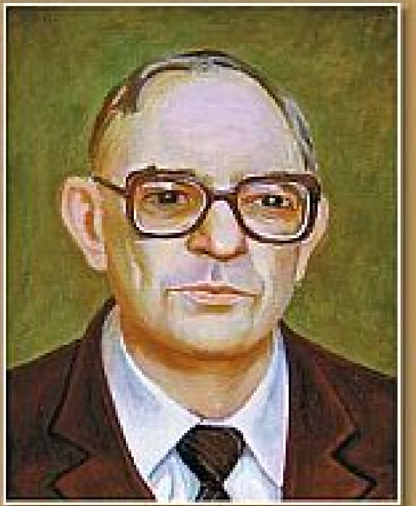

INTERNATIONAL V. SKOROBOHATKO MATHEMATICAL CONFERENCE

(August 25–28, 2015, Drohobych, Ukraine)

ABSTRACTS

НАЦІОНАЛЬНА АКАДЕМІЯ НАУК УКРАЇНИ
ІНСТИТУТ ПРИКЛАДНИХ ПРОБЛЕМ МЕХАНІКИ І МАТЕМАТИКИ
ІМ. Я. С. ПІДСТРИГАЧА
ІНСТИТУТ МАТЕМАТИКИ

КИЇВСЬКИЙ НАЦІОНАЛЬНИЙ УНІВЕРСИТЕТ ІМ. ТАРАСА ШЕВЧЕНКА
ЛЬВІВСЬКИЙ НАЦІОНАЛЬНИЙ УНІВЕРСИТЕТ ІМ. ІВАНА ФРАНКА
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МІЖНАРОДНА МАТЕМАТИЧНА КОНФЕРЕНЦІЯ ІМ. В. Я. СКОРОБОГАТЬКА

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ТЕЗИ ДОПОВІДЕЙ

Львів – 2015

UKRAINIAN NATIONAL ACADEMY OF SCIENCES
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WESTERN SCIENTIFIC CENTER OF NAS AND MES OF UKRAINE

INTERNATIONAL V. SKOROBOHATKO MATHEMATICAL CONFERENCE

(August 25 – 28, 2015, Drohobych, Ukraine)

ABSTRACTS

Lviv – 2015

У збірнику вміщено тези доповідей X Міжнародної математичної конференції ім. В. Скоробогатька. Досліджено властивості розв'язків диференціальних, інтегральних та операторних рівнянь, питання теорії функцій, функціонального аналізу, метричної теорії чисел та обчислювальної математики, описано їх застосування до задач математичної та теоретичної фізики і механіки. Спектр цих досліджень переважно збігається з напрямками наукової діяльності професора Віталія Скоробогатька (1927–1996).

Abstracts of the X International V.Skorobohatko Mathematical Conference are presented. Properties of solutions of the differential, integral and operator equations as well as problems of the function theory, functional analysis, metric number theory and computational mathematics are analyzed. Their applications to problems of mathematical and theoretical physics and mechanics are considered. Range of the present research corresponds mainly to the fields of scientific interests of Prof. Vitaliy Skorobohatko (1927-1996).

Затверджено до друку вченою радою Інституту прикладних проблем механіки і математики ім.Я.С.Підстригача НАН України
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GREEN'S FUNCTION FOR THE THERMOELASTICITY PROBLEM OF A BIMATERIAL BODY

The thermoelastic state of two perfectly contacted half-spaces with point stationary heat source in one of them is investigated in axisymmetric formulation. In the cylindrical coordinate system with the beginning at the interface of materials, expressions for temperature and the potential of thermoelastic displacements in both half-spaces are written. To satisfy the boundary conditions on the phase contact area Love biharmonic functions are determined. Explicit expressions for temperature, displacements and stresses that are relevant Green‘s functions are obtained. These expressions can be used at the determination of thermoselastic state for bimaterial body caused by heating of heat sources distributed along a line, on the surface or on the domain. The plots of axial, radial, circular and tangential stresses on the boundary of the body, depending on the distance of the heat source to this boundary, heat conductivity factors, thermal expansion and modules of elasticity are presented.

Formulas for determination of displacements and stresses in the semi-infinite body are derived by limiting transitions. Boundary of body is load free or rigidly restrained at zero temperature on it, or boundary is insulated.

Further, the obtained results will be used in the process of investigation of the thermoelastic state of the bimaterial body with heat release on a circular domain parallel or perpendicular to his phase contact area.

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ON THE INVERSE PROBLEM FOR A MASS-STRUCTURED POPULATION MODEL

Consider a simple population model (e.g., a model of tumor growth). We assume that during the maturation process individuals (e.g., malignant cells) can accumulate mass, which is measured by a variable x. We make the simplifying assumption that all individuals are created with mass $x = 1$, and their maximum possible mass is $x = 2$ at which they divide into two. At a fixed time t, let $u = u(x,t)$ be the mass density of the population, given in dimensions of individuals per mass. The density evolves according to a conservation law for the number of individuals

$$\frac{\partial u}{\partial t} + \frac{\partial g(x)u}{\partial x} = -m(t)u + f(t), \quad 1 < x < 2,\ t > 0, \tag{1}$$

where $g = g(x)$ gives the mass growth rate, or the rate that mass is accumulated by an individual of mass x, and $m = m(t)$ is the unknown per capita mortality rate, which, in the case of malignant cells, may vary with chemotherapeutic drug concentration.

So equation (1), along with the initial and boundary conditions

$$u(x,0) = u_0(x),\ \ 1 \le x \le 2; \quad u(1,t) = 2u(2,t),\ \ t \ge 0, \tag{2}$$

can serve as a model for the mass-structured population dynamics. The inverse problem is to determine the density $u = u(x,t)$ and the mortality rate $m = m(t)$ subject to (1)-(2) that guarantee the given dynamics $w = w(t)$ of the total mass of individuals (e.g., tumor mass), that is,

$$\int_1^2 xu(x,t)dx = w(t), \quad t \ge 0.$$

It is proved that this inverse problem has a unique solution. The proof uses the method of characteristics and the Banach fixed-point theorem.

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NONLOCAL PROBLEM WITH INTEGRAL CONDITIONS FOR A SYSTEM OF HYPERBOLIC EQUATIONS IN THE CANONICAL FORM

We consider the nonlocal value problem with integral conditions for system of hyperbolic equations in the rectangle $\bar{\Omega} = [0, T] \times [0, \omega]$

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x), \tag{1}$$

$$\int_0^a M(t,x)u(t,x)dx = \psi(t), \qquad t \in [0,T], \tag{2}$$

$$\int_0^b L(t,x)u(t,x)dt = \varphi(x), \qquad x \in [0,\omega], \tag{3}$$

where $u(t,x) = col(u_1(t,x), ..., u_n(t,x))$ is unknown function, the matrices $A(t,x)$, $B(t,x)$, $C(t,x)$ of the size $(n \times n)$, the n-vector function $f(t,x)$ are continuous on $\bar{\Omega}$, the matrices $M(t,x)$, $L(t,x)$ of the size $(n \times n)$ are continuously differentiable with respect to t, x on $\bar{\Omega}$ respectively, the n-vector functions $\varphi(x)$, $\psi(t)$ are continuously differentiable on $[0,\omega]$ and $[0,T]$ respectively, $0 < a \le \omega$, $0 < b \le T$.

The present communication is devoted to questions of existence and uniqueness of the classical solution of the problem (1)-(3) and to approaches to construct the approximate solutions of this problem. The problem (1)-(3), by method of introduction of additional functional parameters [1], is reduced to the Goursat problem for the system of hyperbolic equations with parameters and integral relations. The conditions of the existence of unique classical solutions of the studied problem are obtained in terms of the initial data. An algorithm for construction of approximate solutions is proposed; its convergence is proved.

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SPLITTING METHODS FOR EVOLUTION EQUATIONS, LOCAL ERROR ESTIMATION, AND ADAPTIVITY

For evolution equations (ODEs or PDEs) where the right-hand side splits up into two parts,

$$\begin{aligned} \partial_t\, u(t) &= A(u(t)) + B(u(t)), \\ u(0) &= u_0, \end{aligned}$$

splitting into subproblems often leads to a very efficient approximation algorithm. The simplest cases are the first-order Lie-Trotter scheme, where one step starting from an initial v over a time step h reads

$$v \mapsto \varphi_B(h, \varphi_A(h, v)).$$

Here, $\varphi(t, \cdot)$ and $\varphi(t, \cdot)$ are the subflows associated with the operators A and B. Composition of the Lie-Trotter scheme with its adjoint gives the well known second-oder Strang–Marchuk scheme, and a large variety of higher-order multi-composition schemes have been devised in the literature.

In this talk we address the question of finding coefficients for efficient and accurate higher-order schemes, and the construction of local error estimators for the purpose of adaptive step size control. Numerical results are presented for Schrödinger equations and nonlinear wave equations. The case of splitting into three operators is also considered.

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2. Auzinger W., Koch O., Thalhammer, M. Defect-based local error estimators for splitting methods involving three linear operators, *Numer. Algor.*, published online: 06.11.2014

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EXISTENCE OF THE PERIODIC AND HETEROCLINIC TRAVELLING WAVES FOR THE DISCRETE SINE-GORDON EQUATION ON 2D-LATTICE

We study the discrete sine-Gordon equation on the two-dimensional lattice:

$$\ddot{q}_{n,m} = U'(q_{n+1,m} - q_{n,m}) - U'(q_{n,m} - q_{n-1,m}) +$$
$$+U'(q_{n,m+1} - q_{n,m}) - U'(q_{n,m} - q_{n,m-1}) + K\sin(q_{n,m}), (n,m) \in \mathbb{Z}^2,$$

where $q_{n,m} = q_{n,m}(t)$ is a coordinate of (n,m)-th particle at time t, U is the potential of a neighbor interaction, $K \neq 0$.

Travelling wave is a solution of the form

$$q_{n,m}(t) = u(n\cos\varphi + m\sin\varphi - ct),$$

where the function $u(s), s = n\cos\varphi + m\sin\varphi - ct$, is called the profile function, or simply profile, of the wave and the constant $c \neq 0$ speed of the wave. Making use the travelling wave we obtain the equation

$$c^2u(s) = U'(u(s+\cos\varphi) - u(s)) - U'(u(s) - u(s-\cos\varphi)) +$$
$$+U'(u(s+\sin\varphi) - u(s)) - U'(u(s) - u(s-\sin\varphi)) + K\sin(u(s))$$

for the profile function $u(s)$. This equation has, actually, a variational structure.

We obtain, by means of the critical points method and mountain pass theorem, a result on the existence of nonconstant travelling waves with the periodic condition:

$$u(s+2k) = u(s),\ s \in \mathbb{R},\ k > 0.$$

Moreover, by means of the critical points method and the concentration-compactness principle, we obtain existence of travelling waves with heteroclinic boundary conditions:

$$u(-\infty) = -\pi,\ u(+\infty) = \pi$$

and the quadratic potential $U(r) = c_0^2 r^2/2$ (the linear interaction case).

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SOME TWIN CONVERGENCE REGIONS FOR BRANCHED CONTINUED FRACTIONS OF A SPECIAL FORM

The some Thron's [1] theorems on twin convergence regions for continued fractions $b_0+K(1/b_n)$ are generalized for branched continued fractions of special form

$$b_0 + \mathop{\mathrm{D}}_{k=1}^{\infty} \sum_{i_k=1}^{i_{k-1}} \frac{1}{b_{i(k)}},$$

where $b_0, b_{i(k)}$ are complex numbers, $i(k) = i_1 i_2 \dots i_k$, $1 \le i_s \le i_{s-1}$, $s = \overline{1,k}$, $k \ge 1$, $i_0 = N$, $N \in \mathbb{N}$.

1. Thron W. Twin convergence regions for continued fractions $b_0 + K(1/b_n)$, *American Journal of Mathematics*, **66** (1944), 428–438.

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SPECTRAL PROPERTIES OF THE MULTIPOINT PROBLEM FOR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH AN INVOLUTION

We investigate the spectral properties of the multipoint problem for functional differential equations with an involution $Iy(x) \equiv y(1-x)$

$$Ly \equiv (-1)^n y^{(2n)}(x) + \sum_{j=0}^{2n-1} a_j(x, I) y^{(j)}(x) = f(x),\ x \in (0,1),$$

$$\ell_s y \equiv y^{(m_s)}(0) + b_s y^{(m_s)}(1) + \sum_{q=0}^{m_s} \sum_{r=1}^{k} b_{s,q,r}(I) y^{(q)}(x_r) = 0, \quad s = 1, 2, \ldots, 2n,$$

where $b_s \in \{-1, 1\},\ x_r \in [0,1],\ m_1 \geq m_2 \geq \ldots \geq m_{2n} \geq 0,\ m_{p+2} > m_p,$ $s = 1, 2, \ldots, 2n,\ p = 1, 2, \ldots, 2n-2,\ r = 1, 2, \ldots, k.$

The conditions for operators $a_j(x, I), \quad b_{s,q,r}(I) : L_2(0,1) \longrightarrow L_2(0,1)$ $x \in (0,1)$ and numbers b_s are established when the system of eigenfunctions and associated functions for problems is completed in space $L_2(0,1)$ and its own values are matched with the eigenvalues of the boundary value problem

$$L_0 y \equiv (-1)^n y^{(2n)}(x) = f(x), \quad x \in (0,1),$$

$$\ell_{0,s} y \equiv y^{m_s}(0) + b_s y^{m_s}(1) = 0, \quad s = 1, 2, \ldots, 2n.$$

The conditions for parameters $b_s \in \{-1, 1\}, s = 1, 2, \ldots, 2n,$ are shown, under which this boundary value problem is selfadjoint.

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ON THE NUMBER OF POLYNOMIALS WITH SMALL DERIVATIVES AT A ROOT

Many problems in Diophantine approximation and theory of transcendental numbers are formulated in terms of real, complex or p-adic number sets satisfying the following inequalities:

$$|P(x)| < H(P)^{-w_1}, \quad |P(z)| < H(P)^{-w_2}, \quad |P(\omega)| < H(P)^{-w_3}, \qquad (1)$$

where $w_i > 0$, $x \in \mathbb{R}$, $z \in \mathbb{C}$, $\omega \in \mathbb{Q}_p$, for infinitely many polynomials from some class $\mathcal{P} \subset \mathbb{Z}[x]$. Complexity of these sets motivates the search for their best possible approximations by combinations of simpler sets (real intervals, complex circles or p-adic cylinders).

Let $I = (a, b)$ be a fixed bounded real interval, and let $x \in I$. Consider an ordering of the roots $\alpha_1, \alpha_2, ..., \alpha_n$ of P such that $|x - \alpha_1| \leq |x - \alpha_2| \leq ... \leq |x - \alpha_n|$. Let $Q > Q_0$, where Q_0 is a sufficiently large number, and let $c_1, c_2, ...$ be positive constants which only depend on n. We are going to consider the polynomials lying in the set $\mathcal{P}_n(Q) = \{P \in \mathbb{Z}[x] : H(P) \leq Q, \deg P \leq n\}$.

Solutions of the first inequality of (1) lie in intervals of the form $|x - \alpha_1| < 2^{n-1}|P(x)||P'(\alpha_1)|^{-1}$ which can be quite large for small values of $|P'(\alpha_1)|$. A natural solution to this problem was proposed by R. Baker [1]. For $v \geq 0$ let us define the set $\mathcal{P}_n(Q, v) = \{P \in \mathcal{P}_n(Q) : \exists x \in I, |P(\alpha_1)| < Q^{1-v}\}$. Baker proved that $\#\mathcal{P}_n(Q, v) < c_1 Q^{n+1-\min(v,1)}$ which allowed him to find the exact Hausdorff dimension of the set of real numbers satisfying the first inequality of (1) for infinitely many $P \in \mathbb{Z}[x]$, $\deg P = n$, $w_1 > n$.

We have extended Baker's results by proving the following theorems.

Theorem 1. *If $w_1 > 2$ and $v \leq 3/2$, then $\#\mathcal{P}_n(Q, v) < c_2 Q^{n+1-v}$.*

Theorem 2. *If $v \leq (n+1)/3$, then $\#\mathcal{P}_n(Q, v) > c_3 Q^{n+1-2v}$.*

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MAGNETOHYDRODYNAMIC SIMULATIONS OF RADIATIVE SHOCKS

Evolution of the adiabatic supernova remnants (SNRs) is well described by the Sedov analytical solutions for the strong point explosion. Due to expansion of SNR, the speed and the temperature of the shock wave decreases and the adiabatic conditions are violated because the radiative energy losses increase considerably. After some time, SNRs enters into the radiative stage. The time of the transition from adiabatic to radiative stage is not small and may last as long as the duration of the adiabatic stage. The period of time between the end of the adiabatic and the beginning of the radiative stage is called the post-adiabatic stage.

Our goal is to model the role of magnetic field during this phase. Mathematically, the problem is described by the system of nonstationary equations of magnetohydrodynamics (MHD) with radiative losses.

We have used MHD code PLUTO [1] to solve the system of MHD equations numerically. The numerical integration is performed using the finite volumes approach where the cell averaged values evolve in time. This approach allows one to solve the MHD equation in integral form, so the numerical algorithms can correctly represent MHD flows with strong gradients and discontinuities that are present in our problem.

The influence of value of the magnetic field strength and different orientation of magnetic field (perpendicular and parallel to the shock normal) on the evolution of SNR in post-adiabatic stage is analyzed. We show that the parallel magnetic field does not effect on distribution of hydrodynamic parameters, but the presence of the perpendicular field leads to the significant decrease of the gas compression factor that becomes more apparent for the higher magnetic field values.

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AVERAGING METHOD IN THE PROBLEM OF STRING OSCILATION UNDER THE INFLUENCE OF MULTI-FREQUENCY PERTURBATIONS

We consider the system of equations in the form

$$\frac{\partial^2 u}{\partial \tau^2} = c^2 \frac{\partial^2 u}{\partial x^2} + b(x,\tau)u + f(x,\tau,u_\Lambda,a_\Lambda,\varphi_\Lambda,\varepsilon), \tag{1}$$

$$\frac{da}{d\tau} = A(\tau,a_\Lambda,\varphi_\Lambda,\varepsilon), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + B(\tau,a_\Lambda,\varphi_\Lambda,\varepsilon), \tag{2}$$

where $\tau \in [0,L]$, $\varepsilon \in (0,\varepsilon_0$ is a small parameter, $a \in D \subset \mathbb{R}^n$, $\varphi \in T^m$, $m \geq 1$, $\Lambda = (\lambda_1,\dots,\lambda_r)$, $0 < \lambda_1 < \dots < \lambda_r \leq 1$, $a_\Lambda(\tau) = (a(\lambda_1\tau),\dots,a(\lambda_r\tau)))$.

