$. We denote by $H_{b\mathcal{I}}(\ell_\infty)$ the Fréchet algebra of all entire functions of bounded type on ℓ_∞, generated by the sequence of polynomials $\mathcal{I}$.

We construct a topological isomorphism between the algebras $H_{b\mathcal{I}}(\ell_\infty)$ and $H_{bs}(L^2_\infty[0,1])$. We also establish some general results on countably generated algebras of entire functions of bounded type on a complex Banach space.

Results of the work can be used for investigations of the algebras of symmetric analytic functions on Banach spaces.

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ON THE IMPLEMENTATION OF SOME PARALLEL FILTERING ALGORITHMS ON COMPUTERS WITH SHARED AND DISTRIBUTED MEMORY

Digital filtering is used in applied tasks for processing images, video, signals, etc., where data arrives continuously and requires real-time processing. For this purpose, we proposed a parallel algorithm with limited parallelism, where the number of actually executed threads is specified, and duplication of computations among them is eliminated [1]. In the case of shared memory, this algorithm was implemented in C# using the System and System.Threading libraries. Computations were performed on a computer with multicore Intel Core i5-9600KF processor (6 computing cores) [2]. Although physically the memory is shared, it can be logically divided to study the specifics of parallel computation organization in computers with distributed memory. In this case, particular attention should be paid to the organization of information exchange between computing nodes, which affects overall computational efficiency. One of the approaches used here is the overlapping of communication and computation operations. The correctness of this approach for implementing the proposed limited-parallelism algorithm was verified using a multicore computer.

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SIGMA-DELTA QUANTIZATION AND DIGITAL HALFTONING OF IMAGES

In our work, we consider error diffusion techniques for digital halftoning from the perspective of 1-bit Sigma-Delta quantization. We introduce a method to generate Sigma-Delta schemes for two-dimensional signals as a weighted combination of its one-dimensional counterparts and show that various error diffusion schemes proposed in the literature, can be represented in this framework via Sigma-Delta schemes of 1st order. Under the model of two-dimensional bandlimited signals, which is motivated by a mathematical model of human visual perception, we discuss quantitative error bounds for such weighted Sigma-Delta schemes.

Motivated by the correspondence between existing error diffusion algorithms and first-order Sigma-Delta schemes, we study the performance of the analogous weighted combinations of second-order Sigma-Delta schemes and show that they exhibit superior performance in terms of guaranteed error decay for two-dimensional bandlimited signals. In numerical simulations for real-world images, we demonstrate that with some modifications to enhance stability this high-quality performance also translates to the problem of digital halftoning.

This is joint work with Felix Krahmer from the Technical University of Munich.

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SYMMETRY ASPECTS OF INTEGRABLE NIZHNIK MODELS

The Nizhnik system, which was introduced in [2], has different models that we called Nizhnik models [5]. Depending on whether the product of the parameters of this system is zero or not, we have the symmetric and asymmetric cases. Each of these cases can be considered for dispersion and dispersionless Nizhnik models within the framework of symmetry analysis. Such studies were already carried out in [1, 3, 4] for some dispersionless Nizhnik models.

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BIEM FOR ELASTIC PROBLEM OF TIME-HARMONIC LOADING OF ONE-PERIODIC LATTICE WITH PAIRED COMPLIANT DISK-SHAPED INCLUSIONS

The boundary integral equation method, previously applied to the steady-state interaction of compliant disk-shaped inclusions in a one-periodic array [1], is extended to the case of a time-harmonic loading of one-periodic lattice with paired inhomogeneities of this kind. The identical inclusion pairs are positioned along a single periodic direction with equal distances between adjacent pairs, introducing a one-periodic discontinuity pattern into the elastic matrix. The boundary-integral formulation of the problem is based on the superposition principle, representing the elastic displacement vector as integral sums with displacement discontinuities on each of the scatterers. By enforcing the boundary conditions on the multiply connected domain, in conjunction with the natural symmetry conditions resulting from the periodic arrangement, the problem is reduced to the system of two boundary integral equations. The mode-I dynamic stress intensity factor in the inclusion vicinities depending on the angular coordinate and frequency is calculated and analyzed (following the example of the analysis of periodic cracks [2,3]).

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3. Mykhas'kiv V., Zhbadynskyi I., and Zhang Ch. Elastic wave propagation in metamaterials with non-orthogonal penny-shaped crack lattices, *Int. J. Mech. Sci.*, **302** (2025), 110572.

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INVERSE SCATTERING PROBLEM FOR THIRD ORDER DIFFERENTIAL OPERATORS WITH LOCAL POTENTIAL ON THE WHOLE AXIS

This work is dedicated to the scattering problem for the operator

$$(L_q y)(x) = iy'''(x) + q(x)y(x) \tag{1}$$

on the whole axis, $x \in \mathbb{R}$, where potential $q(x)$ is real and satisfies the condition

$$\int_{-\infty}^{\infty} |q(x)|^2 e^{2a|x|} dx < \infty \tag{2}$$

($a > 0$ is a fixed number).

An important role here is played by the matrix of transition from one system of fundamental solutions $\{u_k(\lambda, x)\}_0^2$ at "$-\infty$" to another system $\{v_k(\lambda, x)\}_0^2$ at "$+\infty$". It is proved that this matrix is J-unitary and unimodular. For the Schrödinger operator, transition from one system of solutions to another is conducted by involution, viz., "taking the complex conjugate", whereas in our case, transition to the dual system takes place as Wronskian is computed (Lie bracket). Presence of three waves at "$-\infty$" and of three waves at "$+\infty$" leads to the scattering problem in which there are two scattering coefficients and one transmission coefficient. It is proved that the numbers corresponding to bound states are situated on rays in a sector, some of which correspond to the positive eigenvalues and others, to the negative eigenvalues. Conditions for "a" from $(0, 2)$ when the number of bound states is finite are found. Using Riemann boundary value problems, problems on jumps on the borders of sectors of Jost solutions holomorphicity are derived. A closed system of singular equations (an analogue of Marchenko equation) for the solution to the inverse problem is found.

The inverse scattering problem is solved. The explicit form of potential is found in terms of solutions to the main equation system. Form of the simplest reflectionless potential (a soliton analogue) is found.

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