The solution of the system (1)-(2) satisfies the initial conditions

$$u(x,0) = v(x), \frac{\partial u(x,0)}{\partial t} = w(x), x \in \mathbb{R},$$
$$a(0,\varepsilon) = y \in D, \varepsilon(0,\varepsilon) = \psi \in \mathbb{R}^m. \tag{3}$$

The case, when the function f does not depend on u, was analysed in [1]. The averaging method for the system (2) is proved in [2].

The system of equations averaged by the rapid variables is obtained according to (1). Existence of a solution of the problem (1)-(3) is proved, the averaging method is justified and the estimate is received in the form

$$|u(x,\tau,\varepsilon) - \overline{u}(x,\tau,\varepsilon)| \leq c\varepsilon^\alpha, \alpha = (mr)^{-1}, x \in \mathbb{R}, \tau \in [0,L].$$

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THE CONSTRUCT OF DIFFUSION PROCESS IN HETEROGENEOUS ENVIRONMENT

We consider the problem of construction of a limited in $D_2^+ = \{(t,r) : t \in (0,\infty); r \in I_2^+ = (0;R_1) \cup (R_1;R_2) \cup (R_2;\infty)\}$ solution of the separative system of differential equations of the parabolic type conductivity:

$$\frac{\partial u_1}{\partial t} + \gamma_1^2 u_1 - a_1^2 \Lambda_{(\mu)}[u_1] = f_1(t,r), \quad r \in (0,R_1),$$

$$\frac{\partial u_2}{\partial t} + \gamma_1^2 u_2 - a_2^2 \frac{\partial^2 u_2}{\partial r^2} = f_2(t,r), \quad r \in (R_1,R_2),$$

$$\frac{\partial u_3}{\partial t} + \gamma_3^2 u_3 - a_3^2 B_{\nu,\alpha}[u_3] = f_3(t,r), \quad r \in (R_2,R_3), \tag{1}$$

with initial conditions

$$u_j(t,x)|_{t=0} = g_j(r), \quad r \in (R_{j-1};R_j), j = 1,2,3, R_0 = 0, \tag{2}$$

and conjugation conditions

$$(L_{j1}^k[u_k(t,r)] - L_{j2}^k[u_{k+1}(t,r)])|_{r=R_k} = \omega_{jk}(t), j,k = 1,2. \tag{3}$$

There are Legandre differential operators $\Lambda_{(\mu)} = \frac{\partial^2}{\partial r^2} + \operatorname{cth} r \frac{\partial}{\partial r} + \frac{1}{4} + \frac{1}{2}\Big(\frac{\mu_1^2}{1-\operatorname{ch} r} + \frac{\mu_2^2}{1+\operatorname{ch} r}\Big)$, Bessel $B_{\nu,\alpha} = \frac{d^2}{dr^2} + (2\alpha+1)r\frac{d}{dr} - (\nu^2-\alpha^2)r^{-2}$, Fourier $\frac{d^2}{dr^2}$ and generalized differential operators of conjugations $L_{jm}^k = \Big(\alpha_{jm}^k + \delta_{jm}^k \frac{\partial}{\partial t}\Big)\frac{\partial}{\partial r} + \beta_{jm}^k + \gamma_{jn}^k \frac{\partial}{\partial t}$, $j,m,k = 1,2$ are involved in equalities.

We obtain the solution of problem (1)-(3) by the Laplace method of the integral transformation on t in the assumption that the desired and specified functions are images on the Laplace method of integral transformation on t.

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MANY-DIMENSIONAL GENERALIZATION OF CONTINUED FRACTIONS

At first, a Japanese mathematician Katahiro Takebe (1664 - 1739) generalized a continued fraction and used this generalization to solve Diophantine inequalities. Lagrange at his time proved that each quadratic irrationality can be expanded into a regular periodic continued fraction. The problem of regular many-dimensional continued fraction constructions into which any algebraic irrationality of the higher order (> 2) is open till now. In this direction one of the most successful and investigated generalizations of continued fractions is the Jacobi-Perron algorithm (a first attempt to find a tree-dimensional regular continued fraction analogue). In 1919 a Norwegian mathematician Viggo Brun generalized Euclid's algorithm, used for the construction of a real number expansion into a regular continued fraction. He used the interpretation of the Euclid algorithm as the algorithm of "differences". Considering Euclid's algorithm as the algorithm of "quotients" E. Podsypanin constructed the algorithm, generally speaking, equivalent to Brun's algorithm.

In 1966 V. Skorobohatko proposed the many-dimensional generalization of continued fractions for functions of many variables, the branched continued fraction (BCF). Later the different special types of BCFs were introduced and investigated, namely two-dimensional continued fractions, BCFs with independent variables, many-dimensional g-fractions [1 - 3].

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JOINT SOLUTION OF THE POISSON AND SCHRÖDINGER EQUATIONS FOR ELECTRON IN NANOHETEROSTRUCTURE

Physics of semiconductor heterostructures is one of the most important parts of the modern solid state physics. It studies the energy spectrum, effective masses, mobilities of quasiparticles, optical and electrical properties in thin films, quantum wires, quantum dots and other low-dimensional systems. Various aspect of the effect of the crystal foundation properties on heterostructures electron and hole states are studied in various investigations. If a semiconductor surface is adjacent to some medium with a lower or higher dielectric permittivity, the presence of the image forces results in an appearance of the presurface region, which is repelling or attracting the charges. Generally the authors of the works considered the heterosystems in witch a dielectric permittivity changes atruptly from one value to the other through the separation boundary. In reality, there always exists a transition layer with the dielectric permittivity changing continuously between the two values corresponding to adjacent semiconductor or dielectric component of the heterostructure. We investigate the effect of the separation boundary with adjacent thin intermediate layer, where dielectric permittivity is a function of a coordinate, on the energy of charged particles in the heterostructure with spherical semiconductor nanocrystal. The functional dependence of the charge potential energy upon the distance is obtained using the Green functions method for solution of the Poisson equation. The electron energy was obtained as eigenvalue of the Hamilton operator. We obtained functional dependence energy of the charge particle from the magnitude intermediate layer, ratio dielectric permittivities of the quantum dot and matrix.

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METHOD OF AVERAGING IN THE DISCRETE OPTIMAL CONTROL PROBLEM WITH THE FAST AND SLOW VARIABLES

The optimal control problem is described by a system of discrete equations with the fast and slow variables and the terminal quality criterion

$$x_{i+1} = x_i + \varepsilon \cdot X\left(i, x_i, y_i, u_i, \varepsilon\right), \qquad x_0 = x^0,$$

$$y_{i+1} = Y\left(i, x_i, y_i, \varepsilon\right), \qquad y_0 = y^0,$$

$$J\left(u\right) = \Phi\left(x_N\right) \to \min_{u \in U}.$$

Here $x_i \in D_x \subset R^n$ are the slow variables, $y_i \in D_y \subset R^m$ are the fast variables, $i \in I = \{0, 1, 2, ..., N\}$ is time of the system, $N = E\left(L\varepsilon^{-1}\right)$, $L = const$, $E\left(c\right)$ is the integer part of c, $\varepsilon > 0$ is a small parameter, $u_i \in U \subset comp\left(R^r\right)$ is a control of the compact subset U.

Suppose that the function

$$X_0\left(x, u\right) = \lim_{h \to \infty} \frac{1}{h} \sum_{i=s}^{s+h-1} X\left(i, x, y\left(i, x, y^0, 0\right), u, 0\right)$$

exists evenly on the $s \geq 0$, $x \in D_x$, $u \in U$, $y^0 \in D_y$ along the solution of the degenerate problem.

Then the averaged optimal control problem is described by a system of discrete equations and terminal quality criterion

$$z_{i+1} = z_i + \varepsilon \cdot X_0\left(z_i, v_i\right), \qquad z_0 = x^0,$$

$$J_0\left(v\right) = \Phi\left(z_N\right) \to \min_{v \in U}.$$

It is proved that the optimal control of the averaged problem is the asymptotically optimal control of the original problem.

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BOUNDARY VALUE PROBLEMS FOR ANISOTROPIC ELLIPTIC-PARABOLIC EQUATIONS WITH VARIABLE EXPONENTS OF NONLINEARITY

Let $\Omega \subset \mathbb{R}^n$ be a (bounded or unbounded) domain with the piecewise smooth boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1$ $(\Gamma_0 \cap \Gamma_1 = \emptyset)$, $\nu = (\nu_1, \ldots, \nu_n)$ be a unit outward pointing normal vector on the $\partial\Omega$, and S be either $(0, T]$, or $(-\infty, 0]$, or $(-\infty, +\infty)$. Put $Q := \Omega \times S$, $\Sigma_0 := \Gamma_0 \times S$, $\Sigma_1 := \Gamma_1 \times S$.

A partial case of considered equations is

$$(b(x)u)_t - \sum_{i=1}^{n} \Big(a_i(x,t)|u_{x_i}|^{p_i(x)-2} u_{x_i} \Big)_{x_i} + a_0(x,t)|u|^{p_0(x)-2}u = f(x,t),$$

$(x,t) \in Q$, where $a_j \in L_\infty(Q)\,(j = \overline{0,n})$ are positive, $b \in L_\infty(\Omega)$ is nonnegative function, while there exists open set $\Omega_0 \subset \Omega$ such that $b(x) > 0$ for a.e. $x \in \Omega_0$, and $b(x) = 0$ for a.e. $x \in \Omega \setminus \Omega_0$, $p_j \in L_\infty(\Omega)$, $\operatorname{ess\,inf}_{x\in\Omega} p_j(x) > 1\,(j = \overline{0,n})$, and f is an integrable function. It is clear, that the equation is parabolic in $\Omega_0 \times S$ and elliptic in $(\Omega \setminus \Omega_0) \times S$.

We consider the following problem for this equation: to find a function $u : \overline{Q} \to \mathbb{R}$, which satisfies (in some sense) the equation, and the boundary conditions

$$u\Big|_{\Sigma_0} = 0, \qquad \sum_{i=1}^{n} |u_{x_i}|^{p_i(x)-2} u_{x_i} \nu_i \Big|_{\Sigma_1} = 0,$$

and, if $S = (0, T]$, initial condition $u(x, 0) = u_0(x)$ in Ω_0, where $u_0 : \Omega \to \mathbb{R}$ are given real-valued function.

In the cases $S = (-\infty, 0]$ and $S = (-\infty, +\infty)$ there are no initial condition, but there may be some additional conditions on behavior of solution as $t \to -\infty$.

We study the existence and uniqueness of a weak solutions for considered problems from generalized Sobolev spaces as well as its continuous dependence on the input data.

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THE ANGLE REGIONS OF CONVERGENCE FOR 1-PERIODIC BRANCHED CONTINUED FRACTION OF THE SPECIAL FORM

We consider 1-periodic branched continued fractions (BCF) of the special form

$$\left(1 + \underset{k=1}{\overset{\infty}{\mathrm{D}}} \sum_{i_k=1}^{i_{k-1}} \frac{c_{i_k}}{1}\right)^{-1}, \tag{1}$$

where $i_0 = N$; $c_j \in \mathbb{C}$, $j = \overline{1, N}$.

Theorem. *Let elements c_j, $j = \overline{1, N}$, of the fraction (1) satisfy next conditions:*

$$-\pi/2 \leq \arg c_1 \leq \ldots \leq \arg c_N \leq 0$$

or

$$0 \leq \arg c_N \leq \ldots \leq \arg c_1 \leq \pi/2.$$

Then

1. *The BCF (1) converges.*
2. *The truncation error bounds hold:*

$$|F_{n+1} - F_n| \leq C \binom{N}{N+n-1} \xi^n,$$

where $n \geq N+1$; $C = \max\limits_{j=\overline{1,N}} \{|c_j|\}$; $\xi = \dfrac{C}{\sqrt{1+C^2}}$; F_n is the n-th approximant of BCF (1).

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NEUMANN PROBLEM IN A DISK FOR FOURTH-ORDER IMPROPERLY ELLIPTIC EQUATIONS

This paper is devoted to the existence of a solution of the Neumann problem in a disk K for fourth-order improperly elliptic differential equations of general form. We propose to carry over the methods of investigations of Babayan [1] and Buryachenko, which were applied for studying the Dirichlet problem, to the Neumann problem. We succeeded in performing this and, as a result, have obtained the sufficient conditions of existence of the classical solution for the Neumann problem in the space $C^4(K) \cup C^{3,\alpha}(\bar{K}), \alpha \in (0, 1)$.

We consider the Neumann problem in a disk for fourth-order improperly elliptic equations with constant complex coefficients:

$$L(\partial_x)u = a_0 \frac{\partial^4 u}{\partial x_1^4} + a_1 \frac{\partial^4 u}{\partial x_1^3 \partial x_2} + a_2 \frac{\partial^4 u}{\partial x_1^2 \partial x_2^2} + a_3 \frac{\partial^4 u}{\partial x_1 \partial x_2^3} + a_4 \frac{\partial^4 u}{\partial x_2^4} = 0, \quad (1)$$

$$u''_{\nu\nu} \mid_{\partial K} = f_1(x), \; u'''_{\nu\nu\nu} \mid_{\partial K} = f_2(x). \quad (2)$$

Here $\vec{\nu}$ is a unit vector of the outer normal, $\partial_x = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})$, $a_k \in \mathbb{C}$, $k = 0, 1, ..., 4$, defined on the boundary ∂K functions $f_1(x) \in C^{1,\alpha}(\partial K)$, $f_2(x) \in C^{\alpha}(\partial K)$, $0 < \alpha < 1$, can be extended to the analytic functions in K and out of K.

Let λ_j, $j = 1, ..., 4$ be complex roots of the equation $L(1, \lambda) = a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$.

We have received sufficient conditions of existence of classical solution of the problem (1), (2) for two classes of the improperly elliptic equations such that

$$\Im\lambda_1 > 0, Im\lambda_2 > 0, \, \Im\lambda_3 > 0, \, \Im\lambda_4 < 0,$$

$$\Im\lambda_1 > 0, \, \Im\lambda_2 > 0, \, \Im\lambda_3 > 0, \, \Im\lambda_4 > 0.$$

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NUMERICAL APPROACH TO SIMULATION OF MECHANICAL BEHAVIOR OF SOLIDS SUBJECTED TO INTENSE TEMPERATURE AND FORCE LOADINGS

A mathematical model for numerical description of thermo-mechanical processes in solids subjected to intense force and temperature loadings was proposed. The model takes into account a temperature dependence of material properties, a heat radiation and an elastic-plastic deformation. Temperature evolution in the solid is governed by the non-stationary heat transfer equation. A non-isothermal theory of thermo-elastic-plastic deformation is used to predict a stress-strain state of the solid.

A corresponding numerical model is developed for description the thermal and elastic-plastic processes in solids subjected to temperature and force loadings. The model is grounded on the finite elements method. The key relations for determination of temperature and mechanical fields in solid are obtained by using weighted residual method. As a result of the standard procedure of finite-element discretization, the problem of heat transfer is reduced to a system of ordinary differential equations for the unknown temperature values at the nodes of the finite element mesh. This problem is solved by use of a unified set of single step algorithms, what allows to carry out calculations for variable steps and orders of the method. Dependencies of deformation curves and of other physical and mechanical material characteristics from temperature are approximated by interpolation splines constructed by the points of experimentally known curves describing the behavior of a solid during the entire temperature loading range.

Appropriate software was developed. It was used for computer simulation a thermo-mechanical behavior of building structures in fire.

A comparative analysis of the computational results for some structures during the fire and after that is provided. It is shown that disregard of elastic-plastic deformation of structures and of the temperature dependence of the material properties at elevated temperatures can lead to significant deviations parameters from the actual ones.

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ELLIPTIC PROBLEMS WITH ADDITIONAL UNKNOWN FUNCTIONS IN BOUNDARY CONDITIONS AND HÖRMANDER SPACES

We discuss applications of isotropic Hörmander inner product spaces to a class of elliptic problems with additional unknown functions in boundary conditions. This class was introduced by B. Lawruk in 1963. It is closed with respect to the transition to a formally adjoint problem, in contrast to the family of usual elliptic boundary-value problems.

We investigate properties of this class in the Hörmander spaces $H^{s,\varphi}$ parametrized with a number $s \in \mathbb{R}$ and a function $\varphi : [1, \infty) \to (0, \infty)$ varying slowly at ∞ in the sense of J. Karamata. By definition, the Hilbert space $H^{s,\varphi}(\mathbb{R}^n)$ consists of all tempered distributions $w \in \mathcal{S}'(\mathbb{R}^n)$ such that $\langle\xi\rangle^s \varphi(\langle\xi\rangle)\mathcal{F}w(\xi) \in L_2(\mathbb{R}^n, d\xi)$. Here $\langle\xi\rangle := (1 + |\xi|^2)^{1/2}$ and $\mathcal{F}w$ is the Fourier transform of w. Analogs of this space for Euclidean domains and smooth manifolds are introduced in the standard way.

We prove that the elliptic problems under consideration are Fredholm in appropriate pairs of Hörmander spaces and induce isomorphisms between some subspaces of finite co-dimension. For the generalized solutions to these problems, we prove a priory estimates and theorems on the global and local regularity in Hörmander spaces. We also find sufficient conditions under which the generalized solutions are classical.

These results are obtained together with A. Murach [1-3].

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APPROXIMATION SCHEMES FOR DIFFERENTIAL-FUNCTIONAL EQUATIONS AND THEIR APPLICATIONS

An approximation algorithm for differential equations with delay by a sequence of systems of ordinary differential equations has been considered in [1]. Further investigation of approximation schemes for systems of the delayed and neutral types differential equations and quasilinear differential-functional equations in different function spaces has been implemented in our papers [2, 3, 4]. The accuracy of the approximation of the solutions of the initial problems for the differential-functional equations by the solutions of the Cauchy problems for the corresponding approximating system of the ordinary differential equations has been investigated.

We offer constructive algorithms for computing the non-asymptotic roots of quasipolynomials and obtain an effective algorithm for the stability analysis of the linear differential equations with delay. As a result, the coefficient stability domains of the linear differential equations with many delays are constructed.

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A SEMIRING IN THE SPECTRUM OF THE ALGEBRA OF ANALYTIC FUNCTIONS OF BOUNDED TYPE ON A BANACH SPACE

Study of spectra of algebras of analytic functions on Banach spaces is a new direction which began to actively develop at the end of the twentieth century. In particular, the spectrum of the algebra of all symmetric analytic functions on Banach spaces that are bounded on bounded sets was studied in [1,2]. In the talk the problem of the extension of complex homomorphisms of the semiring in the spectrum of the algebra of symmetric analytic functions to the corresponding ring is considered.

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ASYMPTOTIC REPRESENTATIONS OF SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS WITH RAPIDLY VARYING NONLINEARITIES

The following differential equation is considered:

$$y'' = \alpha_0 p(t)\varphi(y), \tag{1}$$

where $\alpha_0 \in \{-1, 1\}$, $p : [a, \omega[\longrightarrow]0, +\infty[$ is a continuous function, $-\infty < a < \omega \leq +\infty$, $\varphi : \Delta_{Y_0} \longrightarrow]0, +\infty[$ is the twice continuously differentiable function satisfying the conditions

$$\varphi'(y) \neq 0 \quad \text{if} \quad y \in \Delta_{Y_0}, \qquad \lim_{\substack{y \to Y_0 \\ y \in \Delta_{Y_0}}} \varphi(y) = \begin{cases} \text{or} & 0, \\ \text{or} & +\infty, \end{cases}$$

$$\lim_{\substack{y \to Y_0 \\ y \in \Delta_{Y_0}}} \frac{[\varphi'(y)]^2}{\varphi''(y)\varphi(y)} = 1,$$

Y_0 is either 0 or $\pm\infty$, Δ_{Y_0} is the one-sided neighborhood of Y_0.

Definition. *Solution y of the differential equation (1), that is defined on $[t_0, \omega[\subset \Delta_{Y_0}$, is called the $P_\omega(Y_0, \lambda_0)$-solution, where $-\infty \leq \lambda_0 \leq +\infty$, if it satisfies the conditions*

$$y(t) \in \Delta_{Y_0} \quad \textit{if} \quad t \in [t_0, \omega[, \quad \lim_{t \uparrow \omega} y(t) = Y_0,$$

$$\lim_{t \uparrow \omega} y'(t) = \begin{cases} \textit{or} & 0, \\ \textit{or} & \pm\infty, \end{cases} \qquad \lim_{t \uparrow \omega} \frac{[y'^2(t)]^2}{y''(t)y(t)} = \lambda_0.$$

In this report, the conditions of existence of the $P_\omega(Y_0, \lambda_0)$-solutions of the equation (1) and the asymptotic as $t \uparrow \omega$ representations of these solutions are established.

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A SYSTEM OF THERMAL CONDUCTIVITY EQUATIONS FOR A CONICAL SHELL

Let us consider the axisymmetric truncated conical shell with thickness $2h$ and the angle at the vertex $\pi - \alpha$. The heat exchange with the environment through the facial and the face surfaces is performed by the Newton law.

Using approach [1,2] based on the linear distribution of temperature of conical shell in thickness $t = T_1(x) + \frac{z}{h} T_2(x)$, the system of two differential equations is obtained:

$$\frac{d^2 T_1}{dx^2} + \frac{1}{2\theta} \ln \frac{x+\theta}{x-\theta} \frac{\partial T_1}{\partial x} + \left(\frac{1}{\theta} - \frac{x}{2\theta^2} \ln \frac{x+\theta}{x-\theta} \right) \frac{\partial T_2}{\partial x} -$$

$$-\mu_1 (T_1 - t_1) - \mu_2 (T_2 - t_2) + \frac{1}{2} T_2 \ln \frac{x+\theta}{x-\theta} = 0;$$

$$\frac{d^2 T_2}{dx^2} + 3 \left(\frac{1}{\theta} - \frac{x}{2\theta^2} \ln \frac{x+\theta}{x-\theta} \right) \frac{dT_1}{dx} + 3 \left(-\frac{x}{\theta^2} + \frac{x^2}{\theta^3} \ln \frac{x+\theta}{x-\theta} \right) \frac{dT_2}{dx} -$$

$$-3\mu_1 (T_2 - t_2) - 3\mu_2 (T_1 - t_1) - \frac{3}{2} \frac{x}{\theta} T_2 \ln \frac{x+\theta}{x-\theta} = 0.$$

Here $\theta = \tan \alpha, x = x_1 h$, and x_1 changes along the middle surface generator, $\mu_1(x), \mu(x)_2, t_1(x), t_2(x)$ are defined through the heat exchange coefficients and the temperature of environment on the face surfaces:

$$\mu_1 = \frac{h}{2} (\mu^+ + \mu^-); \ \mu_2 = \frac{h}{2} (\mu^+ - \mu^-); \ t_1 = \frac{1}{2} (t^+ + t^-); \ t_2 = \frac{1}{2} (t^+ - t^-).$$

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THE MINIMUM MODULUS OF ANALYTIC FUNCTIONS IN THE UNIT DISK

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. For an analytic function f on $\mathbb{D}$, we define the minimum modulus $\mu(r, f) = \min\{|f(z)| : |z| = r\}$, $0 < r < 1$, and ($f \not\equiv 0$) for $p \geq 1$ we define

$$m_p(r, f) = \left(\frac{1}{2\pi}\int_0^{2\pi} |\log|f(re^{i\theta})||^p d\theta\right)^{\frac{1}{p}}, \quad 0 < r < 1.$$

We write

$$\rho_p[f] = \limsup_{r \nearrow 1} \frac{\log m_p(r, f)}{-\log(1-r)}.$$

We define the order $\rho_\infty[f]$ of the function f as

$$\rho_\infty[f] = \lim_{p \to +\infty} \rho_p[f].$$

For a measurable set $E \subset [0, 1)$, the upper density of E is defined by $D_1(E) = \limsup_{r \nearrow 1} \frac{\lambda_1(E \cap [r,1))}{1-r}$ where $\lambda_1(E \cap [r, 1))$ denotes the Lebesgue measure of $E \cap [r, 1)$.

Theorem. *Let f be analytic in $\mathbb{D}$, $\rho_\infty[f] = \rho$, $\rho < +\infty$. Then for arbitrary $\epsilon > 0$, there exists $C \in (0, 1)$, and a set $F \subset [0, 1)$, such that*

$$\log \mu(r, f) \geq -\frac{1}{(1-r)^{\rho+\epsilon}} \tag{1}$$

$r \in [0, 1) \setminus F$, $D_1(F) \leq C$.

The ε in (1) could not be omitted [2].

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GENERALIZATION OF THE WIMAN-VALIRON METHOD FOR FRACTIONAL DERIVATIVES OF ENTIRE FUNCTIONS

In contrast to the ordinary differential equations, the analytic theory of the fractional differential equations with variable coefficients was initiated only recently [1, 2]. Such equations are widely used for modeling of the diffusion phenomena and anomalous relaxation.

The Wiman-Valiron method has been successfully applied to obtain an estimate for growth of solutions of the ordinary differential equations in the complex plane [3]. We generalize this method for fractional derivatives.

Given an entire function $f(re^{i\theta})$ and $q > 0$, let $D^q f(re^{i\theta})$ be the Riemann-Liouville fractional derivative of order q of f with respect to r in the domain $\mathbb{C} \setminus \mathbb{R}_-$. For an appropriate choice of the branch z^q in $\mathbb{C} \setminus \mathbb{R}_-$ the function $z^q D^q f(z)$ has an analytic continuation on the whole plane.

We prove that, for $|z| = r$,

$$\frac{z^q}{\nu^q} D^q f(z) \sim f(z)$$

in a neighborhood of the maximum modulus point as $r \to \infty$ outside an exceptional set of values r, where $\nu = \nu(r, f)$ is the central index of f.

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THE LOCAL NONTANGENTIAL GROWTH OF THE POISSON-STIELTJES INTEGRAL IN THE POLYDISC

Let $|z| = \max\{|z_j| : 1 \le j \le n\}$ be the polydisc norm, $U^n = \{z \in \mathbb{C}^n : |z| < 1\}$ be the polydisc and $T^n = \{z \in \mathbb{C}^n : |z_j| = 1, 1 \le j \le n\}$ be the sceleton for $z \in \mathbb{C}^n$, $n \in \mathbb{N}$. For $z \in U^n, z_j = r_j e^{i\varphi_j}, w \in T^n, w_j = e^{i\theta_j}, 1 \le j \le n$, we denote by $\mathcal{P}(z,w) = \prod\limits_{j=1}^{n} P_0(z_j, w_j)$ the multiple Poisson kernel, where $P_0\left(z_j, w_j\right) = \operatorname{Re} \frac{w_j+z_j}{w_j-z_j}$ is the Poisson kernel for the unit disc.

The function $P: \ U^n \to \mathbb{R}$, $P[d\mu](z) = \int_{T^n} \mathcal{P}(z,w)d\mu(w)$, is called the Poisson-Stieltjes integral of complex-valued Borel measure μ, $|\mu|(T^n) < +\infty$. We say that $\mu \in H^{(\beta_1,\ldots,\beta_n)}_{\varphi_1,\ldots,\varphi_n}$, $\beta_j > 0$, $1 \le j \le n$ if $\exists C > 0$

$$|\mu| \left(\left\{ e^{i\theta} \in T^n : \ |\theta_j - \varphi_j| \le \delta^{\frac{1}{\beta_j}}, \ 1 \le j \le n \right\} \right) \le C\delta, \quad 0 < \delta < 1.$$

For $\theta = (\theta_1, \ldots, \theta_n) \in [-\pi; \pi]^n$, $\gamma = (\gamma_1, \ldots, \gamma_n) \in [0; \pi)^n$ we define the Stolz angle $S_\gamma(\theta) = S_{\gamma_1}(\theta_1) \times \ldots \times S_{\gamma_n}(\theta_n)$, where $S_{\gamma_j}(\theta_j)$ is the Stolz angle for the unit disc with the vertex $e^{i\theta_j}$ and opening γ_j, $1 \le j \le n$.

Theorem. *Let μ be a complex-valued Borel measure on T^n, $n \in \mathbb{N}$, $\beta_j > 0$, $1 \le j \le n$. If $\mu \in H^{(\beta_1,\ldots,\beta_n)}_{\varphi_1,\ldots,\varphi_n}$, then*

$$\left| \int\limits_{T^n} P\left(z,w\right) d\mu\left(w\right) \right| = O\left(\log_2^p \frac{1}{\delta} \left[\prod_{j=1}^{n} \left| z_j - e^{i\theta_j} \right|^{\frac{\beta_j}{n}} \right]^{1 - \sum\limits_{j=1}^{n} \max\left\{ \frac{1}{\beta_j}; 1 - \frac{1}{\beta_j} \right\}} \right),$$

where $|z_j| = 1 - \delta^{\frac{1}{\beta_j}}$, $\delta \downarrow 0$, $z \in \mathcal{S}_\gamma\left(\theta\right)$, p is the number of those β_j that equal 2. [1]

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INTERPOLATED SCALES OF LORENTZ-TYPE SPACES OF EXPONENTIAL TYPE VECTORS

In a Banach complex space $(\mathfrak{X}, \|\cdot\|)$ we consider a closed unbounded linear operator A with the norm dense domain $\mathcal{C}^1(A)$. We assume that $0 \in \rho(A)$, where $\rho(A)$ is the resolvent set of A. Denote by $\mathcal{C}^m := \mathcal{C}^m(A)$ $(m \in \mathbb{N})$ the domain of A^m with the norm $\|x\|_{\mathcal{C}^m} = \|A^m x\|$, $x \in \mathcal{C}^m$. We put $\mathcal{C}^0 = \mathfrak{X}$ for the unit operator A^0 and $C^\infty(A) = \bigcap_{k\in\mathbb{N}} \mathcal{C}^k(A)$.

Let $0 < t < \infty$ and $1 \le p, q \le \infty$. Consider the mapping

$$\mathcal{C}^\infty(A) \ni x \longrightarrow \left\{x_k := (A/t)^k x\right\}_{k=0}^\infty$$

whose image is formed by sequences of elements of the Banach space $\mathfrak{X}$. For the indices t, q, p and $m \in \mathbb{Z}_+$ we define the spaces

$$\mathcal{E}^t_{q,p}(\mathcal{C}^m) = \left\{x \in \mathcal{C}^\infty(A)\colon \|x\|_{\mathcal{E}^{t,m}_{q,p}} < \infty\right\},\ \mathcal{E}^t_{q,q}(\mathcal{C}^m) = \mathcal{E}^t_q(\mathcal{C}^m),$$

where

$$\|x\|_{\mathcal{E}^{t,m}_{q,p}} = \begin{cases} \left(\sum_{k=0}^\infty (k+1)^{\frac{p}{q}-1} \|x^*_k\|^p_{\mathcal{C}^m}\right)^{1/p}, & 1 \le p < \infty, \\ \sup_k (k+1)^{1/q} \|x^*_k\|_{\mathcal{C}^m}, & p = \infty. \end{cases}$$

The sequence $\{x^*_k\}_{k=0}^\infty$ consists of elements x_k which are ordered so that $\|x^*_0\|_{\mathcal{C}^m} \ge \|x^*_1\|_{\mathcal{C}^m} \ge \|x^*_2\|_{\mathcal{C}^m} \ldots$.

We call the space $\mathcal{E}^t_{q,p}(\mathcal{C}^m)$ endowed with the norm $\|x\|_{\mathcal{E}^{t,m}_{q,p}}$ the Lorentz-type space of exponential type entire vectors of A. We establish the interpolation properties of such spaces.

Theorem. *If $1 < q < \infty$, $1 \le p \le \infty$ then the following equality holds*

$$\left(\mathcal{E}^t_1(\mathcal{C}^m), \mathcal{E}^t_\infty(\mathcal{C}^m)\right)_{1-1/q,p} = \mathcal{E}^t_{q,p}(\mathcal{C}^m),$$

with equivalent norms.

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BRANCHED CONTINUED FRACTION OF SPECIAL FORM AND THE PARABOLA THEOREM

Research of the parabolic convergence regions for continued fractions started in the mid-twentieth century. The most complete review of the results presented in [5]. Multidimensional generalizations of the parabola theorems for branched continued fractions considered in [1-4].

The following theorem holds.

Theorem. *Branched continued fraction of special form*

$$\Phi_0 + \cfrac{1}{1+\Phi_1+\cfrac{a_{02}}{1+\Phi_2+\cfrac{a_{03}}{1+\Phi_3+\ddots}}}, \quad \Phi_p = \cfrac{1}{1+\cfrac{a_{2p}}{1+\cfrac{a_{3p}}{1+\ddots}}}, \quad p \geq 0,$$

converges uniformly for all a_{rs} in the domain

$$P_M = \{z \in \mathbb{C} : \ |z| - \operatorname{Re} z \leq 1/2, \quad |z| < M\}$$

for every constant $M > 0$.

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DIRECT AND INVERSE PROBLEMs FOR SINGULAR RANK ONE PERTURBATIONs OF A SELFADJOINT OPERATOR

Let H be a separable Hilbert space, A a selfadjoint operator with discrete spectrum acting in H. We denote by H_s, $s \in \mathbb{R}$, the Hilbert space scale generated by A; in particular, H_2 is the domain of A, see [1].

In the talk we study the spectral properties of the operator $B = A + \langle \cdot, \varphi \rangle \psi$, which is a rank one perturbation of the operator A; here φ and ψ are elements of H_{-1} and $\langle \cdot, \cdot \rangle$ denotes the pairing between H_1 and H_{-1}, cf. [1–3]. In particular, the asymptotics of eigenvalues μ_n of the operator B is found and a full description of the spectra for operators B of such type is given. The possibility of reconstructing the Fourier coefficients of φ and φ from the spectra of the operators A and B is studied. The results are applied in the theory of differential equations.

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RENEWAL FUNCTION OF THE STREAM FOR REQUIREMENTS, WHICH WERE SERVICED, AND OF THE STREAM FOR LOST REQUIREMENTS A SINGLE-CHANNEL QUEUING SYSTEM

Let the Poisson process requirements come to the operating unit, i.e it is a recurrent stream, which defines a distribution $F(t) = 0$, provided that $t \leqslant 0$ and $F(t) = 1 - e^{-\lambda t}$, if $t > 0$ $(\lambda > 0)$. The requirement comes to a service, and is served by a random time η and it has the distribution

$$G(t) = \begin{cases} 0, t \leqslant 0, \\ 1 - e^{-\mu t}(\mu > 0), t > 0. \end{cases}$$

Let $v_0(t)$ be the number of requirements, which were serviced during a service time t, $v_l(t)$ is the number of lost requirements during the service time t. $H_0(t)$ and $H_l(t)$ are the respective renewal functions and $H_0(t) = M(v_0(t))$, $H_l(t) = M(v_l(t))$.

Theorem 1. *The renewal function of the stream for requirements, which were serviced, is given in a form*

$$H_0(t) = \frac{\lambda\mu}{\lambda+\mu}t + \frac{\lambda^2}{(\lambda+\mu)^2} - \frac{\lambda^2}{(\lambda+\mu)^2}e^{-(\lambda+\mu)t}.$$

Theorem 2. *The renewal function of the stream for lost requirements*

$$H_1(t) = \frac{\lambda^2}{\lambda+\mu}t - \frac{\lambda^2}{(\lambda+\mu)^2} + \frac{\lambda^2}{(\lambda+\mu)^2}e^{-(\lambda+\mu)t}.$$

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APPROXIMATION OF BOUNDARY VALUE PROBLEM SOLUTIONS FOR INTEGRO-DIFFERENTIAL EQUATIONS WITH DELAY

We consider the following boundary value problem

$$y''(x) = f(x, [y(x)]) + \int_a^b g(x, t, [y(x)])dt, \ x \in [a; b] \tag{1}$$

$$y^{(i)}(x) = \varphi^{(i)}(x), \ i = 0, 1, \ x \in [a^*; a], \ y(b) = \gamma, \tag{2}$$

$$[y(x)] = (y(x), y(x - \tau(x)), y'(x), y'(x - \tau(x))), a^* = \min_{x \in [a;b]} (x - \tau(x)).$$

The existence and uniqueness of solution of the boundary value problem with delay were studied in [1-2]. Analytical solution of the problem (1)-(2) is possible only in the simplest cases. Spline collocation method usage for solving differential-difference equations was investigated in [3]. In this paper an algorithm for finding the approximate solution of the boundary value problem (1)-(2) using cubic splines with defect two is suggested. Sufficient conditions for the iterative process convergence are obtained and numerical simulations to test cases are conducted.

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ASYMPTOTIC REPRESENTATIONS OF SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH REGULARLY VARYING NONLINEARITIES

We consider the differential equation

$$y^{(n)} = \alpha p(t) \prod_{j=0}^{n-1} \varphi_j(y^{(j)}), \tag{1}$$

where $n \geq 2$, $\alpha \in \{-1, 1\}$, $p : [a, \omega[\to]0, +\infty[$ are continuous functions, $\varphi_j : \Delta Y_j \to]0; +\infty[$ is a continuous and regularly varying as $y^{(j)} \to Y_j$ function of order σ_j, $j = \overline{0, n-1}$, $-\infty < a < \omega \leq +\infty$, Y_j is equal to either zero or $\pm\infty$, ΔY_j is some one-sized neighborhood of the point Y_j.

In the paper [1], question of existence of the so-called $\mathcal{P}_\omega-$solutions of particular case of equation (1) and the asymptotic as $t \uparrow \omega$ representations of these solutions were studied by Evtukhov V. and Samoylenko A.

Definition. *A solution y of differential equation (1) is called a $\mathcal{P}_\omega^k(Y_0, \ldots, Y_{n-1}, \lambda_0)-$solution, where $-\infty \leq \lambda_0 \leq +\infty$ and $k \in \{1, \ldots, n\}$, if it is defined on $[t_0, \omega[\subset [a, \omega[$ and satisfies the next conditions:*

$$y^{(j)}(t) \in \Delta Y_j \quad \text{for} \quad t \in [t_0, \omega[\quad (j = \overline{0, n-1}),$$

$$y(t) = \pi_\omega^{k-1}(t)\, [c + o(1)] \ \text{ as } \ t \uparrow \omega, \ c \neq 0, \ \pi_\omega(t) = \begin{cases} t, & \text{if } \omega = +\infty, \\ t - \omega, & \text{if } \omega < +\infty, \end{cases}$$

$$\lim_{t \uparrow \omega} y^{(j)}(t) = Y_j \quad (j = \overline{k, n}), \quad \lim_{t \uparrow \omega} \frac{[y^{(n-1)}]^2}{y^{(n-2)}(t) y^{(n)}(t)} = \lambda_0.$$

In our talk, we present the conditions of existence of the $\mathcal{P}_\omega^k(Y_0, \ldots, Y_{n-1}, \lambda_0)-$ solutions of equation (1) and the asymptotic as $t \uparrow \omega$ representations of these solutions and their derivatives of orders up to $n - 1$.

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ON TRANSFORMATION OPERATORS AND MODIFIED SOBOLEV SPACES IN CONTROLLABILITY PROBLEMS FOR THE WAVE EQUATIONS WITH VARIABLE COEFFICIENTS

The control system

$$w_{tt} = \frac{1}{\rho}\left(kw_x\right)_x + \gamma w, \quad w(0,t) = u(t), \qquad x > 0, \quad t \in (0,T), \tag{1}$$

is investigated in special modified spaces of the Sobolev type introduced and studied in the work. The growth of distributions from these spaces is associated with the equation data ρ and k. Here ρ, k, and γ are given functions on $[0,+\infty)$; $u \in L^\infty(0,\infty)$ is a control; $T > 0$ is a constant. Using some transformation operator introduced and studied in the work, we establish that control system (1) replicates the controllability properties of the auxiliary system

$$z_{tt} = z_{xx} - q^2 z, \quad z(0,t) = v(t), \qquad x > 0, \quad t \in (0,T) \tag{2}$$

and vise versa. Here $q \geq 0$ is a constant and $v \in L^\infty(0,\infty)$ is a control. Necessary and sufficient conditions of (approximate) L^∞- controllability for the main system are obtained from the ones for the auxiliary system (2) [1]. Control problem (1) has been investigated in [2].

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FOURIER QUASICRYSTALS AND LAGARIAS' CONJECTURE

A Fourier quasicrystal is a pure point complex measure μ in $\mathbb{R}^p$ such that its spectrum (Fourier transform in the sense of distributions) $\hat{\mu}$ is also a pure point measure. For example, the sum μ of unit masses at the points of $\mathbb{Z}^p \subset \mathbb{R}^p$ is a Fourier quasicrystal, because $\hat{\mu}$ coincides with μ in this case.

J.Lagarias (2000) conjectured that if μ is a measure with a uniformly discrete support and its spectrum is also a measure with a uniformly discrete support, then the support of μ is a subset of a finite union of shifts of some full-rank lattice. The conjecture was proved by N.Lev and A.Olevskii (2013) in the case p=1, i.e., for measures on the real axis, and in the case of an arbitrary p and a positive measure μ (or $\hat{\mu}$).

On the other hand, A.Cordoba (1989) proved that the support of μ is a finite union of shifts of *several* full-rank lattices under rather weak conditions on the spectrum $\hat{\mu}$.

In my talk I prove that Lagarias' conjecture does not valid in the general case and show the special case when the conjecture is true. Moreover, I show a generalization of Cordoba's result.

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ON CLASSIFICATION OF THE NON-CONJUGATE SUBALGEBRAS OF THE LIE ALGEBRA OF THE POINCARÉ GROUP $P(1,4)$ AND SYMMETRY REDUCTION OF SOME DIFFERENTIAL EQUATIONS

The symmetry reduction is a powerful tool for investigation of partial differential equations. The details can be found, for example, in [1].

The papers [2, 3] are devoted to the symmetry reduction of some differential equations in the spaces $M(1,3) \times R(u)$ and $M(1,4) \times R(u)$, which are invariant with respect to the Poincaré group $P(1,4)$. Here, $R(u)$ is the real number axis of the dependent variable u.

However, it turned out that the reduced equations, obtained with the help of nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ of the given rank, were of different types.

To explain some of the differences in the properties of the above mentioned reduced equations, we suggest to try to investigate the connections between structural properties of nonconjugate subalgebras of the same rank of the Lie algebra of the group $P(1,4)$ and the properties of the reduced equations corresponding to them.

Until now, we have established a connection between the classification of three-dimensional decomposable nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ and symmetry reduction of the Eikonal equation.

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ON SYMMETRY REDUCTION AND EXACT SOLUTIONS OF SOME P(1,4)-INVARIANT D'ALEMBERT EQUATIONS

The linear and nonlinear d'Alembert equations in spaces of different dimensions are used for the solution of various problems of differential geometry, theory of nonlinear waves, theoretical and mathematical physics (see, e.g., [1, 2, 3, 4, 5]).

Let us consider the following differential equations:

1. $\Box_5 u = 0,$
2. $\Box_5 u = \lambda u, \ \lambda \in R, \ \lambda \neq 0,$
3. $\Box_5 u = \sin u,$
4. $\Box_5 u = e^u,$
5. $\Box_5 u = \sinh u,$

where $\Box_5$ is the d'Alembert operator in the five-dimensional Minkowski space $M(1,4)$.

Equations (1)-(5) are invariant with respect to the Poincaré group $P(1,4)$.

Until now, using the subgroup structure of the group $P(1,4)$, we have performed the symmetry reduction for the above mentioned equations and constructed some classes of exact solutions for them.

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SOME NEW PROBLEMS OF A THEORY FOR THE HYPERBOLIC SYSTEMS OF THE FIRST-ORDER QUASILINEAR EQUATIONS

We consider the one-dimensional hyperbolic system of quasilinear equations $\dfrac{\partial u_i}{\partial t} + \lambda_i(x,t,u)\dfrac{\partial u_i}{\partial x} = f_i(x,t,u)$, $i = \overline{1,n}$ with initial and boundary conditions. The correct solvability of new boundary problems are formulated and proved, including:

1) conditions of the generalized solvabilities of the mixed problems for a hyperbolic systems with vertical and horizontal characteristics [1];

2) different statment of problems for regular and singular systems are studied [2];

3) conditions of solvability of optimal control problems of hyperbolic systems are obtained [3];

4) and sufficient conditions of existence and uniqueness of the solution of the boundary problems for a countable hyperbolic equations [4];

5) some issues of the statement of the boundary problem and solvability for these systems are formulated.

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PROBLEM WITHOUT INITIAL CONDITIONS FOR THE COUNTABLE HYPERBOLIC SYSTEM OF DIFFERENTIAL EQUATIONS

Let us consider the problem without initial conditions [1] for countable semilinear hyperbolic systems of differential equations

$$\frac{\partial u_i}{\partial t} + \lambda_i(x,t)\frac{\partial u_i}{\partial x} = f_i(x,t,u_1,u_2,...), \ \ i \in \mathbb{N}. \tag{1}$$

Let $I_0 = \{i|\lambda_i(0,t) > 0\}$, $I_l = \{i|\lambda_i(l,t) < 0\}$ and furthermore $I_0 \cap I_l = \emptyset$, $I_0 \cup I_l = \mathbb{N}$.

We define the boundary conditions for the system (1)

$$\int_0^l \alpha_i(x,t)u_i(x,t) = h_i(t), \quad i \in \mathbb{N}, \quad -\infty < t < \infty. \tag{2}$$

We consider problem (1)-(2) in $G \times C^\infty$, where $G = \{(x,t) : x \in (0,l), t \in (-\infty,\infty)\}$, C^∞ is the space of functions, whose elements are countable set of continuous functions bounded by some constant.

Applying the method of characteristics [2] and the Cantor diagonal method, the conditions of existence and uniqueness of generalized solution of problem (1)-(2) are obtained. The solution belongs to the space of continuous and uniformly bounded functions, which have the property exponential decay with $t \to -\infty$. Also the sufficient conditions of the classical solvability of a initial-boundary value problem for countable semilinear hyperbolic system in halfstrips are established.

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SINGULARLY PERTURBED MIXED BOUNDARY VALUE PROBLEM FOR THE SEMILINEAR SYSTEM OF HYPERBOLIC EQUATION OF THE FIRST ORDER

Let us consider the initial-boundary value problems for the degenerate equations (lack of some derivatives) which describe many processes in mathematical physics. That's why it is interesting to study the singularly perturbed problems, when small parameters are present with the higher order derivatives of the system or the equation. This problems are intermediate for the degenerate equation [1].

On domain $\Omega = \{(x,t): \ 0 < x < 1, \ 0 < t < T\}$, we consider the hyperbolic system

$$\begin{cases} \varepsilon \dfrac{\partial u^\varepsilon}{\partial t} + \dfrac{\partial u^\varepsilon}{\partial x} = F(x,t,u^\varepsilon,v^\varepsilon), \quad (x,t) \in \Omega, \\ \dfrac{\partial v^\varepsilon}{\partial t} - \varepsilon \dfrac{\partial v^\varepsilon}{\partial x} = G(x,t,u^\varepsilon,v^\varepsilon), \quad (x,t) \in \Omega, \end{cases}$$

under the initial and boundary conditions

$$\begin{cases} u^\varepsilon(x,0) = 0, \ v^\varepsilon(x,0) = 0, \quad \text{for } \ x \in (0,1), \\ u^\varepsilon(0,t) = 0, \ v^\varepsilon(1,t) = 0, \quad \text{for } \ t \in (0,T). \end{cases}$$

Under the certain smoothness conditions of initial data and their coordination of the zero and the first order in angular points of domain Ω, the full asymptotic expansion of the solution using the methods from [2] is constructed. The estimation of the asymptotic expansion is obtained.

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SINGULAR PERTURBED INITIAL-BOUNDARY PROBLEMS FOR PARABOLIC EQUATION ON GRAPH

Let Γ be a compact star-like graph in $\mathbb{R}^3$ with n edges $\gamma_1, \ldots, \gamma_n$, and the common vertex a. The rest of vertices forms the boundary $\partial\Gamma$ of the graph. The metric graph Γ can be considered as a model of bundle of n rods of finite length joined in a common point a. We consider heat conduction or diffusion in the system provided the rods possess different thermal conductivity or diffusivity respectively. Temperature of the system is described by a function $u : \Gamma \times (0,T) \mapsto \mathbb{R}$, that is a solution to the problem for parabolic equation

$$\begin{aligned}
&\partial_t u - a(x,\varepsilon)\,\partial_x^2 u + q(x)u = f(x,t), && (x,t) \in \Gamma \times (0,T),\\
&u(x,0) = \varphi(x), && x \in \Gamma,\\
&u(x,t) = \mu(t), && (x,t) \in \partial\Gamma \times (0,T).
\end{aligned}$$

From the physical point of view the solution u should be continuous at the vertex a, i.e. $u_{\gamma_1}(a,t) = u_{\gamma_2}(a,t) = \cdots = u_{\gamma_n}(a,t)$, and satisfies the balance condition of heat currents in roads

$$\big(a_{\gamma_1} \cdot \partial_{\gamma_1} u + a_{\gamma_2} \cdot \partial_{\gamma_2} u + \cdots + a_{\gamma_n} \cdot \partial_{\gamma_n} u\big)(a,t) = 0,$$

where $t \in (0,T)$. Here u_γ is the restriction of u to an edge γ, and $\partial_\gamma u(a,\cdot)$ is the derivative of u with respect to space variable at the vertex a along the edge γ in the direction from the vertex. The coefficient $a(x,\varepsilon)$ is positive on Γ, but it can be equal zero for $\varepsilon = 0$ either on all edges or some of them. Besides the rate of the degeneration $a(x,\varepsilon)$ as $\varepsilon \to 0$ can differ on the different edges.

Under some smoothness conditions on the data, the complete asymptotic expansions of the solution are constructed and justified.

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PROFESSOR S.D.EIDELMAN INVESTIGATION OF PROPERTIES OF SOLUTIONS FOR PARABOLIC EQUATIONS ON UNBOUNDED WITH RESPECT TO TIME VARIABLE INTERVALS AND THEIR DEVELOPMENT

The problems about the investigation of the solutions for parabolic systems in unbounded with respect to time variable domains are quite interesting and a lot of them were engaged by many scientists.

An interesting approach to analyse such problems was suggested by S.Eidelman. He introduced so-called Λ-conditions for studying of the properties of solutions for the Petrovskii parabolic systems of the first order with respect to t variable. These conditions mean that the estimates of the fundamental matrix of solutions of the Cauchy problem are realized on the unbounded intervals of variation of t variable, and their evaluation function tends to zero as $t \to \infty$. Investigations in this area for the Petrovskii parabolic systems of arbitrary order with respect to time was continued in works of S.Eidelman and L.Ivasyshyn. In works of S.Eidelman and his followers, an important applications of the estimates from the conditions for the proof of the theorem of stability and stabilization of solutions of the Cauchy problems, the theorems of Liuville type and for the construction and studying the fundamental matrix of solutions for elliptic systems generated by parabolic systems were found.

S.Edelman research methods were continued to the case of arbitrary order with respect to time variable $2\vec{b}$-parabolic systems in the works of T.Balabushenko and S.Ivasyshen.

To the recent research results in this area one's could include the investigation of some Kolmogorov ultraparabolic equations. Some estimates of the $\Lambda_m^{\pm}$-conditions for them are found in the paper of S.Ivasyshen, G.Ivasyuk and T.Fratavchan. These estimates had similar application for investigation of the properties of solutions of the mentioned above ultraparabolic equations.

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SIMULTANEOUS PADÉ APPROXIMANTS OF THE BASIC HYPERGEOMETRIC SERIES

The exact expressions of the simultaneous Padé approximants for some sets of the basic hypergeometric series are derived.

Theorem. [1] *Simultaneous Padé approximants for the set of functions* $F = \{f_\lambda\}_{\lambda=1}^{\Lambda}$, *where* $f_\lambda(z) = \sum\limits_{k=0}^{\infty} \frac{z^k}{1+\varepsilon_\lambda \gamma^k}$, $\lambda = \overline{1,\Lambda}$, $\varepsilon_\lambda \in (0,\infty)$, $\gamma \in (0,\infty)\backslash\{1\}$, $\log_\gamma \frac{\varepsilon_\mu}{\varepsilon_\lambda} \notin \mathbb{Z}$ *while* $\lambda \neq \mu$, $\lambda,\mu = \overline{1,\Lambda}$, *of index* $R = [M/N]$, $M = (m_1, m_2, \ldots, m_\Lambda)$, $N = (n_1, n_2, \ldots n_\Lambda)$ *such that* $m_\lambda \geq |N| - 1$, $\lambda = \overline{1,\Lambda}$, $|N| = n_1 + n_2 + \ldots + n_\Lambda$ *can be written in the form*

$$[M/N]_F^{(\lambda)}(z) = \frac{P_R^{(\lambda)}(z)}{Q_R(z)}, \quad \lambda = \overline{1,\Lambda},$$

where

$$Q_R(z) = \sum_{k=0}^{|N|} c_k^{(R)} z^{|N|-k},$$

$$P_R^{(\lambda)}(z) = \sum_{k=0}^{|N|} c_k^{(R)} z^{|N|-k} \sum_{p=0}^{m_\lambda+k-|N|} \frac{z^p}{1+\varepsilon_\lambda \gamma^p}, \quad \lambda = \overline{1,\Lambda},$$

and

$$c_k^{(R)} = (-1)^{|N|-k} \cdot \frac{\prod\limits_{\lambda=1}^{\Lambda} \left(-\varepsilon_\lambda \gamma^{m_\lambda-|N|+k+1}; \gamma\right)_{n_\lambda}}{(\gamma;\gamma)_k (\gamma;\gamma)_{|N|-k} \gamma^{(2|N|-k-1)k/2}},$$

$k = \overline{0,|N|}$, *where* $(a,\gamma)_k = (1-a)(1-a\gamma)\cdot\ldots\cdot(1-a\gamma^{k-1})$, $k \geq 1$, $(a;\gamma)_0 = 1$.

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ASYMPTOTICS OF THE SPECTRUM OF ELLIPTIC OPERATOR OF THE FOURTH ORDER WITH SINGULARLY PERTURBED COEFFICIENTS

In this talk, we consider the Dirichlet spectral problem for an elliptic operator of the fourth order with singularly perturbed coefficients. The problem describes eigenmodes of a plate with finite number of stiff and light-weight inclusions of an arbitrary shape.

Let U be a bounded domain in $\mathbb{R}^2$ with smooth boundary and ω is a strictly internal subset of U consisting of a finite number of domains. Set $\Omega = U \setminus \bar{\omega}$. We consider the stiffness coefficient $k_\varepsilon(x)$ being $\varepsilon k(x)$ on Ω and $\varkappa(x)$ on ω, and mass density $r_\varepsilon(x)$ being $r(x)$ on Ω and $\varepsilon^\alpha \rho(x)$ on ω. All the functions are positive and smooth in Ω and ω respectively, and $\alpha > 0$. The main goal is to investigate asymptotic behaviour as $\varepsilon \to 0$ of eigenvalues λ^ε and eigenfunctions u_ε of the problem

$$2(1-\sigma)\frac{\partial^2}{\partial x_1 \partial x_2}\Big(k_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial x_1 \partial x_2}\Big) + \frac{\partial^2}{\partial x_1^2}\Big(k_\varepsilon\Big(\frac{\partial^2 u_\varepsilon}{\partial x_1^2} + \sigma \frac{\partial^2 u_\varepsilon}{\partial x_2^2}\Big)\Big) +$$

$$+ \frac{\partial^2}{\partial x_2^2}\Big(k_\varepsilon\Big(\frac{\partial^2 u_\varepsilon}{\partial x_2^2} + \sigma \frac{\partial^2 u_\varepsilon}{\partial x_1^2}\Big)\Big) = \lambda^\varepsilon r_\varepsilon u_\varepsilon \quad \text{in} \quad U,$$

$$u_\varepsilon = \partial_n u_\varepsilon = 0 \quad \text{on} \quad \partial U, \quad [u_\varepsilon]_{\partial\omega} = [\partial_n u_\varepsilon]_{\partial\omega} = 0,$$

$$[k_\varepsilon(\partial_{nn} u_\varepsilon + \sigma \partial_{\tau\tau} u_\varepsilon)]_{\partial\omega} = 0,$$

$$[\partial_n(k_\varepsilon(\partial_{nn} u_\varepsilon + \sigma \partial_{\tau\tau} u_\varepsilon)) + 2(1-\sigma)(\partial_\tau(k_\varepsilon \partial_{n\tau} u_\varepsilon))]_{\partial\omega} = 0,$$

where $[f]_S$ denote the jump of f on the surface S; $\sigma = \sigma(x)$ is constant in Ω and ω respectively, and $0 \le \sigma(x) < 1$. We write ∂_n and ∂_τ for the normal and tangential derivatives respectively. The number-by-number convergence of the eigenvalues and the corresponding eigenspaces is established.

1. Golovaty Yu., Hut V. Asymptotics of the spectrum of inhomogeneous plate with light-weight stiff inclusions, *Mat. Stud.* **40**:1 (2013), 79–94. (in Ukrainian)

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K. WEIERSTRASS AND HIS INFLUENCE ON THE DEVELOPMENT OF MATHEMATICS IN UKRAINE

On 31st of October, it will be 200 years since the day of birth of K. Weiersrass, one of the greatest mathematicians of the second half of the nineteenth century. His scientific and pedagogical activity signifies the whole epoch in the development of a human thought.

In the mathematical analysis, he succeeded in doing passage to a new level of strictness about which his famous predecessors Newton, Leibnitz, Euler, Lagrange, Cauchy dreamed but did not attain it. Having constructed an example of a nowhere differentiable function and proved, on the other hand, that such a function is a polynomial up to an arbitrarily small number, K. Weierstrass laid the foundation of a new direction in this branch of mathematics, namely the constructive theory of functions. Putting a power series into the basis of definition of an analytic function, he originated the theory of functions analytic in a domain. He founded also the theory of entire functions, developed in the full measure the theory of elliptic and Abelian ones , and started constructing analytic functions of many variables. He obtained significant results in the linear algebra, calculus of variations, the PDE theory, Geometry, too.

His numerous popular lectures at the Berlin University, which became the mathematical Mecca of that time, were attracting attention of many listeners.

We would like to concentrate on the influence of K. Weierstrass on the development of mathematics in Ukraine and on our results concerning the direct and inverse theorems in the approximation theory, differential equations in a Banach space, and partial differential equations, which were suggested by the Veierstrass' ideas.

As for the development of mathematics at the I. Franko University of Lviv in the field of functional analysis, we would like to turn attention to the Weierstrass' criticism regarding the existence of an extremal for a Dirichlet integral, which, probably, above of all gave a rise of functional analysis in Lviv.

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ON SOLUTIONS OF DIFFERENTIAL EQUATIONS IN A BANACH SPACE ON $(-\infty, \infty)$

We consider an equation of the form

$$\left(\frac{d}{dt} - A\right)^n \left(\frac{d}{dt} + A\right)^m, \quad t \in (-\infty, \infty), \tag{1}$$

where $n, m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\},\ n + m \geq 1,\ A$ is the generator of a bounded analytic C_0-semigroup $\left\{e^{tA}\right\}_{t\geq 0}$ in a Banach space $\mathfrak{B}$ with norm $\|\cdot\|$. Denote by $\mathfrak{E}(A)$ the space of entire vectors of the operator A. Let also $\rho(A)$ and $\sigma(A)$ be the resolvent set and the spectrum of A. The following assertion holds true.

Theorem 1. *Suppose $0 \in \rho(A)$. A vector-valued function $y(t)$ is a solution of equation* (1) *on $(-\infty, \infty)$ if and only if it admits a representation*

$$y(t) = \sum_{k=0}^{n-1} t^k \exp(tA) f_k + \sum_{k=0}^{m-1} t^k \exp(-tA) g_k, \quad f_k, g_k \in \mathfrak{E}(A).$$

The vectors $f_k (k = 0, 1, \ldots n-1)$ and $g_k (k = 0, 1, \ldots m-1)$ are uniquely determined by $y(t)$.

Here $\exp(tA) = \sum\limits_{k=0}^{\infty} \frac{t^k A^k}{k!}$ is associated with the semigroup under consideration in the such way: $\exp(tA)x = e^{tA}x$ as $t \geq 0$, and $\exp(tA)x = (e^{-tA})^{-1}x$ as $t < 0$ $(x \in \mathfrak{E}(A))$. The operator function $\exp(zA)$ is entire in $\mathfrak{E}(A)$. Since the space $\mathfrak{E}(A)$ is dense in $\mathfrak{B}$, the set of all the solutions of (1) is infinite-dimensional. For its elements, the following analog of the Phragmen-Lindelöf priciple is fulfilled.

Theorem 2. *Let $s = s(A) = \sup\limits_{\lambda\in\sigma(A)} \mathrm{Re}\lambda$ (it is clear that $s < 0$). If a solution $y(t)$ of equation* (1) *on $(-\infty, \infty)$ satisfies the condition*

$$\exists\gamma \in (0, -s), \exists c_\gamma > 0, \forall t \in (-\infty, \infty) : \|y(t)\| \leq c_\gamma e^{\gamma|t|},$$

then $y(t) \equiv 0$.

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WELL POSSED SOLVABILITY OF THE NONLOCAL MULTI-POINT PROBLEM FOR SINGULARLY EVOLUTION EQUATIONS

Recently the theory of the nonlocal boundary value problems has received wide development. This is because such problems have many applications in mechanics, physics, chemistry, biology, ecology and other natural sciences and arise in the mathematical modeling of various processes. We investigate the nonlocal multi-point on time problem for evolution equations with the pseudo-Bessel operators constructed by constant symbols. We construct a fundamental solution for this problem, study its properties, obtain well-possed solvability of the problem when the boundary function is a generalized function of the distribution type [1].

The relevance of research on such problems with boundary conditions in certain spaces of generalized functions is conditioned due to the fact that the boundary functions may have singularities in one or more points. Depending on the singularity order (a power or higher than the power-law) such functions either allow regularization in spaces of the Sobolev-Schwartz type generalized functions of finite order or are generalized functions of infinite order (for example, ultradistribution, hyperfunction). We have found the class X' of the generalized boundary functions for which the multi-point problem solution is described as a convolution of the boundary function with the fundamental solution of this problem (which is an element of the space X of basic functions), and the solution has the same properties as the fundamental solution.

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DIRECT PROBLEM AT OPTIMIZATION OF TECHNOLOGICAL HEATING OF THE GLASS PIECEWISE-HOMOGENEOUS SHELLS

A direct problem of solid mechanics, which describes the thermal and mechanical processes in piecewise-homogeneous glass shells at optimization of heating regimes used in production processes (defined by heating methods, initial and boundary heat and mechanical conditions) is formulated. A problem of heat transfer is considered as an appropriate two-dimensional one (for precision adopted in shell theory), using a third degree polynomial approximation for temperature distribution on the thickness. This approach is effective in numerical optimization algorithms for the selected criteria (in particular level of stress state) of technological heating regimes of thin piecewise-homogeneous glass shells, which require frequent solving of direct problems.

It is found that temperature disturbance in the vicinity of cross-section coupling can be neglected (for precision adopted in shell theory) and heat transfer problem with homogeneous temperature of environment can be considered as one-dimensional problem. Adopted assumptions lead to significant simplification of initial direct problems of mathematical physics that reduces the complexity of procedures for obtaining the appropriate solution of relevant problems of thermoelasticity in algorithms of numerical optimization of targeted heating regimes and provides time saving of their calculation.

As an example the problem of optimization of heating regimes for piecewise-homogeneous glass shells at different heat conditions on internal surface is considered. A minimum of maximum normal stresses is used as criterion. The temperature of outer surface is used as control function.

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METHODS FOR CALCULATION OF STRESSES IN STRUCTURALLY INHOMOGENEOUS BODIES OF REVOLUTION UNDER THE HEAT TREATMENT

The work offers the methods for calculation the stresses in structurally inhomogeneous, layer in particular, bodies of revolution under heat treatment. The methods of calculation of stresses anticipate: mathematical statement of the problem; development of numerical algorithm of searching the solution; program realization of numerical algorithm. Mathematical statement of the problem involves such stages: analysis of present physical-mechanical processes under heat treatment; selection of parameters of the state; construction of physical-mathematical model of description the existing physical-mechanical processes.

We assume that for considered thermal loads the stressed state of the body does not influence its temperature, the problem on determination the stressed-strained state in the body we formulate in a quasi-static statement (in displacements). In addition the temperature field we describe by the known equations of heat conduction the coefficients of which are described in the approximation of the model of thermoelastic body taking into account the structural inhomogeneity. We restrict ourselves to the case of small deformations. In the region occupied by the body the equations of equilibrium and boundary conditions are to be satisfied. The algorithm of solution the formulated problems is based on numerical methods of weighted residues together with method of finite elements. It allowed us to obtain the efficient approximate solutions to the above formulated problems.

According to the method of finite elements we divide the region occupied by the body into the finite number of elements. The unknown values on the partition element we present by the functions of joum. Using the method of weighted residues the system of initial relations is reduced to a system of nonlinear algebraic eqations which are to be solved by the method of simple iteration.

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INVERSE FUNCTION INTERPOLATING CONTINUED FRACTIONS

Let's have a functional two-dimensional interpolating continued fraction

$$D_{n_*}(x,y) = \frac{P_{n_*}(x,y)}{Q_{n_*}(x,y)} = \left(\Phi_0^{n_*}(x,y) + \underset{k=1}{\overset{n}{\mathrm{K}}} \frac{g_1(x-x_{k-1})g_2(y-y_{k-1})}{\Phi_k^{n_*}(x,y)} \right)^{-1}, \tag{1}$$

where $\Phi_k^{n_*}(x,y) = b_{kk} + \underset{i=k+1}{\overset{n_1}{\mathrm{K}}} \frac{g_1(x-x_{i-1})}{b_{ik}} + \underset{i=k+1}{\overset{n_2}{\mathrm{K}}} \frac{g_2(y-y_{i-1})}{b_{ki}}$; b_{ij} are constants, $n_1 = k_1, n_2 = k_2$, $g_1(x)$ is the continued function in $[\alpha_x, \beta_x]$, $g_2(y)$ the continued function in $[\alpha_y, \beta_y]$ and $g_1(0) = 0, g_2(0) = 0$.

The k-th partial inverse difference can be computed according to the following scheme

$$\{\delta_{ij}^k :\ i = 0,1,\ldots,n_1,\ j = 0,1,\ldots,n_2,\ k = 0,\ldots,N-1,$$

$$N = \max\{n_1,n_2\},\ i,j > k.\}$$

$$\delta_{ij}^k = \frac{g_1(x_{ik}) \cdot g_2(y_{jk})}{\delta_{ij}^{k-1} + \theta_j^k \cdot \delta_{ik}^{k-1} + \theta_i^k \cdot \delta_{kj}^{k-1} + \theta_i^k \cdot \theta_j^k \cdot \delta_{kk}^{k-1}},$$

$$\delta_{ij}^{-1} = c_{ij}^{-1}, \quad \theta_s^t = \begin{cases} -1, & \text{if} \quad s > t, \\ 0, & \text{if} \quad s \leqslant t, \end{cases}$$

$$g_1(x_{ij}) = \begin{cases} g_1(x_i - x_j), i > j, \\ 1, \quad i \leqslant j, \end{cases} \quad g_2(y_{ij}) = \begin{cases} g_2(y_i - y_j), & \text{with} \quad i > j, \\ 1, & \text{with} \quad i \leqslant j. \end{cases}$$

Theorem 1 *The coefficient of the two-dimensional interpolating continued fraction (1) will be determined by the correlation*

$$b_{ij} = \delta_{ij}^{s-1}, \quad \textit{where}\ i = 0,1,\ldots,n_1,\ j = 0,1,\ldots,n_2,\ s = \max\{i,j\}. \tag{2}$$

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A HAMILTONIAN STRUCTURE OF THE SUPERSYMMETRIC mKP-HIERARCHY ON THE EXTENDED PHASE SPACE

In the previous reporter papers, the existence problem of Hamiltonian representation for the supersymmetric KP-hierarchy, coupled with the corresponding evolutions of eigenfunctions of the associated spectral problems, has been considered. This problem has been solved by use of some Backlund transformation on a suitably extended phase space. In the present report, the existence of the Hamiltonian representation is established for the extended supersymmetric mKP-hierarchy in the form

$$\begin{aligned}
&L_{t_n} = [L^n_{\geq 1},\ L],\\
&q_{i,t_n} = L^n_{\geq 1} q_i, \quad q^*_{i,t_n} = -D_\theta^{-1}(L^n_{\geq 1})^* D_\theta q^*_i,\\
&\varphi_{i,t_n} = L^n_{\geq 1}\varphi_i, \quad \varphi^*_{i,t_n} = -D_\theta^{-1}(L^n_{\geq 1})^* D_\theta \varphi^*_i,
\end{aligned}$$

where $L := f_1^{-1} l f_1 = \partial^p + \sum_{0<m<2p} v_m D_\theta^m + q_1 + D_\theta^{-1} q_1^* D_\theta +$ $+\sum_{i=2}^N (q_i D_\theta^{-1} q_i^* D_\theta + \varphi_i D_\theta^{-1}\varphi_i^* D_\theta)$, $l := \partial^p + \sum_{0\leq k<2p} u_k D_\theta^k +$ $+\sum_{i=1}^N (f_i D_\theta^{-1} f_i^* + \phi_i D_\theta^{-1}\phi_i^*)$, $v_{2r}, u_{2s}, q_i, q_i^*, f_i, \phi_i^* \in C^\infty(\Lambda_0\times\Lambda_1;\Lambda_0)$, $v_{2r+1}, u_{2s+1}, \varphi_j, \varphi_j^*, f_i^*, \phi_i \in C^\infty(\mathbb{S}\times\Lambda_1;\Lambda_1)$, $r=\overline{1,p-1}$, $s=\overline{0,p-1}$, $i=\overline{1,N}$, $p,N\in\mathbb{N}$, $\partial := \partial/\partial x$, $D_\theta := \partial/\partial x + \theta\partial/\partial\theta$, $(x,\theta)\in\mathbb{S}\times\Lambda_1$, $\mathbb{S}\simeq\mathbb{R}/2\pi\mathbb{Z}$, $\Lambda=\Lambda_0\oplus\Lambda_1$ is the Grassmann algebra over $\mathbb{C}$, $\Lambda_0\supset\mathbb{C}$, the index "$\geq 1$" denotes the pure differential part of the corresponding super-integro-differential operator, $t_n\in\mathbb{R}$, $n\in\mathbb{N}$, by means of the Backlund transformation connecting the KP- and mKP-hierarchies on the extended phase spaces and being generated by the gauge transformation on the dual space to the Lie algebra of super-integro-differential operators.

The Hamiltonian representations for the additional symmetries of the extended supersymmetric mKP-hierarchy are obtained as the result of the composition of two Backlund transformations: one of them is the Backlund transformation on the extended phase space of the KP-hierarchy, another is the Backlund one generated by the gauge transformation.

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ON UPPER-LOWER SPLITTING IN THE PALEY-WIENER SPACE

We denote by W^1_σ, $\sigma > 0$, the Paley-Wiener space of entire functions f satisfying the condition

$$\sup_{\varphi\in(\alpha;\beta)} \left\{ \int_0^{+\infty} |f(re^{i\varphi})| e^{-\sigma r|\sin\varphi|} dr \right\} < +\infty.$$

Each function $f \in W^1_\sigma$ is representable as

$$f(z) = \sum_{k=-\infty}^{+\infty} (-1)^k c_k \frac{\pi \sin \sigma z}{\sigma z - \pi k},\ c_k \in l^1. \tag{1}$$

Problem. Does every function $f \in W^1_\sigma$ admit a splitting $f = f_1 + f_2$, where functions f_1 and f_2 are analytic in $\mathbb{C}_+$, f_1 obeys condition $B(0; \frac{\pi}{2})$ and f_2 obeys condition $B(-\frac{\pi}{2}; 0)$? Here we say that a function f obeys condition $B(\widehat{\alpha}; \widehat{\beta})$ if

$$\sup_{\varphi\in(\widehat{\alpha};\widehat{\beta})} \left\{ \int_0^{+\infty} |f(re^{i\varphi})| dr \right\} < +\infty.$$

In [1] the solution of a narrower problem was obtained.

Theorem. *If the function $f \in W^1_\sigma$ satisfies the condition*

$$\int_1^{+\infty} \left| \sum_{k=-\infty}^{+\infty} c_k \frac{k}{\left(x + \frac{\pi}{\sigma}\left(\frac{i}{2} - k\right)\right)\left(x + \frac{\pi}{\sigma}\left(\frac{i}{2} - ik\right)\right)} \right| dx < +\infty,$$

where coefficients c_k defined by equality (1), then the Problem is positively solvable.

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ON STABILITY TO PERTURBATIONS OF BRANCHED CONTINUED FRACTIONS WITH POSITIVE ELEMENTS

We consider the problems of the stability to perturbations of the branched continued fractions (BCF) with positive elements

$$a_0\left(b_0+\mathop{\mathrm{D}}_{k=1}^{\infty}\sum_{i_k=1}^{N}\frac{a_{i(k)}}{b_{i(k)}}\right)^{-1}, \tag{1}$$

where $N\in\mathbb{N}$, $i(k)$ is the multi-index: $i(0)=i_0=0$, $i(k)=(i_1i_2\dots i_k)$, $k=1,2,\dots$ Let $I_0=\{0\}$, $I_k=\left\{i(k):1\le i_p\le N, p=\overline{1,k}\right\}$, $k=1,2,\dots$

Theorem. *Let the relative errors* $\alpha_{i(k)}$, $\beta_{i(k)}$ *of the elements* $a_{i(k)}$, $b_{i(k)}$ *of BCF (1) satisfy following conditions:* $\left|\alpha_{i(k)}\right|\le\alpha$, $\left|\beta_{i(k)}\right|\le\beta$, $0\le\alpha<1$, $0\le\beta<1$, $\alpha+\beta\ne0$, $i(k)\in I_k$, $k=0,1,2,\dots$ *Then the sequence of sets* $\Omega_0=\left(0,\mu_0^{(2)}\right]\times\left[\nu_0^{(1)},+\infty\right)$, $\Omega_{i(k)}=\left[\mu_k^{(1)},\mu_k^{(2)}\right]\times\left[\nu_k^{(1)},\nu_k^{(2)}\right]$, $0<\mu_k^{(1)}<\mu_k^{(2)}$, $0<\nu_k^{(1)}<\nu_k^{(2)}$, $i(k)\in I_k$, $k=1,2,\dots$, is the sequence of sets of the relative stability to perturbations of the BCF (1), if the sequence $\sum\limits_{n=1}^{s}\dfrac{\mu_n^{(2)}}{r_{n-1}^{(s)}r_n^{(s)}}\prod\limits_{k=1}^{n-1}\left(1+\dfrac{\nu_{k-1}^{(1)}r_k^{(s)}}{N\mu_k^{(2)}}\right)^{-1}$, $s=1,2,\dots$, is bounded, where $r_k^{(s)}=\nu_k^{(1)}+\dfrac{N\mu_{k+1}^{(1)}}{\nu_{k+1}^{(2)}}+\dfrac{N\mu_{k+2}^{(2)}}{\nu_{k+2}^{(1)}}+\ldots+\dfrac{N\mu_s^{(p_2(s,k))}}{\nu_s^{(p_1(s,k))}}$, $k=\overline{0,s-1}$, $r_s^{(s)}=\nu_s^{(1)}$, $s=1,2,\dots$, $p_j(s,k)=j+(-1)^{j+1}\left(s-k-2\left[(s-k)/2\right]\right)$, $k=\overline{0,s-1}$, $s=1,2,\dots$, $j\in\{1,2\}$. For the relative errors of approximants of the BCF (1), the estimate

$$\left|\varepsilon^{(s)}\right|\le\alpha+(1+\alpha)\left(\frac{\beta}{1-\beta}+\frac{\alpha N}{1-\alpha}\sum_{n=1}^{s}\frac{\mu_n^{(2)}}{r_{n-1}^{(s)}r_n^{(s)}}\prod_{k=1}^{n-1}\left(1+\frac{\nu_{k-1}^{(1)}r_k^{(s)}}{N\mu_k^{(2)}}\right)^{-1}\right),$$

$s=0,1,2,\dots$, is valid.

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PADÉ TYPE APPROXIMANTS FOR SOME SPECIAL POWER SERIES OF THE TWO VARIABLES

The spread of Dzyadyk's method of generalized moment representations to two-dimensional numerical sequences has been carried out in [1]. In this report the case of two-dimensional numerical sequences $\{s_{k,m}\}_{k,m=0}^{\infty}$ is considered having two-dimensional moment representations of the form

$$s_{k,m} = \langle A^k B^m x_{0,0}, y_{0,0} \rangle, \quad k, m \in \mathbb{Z}_+,$$

where A is integral operator $(A\varphi)(t) = \int_0^t \varphi(\tau) d\tau$ in the space $\mathcal{X}_\alpha = \left\{ x(t) : \sup_{t\in[0,1]} |x(t)\, t^\alpha| < \infty \right\}$, $B = A^p,\ p \geq 2,$ and $x_{0,0}(t) = t^\nu$, $y_{0,0}(t) = (1-t)^\sigma,$ where $\nu, \sigma > -\alpha$.

By using this representations two-dimensional Padé type approximants are constructed and studied for functions of the form

$$f(z,w) = \frac{z^p}{z^p - w} \widetilde{f}(z) - \frac{w^{1/p}}{p} \sum_{r=0}^{p-1} \frac{\xi_r^{(p)} \widetilde{f}(w^{1/p} \xi_r^{(p)})}{z - w^{1/p} \xi_r^{(p)}},$$

where $\widetilde{f}(z) = \frac{\Gamma(\nu+1)\Gamma(\sigma+1)}{\Gamma(\nu+\sigma+2)}\, {}_1F_1(1; \nu+\sigma+2; z)$, a ${}_1F_1(a;b;z)$ – confluent hypergeometric function and $\xi_r^{(p)} = e^{2\pi i r/p}$, $r = \overline{0, p-1}$, – the p-th roots of unity.

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Fourier-Bessel transform of set $\dot{S}_{m_{2k,2q}}$

Properties of Fourier transform of S type spaces were proved in [1]. Here, analogous properties of Fourier-Bessel transform are obtained.

Let $\dot{S}_{m_{2k,2q}}$, $\{k,q\} \subset \mathbb{Z}_+$, be the set of paired infinitely differentiable functions φ, for which:

1) $\exists C > 0\ \exists A > 0\ \exists B > 0\ \forall x \in [0;+\infty]\ \forall\{k,q\} \subset \mathbb{Z}_+ :$

$$|x^{2k}\varphi^{(2q)}(x)| \le CA^{2k}B^{2q}m_{2k,2q}; \tag{1}$$

2) the Bessel operator $B_p := \frac{d^2}{dx^2} + \frac{2p+1}{x}\frac{d}{dx}$, $p > -\frac{1}{2}$, is applicable.

Theorem 1. *For functions $\varphi \in \dot{S}_{m_{2k,2q}}$ the condition (1) is equivalent to the condition*

1) $\exists C = C(p) > 0\ \exists A > 0\ \exists B = B(p) > 0\ \forall x \in [0;+\infty]\ \forall\{k,q\} \subset \mathbb{Z}_+ :$

$$|x^{2k}B_p^q\varphi(x)| \le CA^{2k}B^{2q}m_{2k,2q}. \tag{2}$$

Direct and inverse Fourier-Bessel transforms are determined for functions of $\dot{S}_{m_{2k,2q}}$:

$\phi(\sigma) := F_{B_p}[\varphi](\sigma) = \int\limits_0^{+\infty} \varphi(x)j_p(\sigma x)x^{2p+1}dx,$

$\varphi(x) := F_{B_p}^{-1}[\phi](x) = \frac{1}{2^{2p}\Gamma^2(p+1)} \int\limits_0^{+\infty} \phi(\sigma)j_p(\sigma x)\sigma^{2p+1}d\sigma,$

where j_p is the normalized Bessel function.

Theorem 2. $F_{B_p}[\dot{S}_{m_{2k,2q}}] = \dot{S}_{m_{2q,2k}}$, *if numbers $m_{2k,2q}$ are such that*

1) $\exists \gamma > 0\ \exists \theta \le 1\ \forall\{k,q\} \subset \mathbb{N} : 2q \cdot 2k\frac{m_{2k-1,2q-1}}{m_{2k,2q}} \le \gamma(2k+2q)^\theta;$
2) $\forall \varepsilon > 0\ \exists \mu_\varepsilon > 0\ \forall\{k,q\} \subset \mathbb{N} : \frac{m_{2k+2,2q}}{m_{2k,2q}} \le \mu_\varepsilon(1+\varepsilon)^{(2k+2q)}.$

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ON SINGULAR PERTURBED SPECTRAL PROBLEMS FOR THE LAPLACIAN ON GEOMETRICAL GRAPHS

Netlike structures in physics, engineering, etc. (such as networks of flexible strings, networks of waveguides, electrical and hydraulic networks, complex molecules, neurons and all that) can be often formalized as the one-dimensional continuum connecting and interacting by nodes. The processes occurring in such systems can be usually described by classical mathematical models realized on geometrical graphs.

Focusing on mechanical interpretation, we study the asymptotics of the spectrum of the eigenvibrations problem for flexible string networks with singularly perturbed density or stiffness. For problems with local density perturbations in the vicinity of unfixed nodes, the density function has the form $\varepsilon^{-m}q(\varepsilon^{-1}(x-a))$, $m \in \mathbb{R}$, near such a node a. Here ε is a small parameter, and parameter m specifies the perturbation power. The effect of m on the behavior of the spectrum has been settled. There are following typical cases: 1) $m < 1$; 2) $m = 1$; 3) $1 < m < 2$; 4) $m = 2$; 5) $m > 2$. In the cases 1)–3) limiting problems have been stated and convergence theorems have been proved. The case 4) was concidered for the star-like graph; the limiting spectral problem, which is nonself-adjoint as distinct from perturbed problem, have been particularly analysed.

For two-component string networks, the stiff problem and the eigenvibration problem of network with contrast density have been considered. In the first problem, the stiffness coefficients of the components have a different order of smallness. In the second, the component densities are very different. In order to study the asymptotics of the spectrum of the stiff problem, the technique [1] has been used. It is based on a reduction of the original problem to a spectral problem on the less stiff part of the system, and on the resolvent convergence of operators.

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INVERSE SCATTERING FOR SINGULAR ENERGY-DEPENDENT SCHRÖDINGER EQUATIONS

We discuss the direct and inverse scattering theory for the one-dimensional energy-dependent Schrödinger equations

$$-y'' + q(x)y + 2kp(x)y = k^2 y \tag{1}$$

on the half-line under minimal assumptions on the real-valued potentials p and q. Namely, the potential q is a distribution from the space $H^{-1}_{2,\mathrm{loc}}(\mathbb{R}_+)$ enjoying certain integrability conditions, while p belongs to $L_1(\mathbb{R}_+) \cap L_2(\mathbb{R}_+)$. The above equation (1), being subject to some boundary conditions, leads to a non-standard spectral problem involving both the spectral parameter k and its square k^2. Due to this, the spectrum of the problem is no longer real and may contain pairs of complex conjugate eigenvalues.

In the talk, we introduce the scattering data for (1) and give a complete description of such data for the considered class of problems. Also, a procedure reconstructing the potentials p and q from the scattering data is described and continuity of the mapping between the potentials p and q and the corresponding scattering data is established.

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DETERMINATION OF THE COEFFICIENT AT THE FIRST DERIVATIVE IN A STRONGLY DEGENERATE PARABOLIC EQUATION

In the domain $Q_T = \{(x,t) : 0 < x < h, 0 < t < T\}$ we consider an inverse problem of the identification of the time-dependent coefficient $b = b(t)$ in the one-dimensional parabolic equation

$$u_t = a(t)t^{\beta}u_{xx} + b(t)u_x + c(x,t)u + f(x,t) \tag{1}$$

with the initial condition

$$u(x,0) = \varphi(x),\ 0 \le x \le h, \tag{2}$$

boundary conditions

$$u(0,t) = \mu_1(t),\ u(h,t) = \mu_2(t),\ t \in [0,T] \tag{3}$$

and overdetermination condition

$$\int_0^h u(x,t)dx = \mu_3(t),\ t \in [0,T]. \tag{4}$$

The solution of the problem (1)-(4) is a pair of functions $(b,u) \in C[0,T] \times C^{2,1}(Q_T) \cap C^{1,0}([0,h] \times (0,T])$ which satisfies the conditions (1)-(4).

We establish the conditions of the existence and uniqueness of the solution to the named problem in a case of strong degeneration ($\beta \ge 1$). Note that the case of weak power degeneration ($0 < \beta < 1$) to the problem (1)-(4) has been investigated in [1].

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NONLOCAL BOUNDARY VALUE PROBLEM FOR A SYSTEM OF DIFFERENTIAL-OPERATOR EQUATIONS IN THE REFINED SOBOLEV SCALE

In the cylindrical domain $D^p = [0,T] \times \mathcal{S}^p$, $\mathcal{S} \subset \mathbb{C}\backslash\{0\}$, $T > 0$, $p \geq 2$, the non-local boundary value problem for a system of n order ($n \geq 1$) partial differential equations with operator coefficients

$$\sum_{s_0+|s|\leq n} A_{s_0,s} B^s \frac{\partial^{s_0} u}{\partial t^{s_0}} = 0, \tag{1}$$

$$\mu \frac{\partial^j u}{\partial t^j}\bigg|_{t=0} - \frac{\partial^j u}{\partial t^j}\bigg|_{t=T} = \varphi_j, \qquad j = 0, 1, \ldots, m-1, \tag{2}$$

where $s = (s_1, \ldots, s_p) \in \mathbb{Z}_+^p$, $|s| = s_1 + \ldots + s_p$, $A_{s_0,s}$ ($A_{n,0} = I$) are the square matrices of order $m \geq 1$, $B^s = B_1^{s_1} \cdots B_p^{s_p}$, $\varphi_j = \text{col}\,(\varphi_{j1}(z), \ldots, \varphi_{jm}(z))$ are given vector-functions, $u = \text{col}\,(u_1(t,z), \ldots, u_m(t,z))$ is some unknown vector-function, $\mu \neq 0$ the complex parameter, is examined. The operator $B = (B_1, B_2, \ldots, B_p)$ is composed of the operators of generalized differentiations, namely $B_j \equiv z_j \dfrac{\partial}{\partial z_j}$, in particular, $B_j z^k = k_j z^k$, $B_j^0 u \equiv u$, $B_j^l u = B_j(B_j^{l-1} u)$, where $j = 1, \ldots, p$.

The nonlocal problem (1), (2) is considered in the Hörmander spaces of several complex variables functions which form a refined Sobolev scale. The smoothness of the functions of this scale is determined by two parameters: numerical (real number) and functional (positive slowly varying function at infinity). A refined Sobolev scale allows us to characterize the smoothness of functions more finely.

The problem (1), (2) is incorrect in the Hadamard sense and its solvability depends on the small denominators arising in the construction of the solution. By using of metric approach, theorems about lower estimations of small denominators was proved. Solvability conditions of the problem for almost all vectors composed of equations coefficients and boundary conditions parameter are established.

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INITIAL-BOUNDARY VALUE PROBLEMS FOR THE PARABOLIC DELAY EQUATIONS WITH VARIABLE EXPONENTS OF NONLINEARITY

Suppose $\Omega \subset \mathbb{R}^n$ is a bounded domain with the piecewise smooth boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1$ ($\Gamma_0 \cap \Gamma_1 = \emptyset$), $\nu = (\nu_1, \dots, \nu_n)$ is a unit outward pointing normal vector on the $\partial\Omega$, $T > 0$. Put $Q := \Omega \times (0,T)$, $\Sigma_0 := \Gamma_0 \times (0,T)$, $\Sigma_1 := \Gamma_1 \times (0,T)$. For $r \in L_\infty(\Omega)$, $r(x) \geq 1$ for a.e. $x \in \Omega$, $L_{r(\cdot)}(Q)$ is a generalized Lebesgue space, which consists of the functions $v \in L_1(Q)$ such that $\rho_r(v) < \infty$, where $\rho_r(v) := \iint_Q |v(x,t)|^{r(x)}\, dxdt$. This is a Banach space with respect to the norm $||v||_{L_{r(\cdot)}(Q)} := \inf\{\lambda > 0 \mid \rho_r(v/\lambda) \leq 1\}$.

Let $p = (p_0, \dots, p_n) : \Omega \to \mathbb{R}^{n+1}$ be a measurable function such that $p_j(x) > 1$ for a.e. $x \in \Omega$. Denote by $W^{1,0}_{p(\cdot)}(Q)$ a space of functions $w \in L_{p_0(\cdot)}(Q)$ such that $w_{x_1} \in L_{p_1(\cdot)}(Q)$, $\dots$, $w_{x_n} \in L_{p_n(\cdot)}(Q)$, with the norm $||w||_{W^{1,0}_{p(\cdot)}(Q)} := ||w||_{L_{p_0(\cdot)}(Q)} + \sum_{i=1}^n ||w_{x_i}||_{L_{p_i(\cdot)}(Q)}$. Define $\widetilde{W}^{1,0}_{p(\cdot)}(Q)$ be the subspace of the space $W^{1,0}_{p(\cdot)}(Q)$ that is the closure of $\widetilde{C}^{1,0}(\overline{Q}) := \big\{w \in C(\overline{Q}) \bigm| w_{x_i} \in C(\overline{Q})\ (i = \overline{1,n}),\ \ w|_{\Sigma_0} = 0\big\}$ in $W^{1,0}_{p(\cdot)}(Q)$.

The modeling case of considered problem is finding function $u \in \widetilde{W}^{1,0}_{p(\cdot)}(Q) \cap C\left([-\tau_0, T]; L_2(\Omega)\right)$ such that

$$u_t - \sum_{i=1}^n \Big(|u_{x_i}|^{p_i(x)-2} u_{x_i}\Big)_{x_i} + |u|^{p_0(x)-2}u + \int\limits_{t-\tau(t)}^{t} c(x,t,s)u(x,s)ds = f \text{ in } Q,$$

$$u\Big|_{\Sigma_0} = 0, \quad \sum_{i=1}^n |u_{x_i}|^{p_i(x)-2} u_{x_i} \nu_i(x)\Big|_{\Sigma_1} = 0, \ \ u = u_0 \text{ on } \Omega \times [-\tau_0, 0].$$

Here $\tau : [0,T] \to \mathbb{R}$ is continuous function such that $\tau(t) \geq 0$ for all $t \in [0,T]$, $\tau_0 := -\inf\limits_{t\in[0,T]} (t - \tau(t))$, and $c : Q \times [-\tau_0, T] \times \mathbb{R} \to \mathbb{R}$, $f : Q \to \mathbb{R}$, $u_0 : \Omega \times [-\tau_0, 0] \to \mathbb{R}$ are given real-valued functions.

Under certain additional conditions on the data-in, the existence and uniqueness of the weak solution of this problem are proved. The estimate of this solution is obtained.

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AN INVERSE PROBLEM FOR A 2D PARABOLIC EQUATION

We consider an inverse problem for an anisotropic heat equation

$$u_t = a(y,t)u_{xx} + b(x,t)u_{yy} + f(x,y,t), \quad (x,y,t) \in Q_T \tag{1}$$

with initial, boundary and overdetermination conditions

$$u(x,y,0) = \varphi(x,y), \quad (x,y) \in \overline{D}, \tag{2}$$
$$u(0,y,t) = \mu_1(y,t), \quad u(h,y,t) = \mu_2(y,t), \quad (y,t) \in [0,l] \times [0,T], \tag{3}$$
$$u(x,0,t) = \mu_3(x,t), \quad u(x,l,t) = \mu_4(x,t), \quad (x,t) \in [0,h] \times [0,T], \tag{4}$$
$$a(y,t)u_x(0,y,t) = \mu_5(y,t), \quad (y,t) \in [0,l] \times [0,T], \tag{5}$$
$$b(x,t)u_y(x,0,t) = \mu_6(x,t), \quad (x,t) \in [0,h] \times [0,T], \tag{6}$$

where $D := \{(x,y) : 0 < x < h, 0 < y < l\}, Q_T := D \times (0,T), a(y,t), b(x,t)$ – unknown coefficients.

Suppose that the following assumptions hold:

(A1) $\varphi \in C^{2+\gamma}(\overline{D}), \mu_i \in C^{2+\gamma,1+\gamma/2}([0,l] \times [0,T]), i \in \{1,2\}, \mu_k \in C^{2+\gamma,1+\gamma/2}([0,h] \times [0,T]), k \in \{3,4\}, \mu_5 \in C^{\gamma,\gamma/2}([0,l] \times [0,T]), \mu_6 \in C^{\gamma,\gamma/2}([0,h] \times [0,T]), f \in C^{\gamma,\gamma/2}(\overline{Q}_T)$;

(A2) $\varphi_x(x,y) > 0, \varphi_y(x,y) > 0, (x,y) \in \overline{D}, \mu_5(y,t) > 0, (y,t) \in [0,l] \times [0,T], \mu_6(x,t) > 0, (x,t) \in [0,h] \times [0,T]$;

(A3) consistency conditions of the zero and the first order.

Conditions of existence and uniqueness of solution are given by the following theorems:

Theorem 1. Suppose that the assumptions **(A1)-(A3)** hold. Then for some $T_0 \in (0,T]$ there exists a solution of the problem (1)-(6) in the space $\in C^{\gamma,\gamma/2}([0,l] \times [0,T_0]) \times C^{\gamma,\gamma/2}([0,h] \times [0,T_0]) \times C^{2+\gamma,1+\gamma/2}(\overline{Q}_{T_0})$, $a(y,t) > 0, (y,t) \in [0,l] \times [0,T_0], b(x,t) > 0, (x,t) \in [0,h] \times [0,T_0]$.

Theorem 2. Suppose that $m_5(y,t) \neq 0, (y,t) \in [0,l] \times [0,T], m_6(x,t) \neq 0, (x,t) \in [0,h] \times [0,T]$. Then a solution of the problem (1)-(6) is unique.

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LIFE AND WORK OF PROFESSOR S. D. EIDELMAN

On June 8, 2005, the life path of S.D.Eidelman has broken. He was 85 years old, a famous Ukrainian mathematician, Doctor of Science, professor, who graduated from Chernivtsi University. He was the head of the department of differential equations in this university and became a founder of well known scientific school in partial differential equations.

Life and work of S.D.Eidelman are shortly described in this lecture.

Detailed information about the life, main achievements of S.D.Eidelman and memories about him may be found in books [1, 2].

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ON PARABOLIC INITIAL PROBLEMS OF SOLONNIKOV–EIDELMAN

The class of parabolic systems of partial differential equations, which generalize the known classes of systems, which are parabolic by S. Eidelman and V. Solonnikov is considered. The setting of initial problems is described for this class, namely, the Solonnikov-Eidelman parabolic initial problems (SEPIP). Their feature is that the initial conditions are specified with the usage of the matrix differential expression, which must satisfy the relevant conditions of conjugation.

The main results on SEPIP concerning the correct solvability of these problems in the Hölder spaces of rapidly growing functions and the analogous results for more narrow class of such problems in the corresponding Sobolev-Slobodeckij spaces are presented in papers [1] and [2] respectively. The proof of the corresponding theorem is carried out under the scheme: reducing SEPIP to the problem with zero initial data, which have the unique solution in a thin layer and appropriate estimates of their solutions and based on a detailed study of the structure and properties of regularizer for this problem. The regularizer is constructed using the operators that solve the corresponding SEPIP with constant coefficients and which contain only a group of the leader members of both the system and the initial condition (a model SEPIP). Solutions of the model SEPIP are expressed through the potentials which generated by the fundamental solution of a parabolic by Eidelman equation of an arbitrary order with constant coefficients.

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PROBLEM FOR THE NONHOMOGENEOUS EVOLUTION EQUATION OF THE SECOND ORDER WITH HOMOGENEOUS INTEGRAL CONDITIONS

Let H be a linear space and A be a given linear operator acting in it ($A\colon H \to H$). Arbitrary powers A^j, $j = 2, 3, \ldots$, are also defined in H.

We consider the following problem:

$$\left[\frac{d^2}{dt^2} - 2a(A)\frac{d}{dt} + a^2(A)\right] U(t) = f(t), \tag{1}$$

$$\int_0^T U(t)dt = 0, \qquad \int_0^T tU(t)dt = 0, \tag{2}$$

where $a(A)$ is a linear operator with entire symbol $a(\lambda)$, $U\colon [0,T] \to H$ is an unknown vector-function, $f\colon [0,T] \to H$ is a given vector-function from N, i.e. can be represented in the form of a certain Stieltjes integral over a certain measure.

We show that in this case the solution of problem (1), (2) is also expressed in a similar form. We also extend the proposed method to the case of the problem with homogeneous time integral conditions for PDE of second order in time and, in general, infinite order in spatial variable (this representation could also be obtained by means of the differential-symbol method [1]).

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ON LOCALLY CONNECTED BAIRE-ONE RETRACTS

A subset E of a topological space X is a retract of X if there exists a continuous mapping $r : X \to E$ such that $r(x) = x$ for all $x \in E$. Different modifications of this notion in which r is allowed to be discontinuous were considered in [1, 2]. The author introduced in [3] the notion of B_1-retract: a subset E of a topological space X is called *a B_1-retract of* X if there exists a sequence of continuous mappings $r_n : X \to E$ such that $r_n(x) \to r(x)$ for all $x \in X$ and $r(x) = x$ for all $x \in E$; the mapping $r : X \to E$ is called *a B_1-retraction* of X onto E;

The following result was obtained in [3].

Theorem 1. *Let X be a completely metrizable space and let E be an arcwise connected and locally arcwise connected G_δ-subspace of X. Then E is a B_1-retract of X.*

It is natural to ask is any arcwise connected G_δ-subspace E of a completely metrizable space X is a B_1-retract of this space?

We will denote by $LC(X)$ the set of all points of local connectedness of X.

Theorem 2. *Let X be a locally connected Baire space and E be a metrizable B_1-retract of X. Then the set $E \setminus LC(E)$ is of the first category in X. If, moreover, X has a regular G_δ-diagonal and E is dense in X then $LC(E)$ is a dense G_δ-subset of X.*

The following example gives the negative answer to the posed question.

Example. *Let $\mathbb{I}$ be the set of all irrational numbers and $X = \mathbb{I} \cap [0, 1]$. Define $E = \{(xt, t) : x \in X, t \in [0, 1]\}$. Then E is not a B_1-retract of $\mathbb{R}^2$.*

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THE IMLICIT FUNCTION THEOREM FOR SYSTEM OF INEQUALITIES

Consider strictly differentiable functions $f_i(x, y), i \in [1 : k]$ on the space R^{n+m}, where $x \in R^n, y \in R^m$.

Suppose $(x_0, y_0) \in R^{n+m}$ is a point and $f_i(x_0, y_0) \leq 0, i \in [1 : k]$.

Denote

$$I(x_0, y_0) = \{i \in [1 : k] : f_i(x_0, y_0) = 0\}.$$

Set

$$K = \{(\overline{x}, \overline{y}) \in R^{n+m} : \left(\frac{\partial f_i(x_0, y_0)}{\partial x}, \overline{x}\right) + \left(\frac{\partial f_i(x_0, y_0)}{\partial y}, \overline{y}\right) \leq 0, i \in [1 : k]\},$$

$$D(\overline{x}) = \{\overline{y} \in R^m : (\overline{x}, \overline{y}) \in K\}.$$

Theorem. *Assume that the above mentioned conditions hold and there existes a vector $w \in R^m$ such that*

$$\left(\frac{\partial f_i(x_0, y_0)}{\partial y}, w\right) < 0, i \in I(x_0, y_0).$$

Then for any vector $(\hat{x}, \hat{y}) \in K$ there exists a continous mapping $y : R^n \to R^m$ defined in some neighborhood V of the point x_0 such that
a) $f_i(x, y(x)) \leq 0, i \in [1 : k], x \in V, y(x_0) = y_0$,
b) the derivative of the mapping $y^{'}(x_0, \overline{x})$ at x_0 exists along with arbitrary direction $\overline{x} \in R^n$ and $y^{'}(x_0, \hat{x}) = \hat{y}, y^{'}(x_0, \overline{x}) \in D(\overline{x}), \overline{x} \in R^n$.

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BASIS PROPERTIES OF SYSTEMS OF THE BESSEL AND MITTAG-LEFFLER-TYPE FUNCTIONS

Let $J_\nu(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$ be Bessel's function of the first kind of order ν and $E_\rho(z;\mu) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\mu+k/\rho)}$ be the Mittag-Leffler-type function of order ρ. It is well known the following statement: *Let $\nu > -1$ and $(\rho_k : k \in \mathbb{N})$ be a sequence of positive zeros of J_ν. Then the system $(\sqrt{x\rho_k} J_\nu(x\rho_k) : k \in \mathbb{N})$ forms a basis in $L^2(0;1)$.*

Let $\chi \in \mathbb{R}$ and S_χ be the class of entire functions G of exponential type $\sigma \le 1$ satisfying the following conditions (see [1]): a) $c_1(1+|z|)^\chi e^{|\Im z|} \le |G(z)| \le c_2(1+|z|)^\chi e^{|\Im z|}$ if $|\Im z| \ge s_0$ for some $s_0 > 0$; b) G has an infinite number of roots $\{\rho_k : k \in \mathbb{Z} \setminus \{0\}\}$, all the roots are simple and different from zero; c) $\inf\{|\rho_k - \rho_n| : k \ne n\} > 0$; d) $c_3(1+|z|)^\chi e^{|\Im z|} \le |G(z)| \le c_4(1+|z|)^\chi e^{|\Im z|}$ if $z \notin \cup\{z : |z-\rho_k| < \delta\}$ for some $\delta > 0$; e) $|G'(\rho_k)| \ge c_5(1+|\rho_k|)^\chi$; f) $\sum_{k\in\mathbb{Z}\setminus\{0\}}(1+|\rho_k|)^{-\alpha} < +\infty$ for each $\alpha > 1$. Here by c_j we denote positive constants. We obtain the following result.

Theorem [2]. *Let $\nu \ge -1/2$, $\chi = -\nu - 1/2$ and $(\rho_k : k \in \mathbb{N})$ be a sequence of complex numbers such that $\rho_k^2 \ne \rho_n^2$ for $k \ne n$. If the sequence $(\rho_k : k \in \mathbb{Z} \setminus \{0\})$, $\rho_{-k} := -\rho_k$, $k \in \mathbb{N}$, is a sequence of zeros of some even function $G \in S_\chi$, then the system $(\sqrt{x\rho_k} J_\nu(x\rho_k) : k \in \mathbb{N})$ forms a basis in $L^2(0;1)$. Moreover, if $\nu \in [-1/2;1)$ and $\mu = \nu + 3/2$, then the system $(\sqrt{t\rho_k} J_\nu(t\rho_k) : k \in \mathbb{N})$ forms a basis for the space $L^2(0;1)$ if and only if the system $(t^{\nu+1/2} E_{1/2}(-t^2 z_k;\mu) : k \in \mathbb{N})$, $z_k := \rho_k^2$, is a basis in this space.*

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MULTIPOINT PROBLEM FOR STRONG LOADED HYPERBOLIC EQUATIONS

Let $n \in \mathbb{N}$, $\Omega^p = (\mathbb{R}/2\pi\mathbb{Z})^p$, $x = (x_1, \dots, x_p)$, $k = (k_1, \dots, k_p) \in \mathbb{Z}^p$, $|k| = |k_1| + \ldots + |k_p|$, $D = (-i\partial_{x_1}, \dots, -i\partial_{x_p})$, $L(\partial_t, D) \equiv \partial_t^n + \sum_{j=0}^{n-1} \sum_{|s|=n-j} a_{(j,s)} D^s \partial_t^j$ be a strongly hyperbolic differential expression, $a_{(j,s)} \in \mathbb{R}$, $D^s = \partial^{|s|}/(\partial x^{s_1} \ldots \partial x^{s_p})$, H_α ($\alpha \in \mathbb{R}$) be a space of the trigonometric series $\varphi = \sum \varphi_k \exp(ik, x)$ for which the norm $\|\varphi(x); H_\alpha\| = \sqrt{\sum\limits_{k \in \mathbb{Z}^p} |\varphi_k|^2 (1 + |k|)^{2\alpha}}$ is a finite.

We consider the problem

$$\begin{aligned} &L(\partial_t, D)\, u(t,x) = F(t,x) + N(D)[u(t,x)], \quad t \in (0;T), \quad x \in \Omega^p, \\ &u(t_j, x) = \varphi_j(x), \quad j = \overline{1,n}, \quad x \in \Omega^p, \end{aligned} \tag{1}$$

where $N(D)[u(t,x)] = \sum\limits_{j=1}^{m} \sum\limits_{r=0}^{n-1} \sum\limits_{|s| \le M} b_{jr}^s D^s \partial_t^r u(t,x)|_{t=\tau_{r,j}}$, $M < n$, $\tau_{r,j}$, $j = \overline{1,m}$, $q = \overline{1,m}$ and t_j, $j = \overline{1,m}$, are different points from $(0;T)$.

Let $\lambda_q(k)$, $q = \overline{1,n}$, be the λ-roots of equation $L(\lambda, k) = 0$, $k \in \mathbb{Z}^p$, $y_{qk}(t) = t^{q-1}$, $q = \overline{1,n}$, if $k = \vec{0}$, $y_{qk}(t) = \exp(i\lambda_q(k)t)$, $q = \overline{1,n}$, if $k \neq \vec{0}$, $\Delta(k) = \det \|y_{qk}(t_j)\|_{j,q=1}^n$, $k \in \mathbb{Z}^p$, $\Gamma(k) = 1 - N(k)[\int_0^T G_k(t,\tau)d\tau]$, $k \in \mathbb{Z}^p$, where $G_k(t,\tau)$ is the Green function of the multipoint problem $L(d/dt, k)y(t) = f(t)$, $y(t_j) = 0$, $j = \overline{1,n}$.

Theorem. *Suppose that the inequalities $|\Delta(k)| \neq 0$, $|\Upsilon(k)| \neq 0$ are satisfied for all $k \in \mathbb{Z}^p$ and suppose that constants ω_1, ω_2 and γ exist such that for all (except a finite number) vectors $k \in \mathbb{Z}^p$ the following inequalities $|\Delta(k)| \ge (1 + |k|)^{-\omega_1}$, $|\Gamma(k)| \ge (1 + |k|)^{-\omega_2}$ and $|\sum_{|s|=n} a_{(0,s)} k_1^{s_1} \cdot \ldots \cdot k_p^{s_p}| \ge (1 + |k|)^{-\gamma}$ are hold. If $F \in C([0;T], H_{\eta-2})$, $\varphi_j \in H_{\eta-1}$, $j = \overline{1,n}$, $\eta = \alpha + 2\omega_1 + \omega_2 + \gamma + 2n + M$, then the unique solution of the problem (1) exists in the space $C([0;T], H_\alpha)$ and continuously depends on $F(t,x)$ and $\varphi_j(x)$, $j = \overline{1,n}$.*

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ASYMPTOTICALLY OPTIMAL CONTROL IN DISCRETE PROBLEM WITH VARIABLE DELAY

The optimal control problem is described by a system of discrete equations with variable delay and the terminal quality criterion

$$x_{i+1} = x_i + \varepsilon \cdot \left[f\left(i, x_i, x_{s(i)}\right) + A\left(x_i, x_{s(i)}\right) \cdot \varphi\left(i, u_i, u_{ss(i)}\right) \right], \quad x_0 = x^0,$$

$$J(u) = \Phi(x_N) \to \min_{u \subset U}.$$

Here $x_i \in D \subset R^n$ is a state of the system, $i \in I = \{0, 1, 2, ..., N\}$ is time of the system, $N = E\left(L\varepsilon^{-1}\right)$, $L = const$, $E(c)$ is the integer part of c, $\varepsilon > 0$ is a small parameter, $u_i \in U \subset comp(R^r)$ is a control of the compact subset U. The functions $s(i) \in I_s = \{0, 1, 2, ..., i\}$ and $ss(i) \in I_s = \{0, 1, 2, ..., i\}$ describe the state delay and control delay.

Let the limits

$$f_0\left(w^1, w^2\right) = \lim_{h \to \infty} \frac{1}{h} \sum_{j=q}^{q+h-1} f\left(j, w^1, w^2\right), V = \lim_{h \to \infty} \frac{1}{h} \sum_{j=q}^{q+h-1} \varphi(j, U, U).$$

exist evenly on the $q \geq 0$, $w^1, w^2 \in D$.

Then the averaged optimal control problem may be described by a system of the discrete equations and the terminal quality criterion in slow time

$$\xi_{k+1} = \xi_k + \varepsilon h \cdot \left[f_0\left(\xi_k, \xi_{m(k)}\right) + A\left(\xi_k, \xi_{m(k)}\right) \cdot v_k \right], \quad \xi_0 = x^0,$$

$$J_0(v) = \Phi\left(\xi_{N_k}\right) \to \min_{v \subset V},$$

$$k \in I_k = \{0, 1, 2, ..., N_k\}, \quad N_k = E\left(\frac{L}{\varepsilon h}\right), \quad m(k) = E\left(\frac{s(kh)}{h}\right).$$

The asymptotically optimal control of the original problem is constructed on the optimal control of the averaged problem.

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INVERSE PROBLEM FOR A PARABOLIC EQUATION WITH A NONLOCAL OVERDETERMINATING CONDITION

The inverse problem of finding a solution pair $(a(t), u(x,t))$ of the parabolic equation:

$$u_t = a(t)u_{xx} + b(x,t)u_x + c(x,t)u + f(x,t), \quad (x,t) \in Q_T \tag{1}$$

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

$$u(0,t) = \mu_1(t), \quad u(h,t) = \mu_2(t), \quad t \in [0,T] \tag{3}$$

$$\nu_1(t)u_x(0,t) + \nu_2(t)u_x(h,t) = \mu_3(t), \quad t \in [0,T], \tag{4}$$

is investigated in the domain $Q_T := \{(x,t) : 0 < x < h, 0 < t < T\}$.

The following theorems are proved.

Theorem 1. *Under the conditions*
(A1) $b, c, f \in C^{1,0}(\overline{Q_T}), \quad \varphi \in C^2([0,h]), \quad \mu_1, \mu_2, \mu_3, \nu_1, \nu_2 \in C^1([0,T])$;
(A2) $\varphi''(x) > 0, \quad x \in [0,h], \quad \nu_1(t) < 0, \nu_2(t) > 0, \nu_1(t) + \nu_2(t) > 0,$ $\mu_3(t) - \frac{\nu_1(t)+\nu_2(t)}{h}(\mu_2(t) - \mu_1(t)) > 0, \quad b(0,t) > 0, \quad \mu_1'(t) - c(0,t)\mu_1(t) - f(0,t) - \frac{b(0,t)\mu_3(t)}{\nu_2(t)+\nu_1(t)} > 0, \quad b(h,t) > 0, \quad \mu_2'(t) - c(h,t)\mu_2(t) - f(h,t) - \frac{b(h,t)\mu_3(t)}{\nu_2(t)+\nu_1(t)} > 0, \quad t \in [0,T]$;
(A3) $\varphi(0) = \mu_1(0), \qquad \varphi(h) = \mu_2(0), \quad \nu_1(0)\varphi'(0) + \nu_2(0)\varphi'(h) = \mu_3(0)$
the problem (1)-(4) has at least one solution $(a,u) \in C([0,t^*]) \times C^{2,1}(\overline{Q}_{t^*})$, *where* $t^* \in (0,T]$ *is determined from the initial data.*

Theorem 2. *If* $b, c \in C^{\alpha,0}(\overline{Q}_{t^*})$ *and the condition (A2) holds, the solution of the problem* (1)-(4) *is unique in* $C([0,t^*]) \times C^{2+\alpha,1}(\overline{Q}_{t^*})$.

In the proof of Theorem 1, the theory of Green's functions is utilized to reformulate problem (1)-(4) as an equivalent system of integral equations. The Schauder fixed point theorem is used to prove the existence of its solution.

In Theorem 2, the problem (1)-(4) is again reduced to a system of integral equations and the uniqueness of the solution is obtained due to properties of Volterra integral equations of the second kind with integrable kernels.

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BIFURCATION ANALYSIS OF BOUNDARY VALUE HYPERBOLIC PROBLEMS

In the talk, we will concentrate on boundary value problems for general one-dimensional semilinear first-order hyperbolic systems. We state conditions for Hopf bifurcation [1], i.e., for existence, local uniqueness (up to phase shifts), smoothness and smooth dependence on control parameters of time-periodic solutions bifurcating from the zero stationary solution. The proof is done by means of a Lyapunov-Schmidt reduction procedure, where the Fredholm property of the linearization [2] and a generalized implicit function theorem [3] are essentially used. The problem of small denominators arising here is prevented by a certain non-resonance condition formulated in terms of some coefficients of PDEs and boundary conditions. It should be noted that, among others, the question if a non-degenerate time-periodic solution depends smoothly on the system parameters is much more delicate for hyperbolic PDEs than in the case of parabolic PDEs or ODEs.

This is a joint work with Lutz Recke.

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FREDHOLMNESS OF DISSIPATIVE BOUNDARY VALUE PROBLEMS FOR FIRST-ORDER HYPERBOLIC SYSTEMS

We investigate a large class of linear boundary value problems for the general first-order one-dimensional hyperbolic systems in the strip, namely

$$\partial_t u_j + a_j(x,t)\partial_x u_j + \sum_{k=1}^{n} b_{jk}(x,t)u_k = f_j(x,t), \quad (x,t) \in (0,1)\times\mathbb{R}, \quad j \le n, \tag{1}$$

$$\begin{aligned} u_j(0,t) &= (Ru)_j(t), \quad 1 \le j \le m,\ t \in \mathbb{R}, \\ u_j(1,t) &= (Ru)_j(t), \quad m+1 \le j \le n,\ t \in \mathbb{R}, \end{aligned} \tag{2}$$

where $R = (R_1, ..., R_n)$ is a linear bounded operator in the space of bounded and continuous functions.

We state natural conditions on the data such that the operators of the problems satisfy the Fredholm alternative theorem in the spaces of continuous and bounded functions. Crucial for our analysis is a dissipativity condition formulated in terms of the data responsible for the bijective part of the Fredholm operator. We state the dissipativity condition in terms of coefficients of the differential and the diagonal zero-order parts of the hyperbolic systems as well as the boundary data. We provide sharp dissipative (non-resonance) conditions in the case of reflection boundary conditions.

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SOME NEW RESULTS FOR BERNOULLI POLYNOMIALS AND CAUCHY POLYNOMIALS

The integral values of Bernoulli polynomials are expressed in terms of r-Stirling numbers of the second kind, and the integral values of Cauchy polynomials are expressed in terms of r-Stirling numbers of the first kind. Several relations between the integral values of Bernoulli polynomials and those of Cauchy polynomials are obtained in terms of r-Stirling numbers of both kinds. We also define q-multiparameter-Bernoulli polynomials and q-multiparameter-Cauchy polynomials by Jackson's integrals, and investigate their properties connected with usual Stirling numbers and weighted Stirling numbers. We also give the relations between generalized poly-Bernoulli polynomials and two kinds of generalized poly-Cauchy polynomials.

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ON PASTING OF TWO DIFFUSION PROCESSES ON A LINE WITH NONLOCAL CONJUGATION CONDITION OF THE FELLER-WENTZELL TYPE

The problem of pasting of two one-dimensional processes with different variants of general conjugation condition of Feller-Wentzell type [1] that includes integral term, was studied by us in the paper [2]. For the investigation we have used an analytical approach according to which the problem of existence of required Feller semigroup that generates sought-for process is narrowed down to solution of a corresponding conjugation problem for a linear parabolic equation of second kind with break in pasting point. As for the last problem, the parabolic conjugation problem, it's classical solvability was ascertained by us using boundary integral equations method with help of simple-layer potential.

A problem of existence of transition probability density that corresponds to described processes is studied here. It is known that solution of the problem is directly linked to the possibility of solving mentioned parabolic conjugation problems using Green's function method [3].

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ON THE TWO-PARAMETER FELLER SEMIGROUP THAT CORRESPONDS TO THE MULTIDIMENSIONAL NONHOMOGENEOUS DIFFUSION PROCESS WITH THE GENERALIZED DRIFT VECTOR AND DIFFUSION MATRIX

Diffusion processes with generalized drift vector and diffusion matrix in particular appear while solving a so called problem of pasting of two diffusion processes in finite-dimensional euclidian spaces when general Wentzel conjugation condition is given on a hypersurface of breach of initial diffusion characteristics of the process [1]. Processes of this kind can be a mathematical model for a diffusion in a media with a membrane (see [2]).

One of special cases for pasting of two multidimensional diffusion processes problem was studied by us earlier in paper [3]. There we had assumed that the medium where the diffusion process is considered has homogeneous diffusion characteristics and those characteristics together with diffusion coefficients from Wentzel boundary conditions depends only on space variable. An analytical approach with use of classical potential theory was used to solve the problem. Now the result of paper [3] is generalized for a case when all the parameters of the problem are dependent on space and time variables. This means that sought processes will be nonhomogeneous with respect to time variable.

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ON EXISTENCE AND UNIQUENESS OF SOLUTION OF THE NONLOCAL CONJUGATION PROBLEM FOR A PARABOLIC EQUATION WITH DISCONTINUOUS COEFFICIENTS

In the report we introduce the results concerned with an application of the method of potential theory to solution of the conjugation problem for the one-dimensional (with respect to spatial variable) linear parabolic equation of the second order with discontinuous coefficients assuming that different variants of the nonlocal Feller-Wentzell conjugation condition [1] are given on the curve of discontinuity of the equation coefficients. Note (see, for instance, [2]) that the parabolic conjugation problem with the Feller-Wentzell conjugation condition arises particularly in the theory of stochastic processes in investigation og the so-called problem of pasting together the two diffusion processes on a line using analytical methods.

The classical solvability of the problem described here is established for the first time by the boundary integral equations method with the use of the ordinary simple-layer potential under some conditions on its output data. The application of obtained results in studying the already mentioned problems of the theory of diffusion processes is also considered.

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INITIAL BOUNDARY VALUE PROBLEMS FOR THE SECOND ORDER PARABOLIC EQUATION WITH THE WENTZELL TYPE BOUNDARY AND CONJUGATION CONDITIONS

We investigate the conjugation problems for the second order linear parabolic equation in non-cylindrical bounded domains of the finite-dimensional Euclidean space under the assumption that on the common part of domains boundaries Wentzell type conjugation condition [1,2] is given. Besides the first order derivatives with respect to spatial variables, this condition contains the derivative with respect to time variable and the second order derivatives with respect to tangent directions. Concerning the boundary conditions given on the external part of the boundary of one of the domains, we consider here different variants of general Wentzell type boundary condition. The classical solvability of these boundary value problems is established in Hölder spaces by the boundary integral equations method with the use of the ordinary simple-layer potential. We also study the problem on application of the obtained results to construction of integral representations of two-parameter operator semigroups describing the diffusion processes in medium with sticky membranes placed on a fixed surfaces [3].

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ESTIMATES OF EQUIMEASURABLE REARRANGEMENTS OF FUNCTIONS IN TERMS OF THE RELATIVE MEAN OSCILLATION

An estimate of the equimeasurable rearrangement of a function in terms of relative integral mean oscillations is obtained. This estimate implies the known Gurov-Reshetnyak Lemma and its generalizations on the case of the Orlicz classes. In addition, it enables us to obtain estimates on the uniform modulus of continuity of a function by its mean oscillations, and also some other results.

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ON SIMULTENEOUS DIOPHANTINE APPROXIMATION AND THE PROBLEM OF SMALL DIVISORS

One of the applications of the Diophantine approximation is the solving of the problem of small divisors in partial differential equations [1]. Recently, mathematicians from Vladimirov's school have proposed a new line of research which consists in consideration of the problem in the field $\mathbb{Q}_p$ of p-adic numbers. We obtain the result on simultaneous Diophantine approximation in regard to the Archimedean and non-Archimedean valuations.

Let $P = P(t) = a_n t^n + \cdots + a_1 t + a_0 \in \mathbb{Z}[t]$, $a_n \neq 0$. Assume that $H = \max(|a_n|, \ldots, |a_0|)$ increases. Let $p_j \geq 2$ be different prime numbers, $\mathbb{Q}_{p_j}$ be the field of p_j-adic numbers, $|\cdot|_{p_j}$ be the p_j-adic valuation $(j = 1, 2, 3)$. Let $\mathcal{O} = \mathbb{R} \times \mathbb{C} \times \prod_{j=1}^{3} \mathbb{Q}_{p_j}$. We define a measure $\overline{\mu}$ in $\mathcal{O}$ as a product of the Lebesque measure μ_1 in $\mathbb{R}$, the Lebesque measure μ_2 in $\mathbb{C}$ and the Haar measure μ_j in $\mathbb{Q}_{p_j}$ $(j = 1, 2, 3)$. Let $\Psi : \mathbb{N} \to \mathbb{R}^+$ be a monotonic decreasing function, $\Lambda = (\lambda_1; \lambda_2; \lambda_{31}, \lambda_{32}, \lambda_{33})$ and $V = (v_1; v_2; v_{31}, v_{32}, v_{33})$ be nonnegative vectors in $\mathbb{R}^5$. We consider the system of inequalities

$$|P(x)| < H^{-v_1} \Psi(H)^{\lambda_1}, \quad |P(z)| < H^{-v_2} \Psi(H)^{\lambda_2},$$
$$|P(\omega_j)|_{p_j} < H^{-v_{3j}} \Psi(H)^{\lambda_{3j}} \quad (j = 1, 2, 3) \tag{1}$$

where $(x; z; \omega_1, \omega_2, \omega_3) \in \mathcal{O}$, $v_1 + 2v_2 + v_{31} + v_{32} + v_{33} = n - 4$ and $\lambda_1 + 2\lambda_2 + \lambda_{31} + \lambda_{32} + \lambda_{33} = 1$.

Let $M_n(V, \Psi, \Lambda)$ be a set of points in $\mathcal{O}$ such that the system (1) has infinitely many solutions in the polynomials P. Using Sprindẑuk's method of the essential and nonessential domains we prove

Theorem. *If* $n \geq 4$ *and* $\sum_{H=1}^{\infty} \Psi(H) < \infty$ *then*

$$\overline{\mu}(M_n(V, \Psi, \Lambda)) = 0.$$

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RESIDUAL STRUCTURAL AND STRESS STATE OF THE STEEL PLATES STIPULATED BY MOVING DISTRIBUTED HEAT SOURCES

The problems of the mechanics of deformable solid for definition and optimization of residual stresses and phase distributions in thin low-carbon low-alloyed steel plates subjected to high-temperature heating system of moving distributed heat sources are formulated.

Method and parametric optimization problems of moving distributed heat sources relatively to percentage of martensite content (with minimum criterion of maximum martensite content) with decreasing values of residual stress to improve the plastic properties of heat affected zone the is proposed.

On this basis, heating features in the presence of one (main) source and one additional in various (including optimal) their localization centers and power parameters is analyzed.

On the analysis of the obtained solutions a number of new laws in the distribution of residual stresses and phase components in the plate at different thermal parameters of the heating mode are defined.

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AVERAGING METHOD IN THE MULTIFREQUANCY NOETHER BOUNDARY PROBLEMS

The averaging method for extensive classes of multifrequency problems was justified in the works of A.Samoilenko and R.Petryshyn [1]. The Noether boundary problems were analysed in [2]. Some questions of the m-frequent Noether boundary problems were studied in [3].

Consider the multifrequancy system with linearly transformed arguments and boundary conditions

$$\begin{aligned} \frac{da}{d\tau} &= X(\tau, a_\Lambda, \varphi_\Theta), & \frac{d\varphi}{d\tau} &= \frac{\omega(\tau)}{\varepsilon} + Y(\tau, a_\Lambda, \varphi_\Theta), \\ A_0 a|_{\tau=0} + A_1 a|_{\tau=L} &= d_1, & B_0\varphi|_{\tau=0} + B_1\varphi|_{\tau=L} &= d_2, \end{aligned}$$

where λ_i and θ_j are numbers from semi-interval $(0,1]$, $0 < \lambda_1 < \cdots < \lambda_{r_1} \leq 1$, $0 < \theta_1 < \cdots < \theta_{r_2} \leq 1$, $a_{\lambda_i}(\tau) = a(\lambda_i\tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j\tau)$, $a_\Lambda = (a_{\lambda_1}, \ldots, a_{\lambda_{r_1}})$, $\varphi_\Theta = (\varphi_{\theta_1}, \ldots, \varphi_{\theta_{r_2}})$; $a \in D$, $\varphi \in \mathbb{R}^m$, $\tau \in [0, L]$, vector-functions X and Y are 2π-periodic with components φ_Θ; A_0, A_1 are constant $(q_1 \times n)$-matrixes $q_1 \geq n$; B_0, B_1 are constant $(q_2 \times m)$-matrixes, $q_2 \geq m$; d_1, d_2 are the preset n- and m-vectors.

Together with the original problem, we consider the boundary-value problem averaged over all angular variables. The solution of the averaged problem is much simpler than the solution of the original problem.

In this report, we show that the averaging method can be efficiently applied to analysis of the Noether boundary-value problem.

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ON LINEAR EXTENDERS AND LINEAR HOMEOMORPHISMS OF FUNCTION SPACES

For a Tychonoff space X we denote by $C_p(X)$ the space of the continuous real valued functions on X in the topology of the pointwise convergence. We say that a subspace Y of X is l-embedded in X if there exists a continuous linear mapping $\varphi\colon C_p(Y) \to C_p(X)$ such that $\varphi(g)|_Y = g$ for all $g \in C_p(Y)$. Topological spaces X and Y are called l-equivalent if linear topological spaces $C_p(X)$ and $C_p(Y)$ are linearly homeomorphic.

Theorem 1. *Let X and Y be closed subspaces of topological space $X \cup Y$ such that $X \cap Y$ is l-embedded in $X \cup Y$. Then*

$$X \oplus Y \overset{l}{\sim} (X \cup Y) \oplus (X \cap Y).$$

Let Y be a subspace of a space X and $\sim_Y$ be an equivalence relation defined on Y. We can extend this relation to the relation $\sim_X$ on X putting $x \sim_X y$ if and only if $x = y$ or $x, y \in Y$ and $x \sim_Y y$. Sometimes we shall write $\sim_X$ instead of $\sim_Y$.

Theorem 2. *Let X be a space, K_1, K_2 its l-embedded subspaces such that $X/K_1 \overset{l}{\sim} X/K_2$. Let $\sim_1$ and $\sim_2$ be equivalence relations on K_1 and K_2 respectively such that $(K_1/\sim_1) \overset{l}{\sim} (K_2/\sim_2)$. Then $(X_1/\sim_1) \overset{l}{\sim} (X_2/\sim_2)$.*

We can modify theorems 1 and 2 for the spaces $C_p^*(X)$ of continuous real valued bounded functions by introducing the notions of l^*-equivalent spaces and l^*-embedded subspaces.

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ASYMPTOTIC STABILITY ANALYSIS OF THE EULER–BERNOULLI BEAM MODEL

This talk is devoted to the study of a spectral problem

$$A\xi = \lambda\xi, \tag{1}$$

where A is the infinitesimal generator of a C_0-semigroup on the Hilbert space $X = \overset{\circ}{H}{}^2(0,l) \times L^2(0,l) \times \mathbb{R}^2$. The problem (1) is obtained by applying LaSalle's invariance principle to the stability problem for the Euler–Bernoulli beam model.

The asymptotic distribution of eigenvalues of A is described by the following transcendent equation:

$$-\frac{m}{8}\left(\sqrt{2}\cos\left(\frac{\pi}{4} - \mu l\right) - \cos(\mu(l - 2l_0))\right)\frac{e^{\mu l}}{\mu} + o\left(\frac{e^{\mu l}}{\mu}\right) = 0, \tag{2}$$

as $\mu \to +\infty$. Here $\mu = \lambda^{1/4}$ is the spectral parameter.

By studying the above equation, we establish the following limit distribution of eigenvalues:

$$\overline{\lim_{y\to\infty}}\ \overline{\lim_{z\to\infty}}\ \frac{n[y, y+z)}{z} = 0, \tag{3}$$

where $n[y, y+z)$ is the number of eigenvalues λ in the interval $[y, y+z)$.

We show that the system of eigenfunctions of A is linearly independent. By applying the LaSalle invariance principle, we conclude that the trivial solution of the corresponding Euler-Bernoulli equation is asymptotically stable.

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ON THE l-INDEX BOUNDEDNESS OF THE ENTIRE RIDGE FUNCTONS

An entire function $\varphi \not\equiv$ const is called the ridge provided if the inequality $|\varphi(z)| \le |\varphi(i\mathrm{Im}\, z)|$ holds for all $z \in \mathbb{C}$.

Let Λ be a class of positive continuous functions on $[0, +\infty)$. If $l \in \Lambda$, an entire function f is called [1, p.5] a function of bounded l-index provided there exists $N \in \mathbb{Z}_+$ such that

$$\frac{|f^{(n)}(z)|}{n! l^n(|z|)} \le \max\left\{\frac{|f^{(k)}(z)|}{k! l^k(|z|)} : 0 \le k \le N\right\} \qquad (1)$$

for all $n \in \mathbb{Z}_+$ and $z \in \mathbb{C}$. If (1) holds for $l(r) \equiv 1$ then f is called a function of bounded index.

Theorem. *If an entire ridge function φ of finite order has only real zeros and its positive zeros a_k satisfy the condition $a_k^2 - a_{k-1}^2 \nearrow +\infty$ as $k \to \infty$ then φ is a function is of bounded l-index with $l(r) = r$ for $r \ge r_0 > 0$. The condition $a_k^2 - a_{k-1}^2 \nearrow +\infty$ as $k \to \infty$ can not be replaced by the condition $a_k^2 - a_{k-1}^2 \to +\infty$ as $k \to \infty$.*

Consider a function depicted finite transformation Laplace-Stiltyes

$$f(z) = \int_{-\sigma}^{\sigma} e^{zx} dv(x), \qquad (2)$$

when $0 < \sigma < +\infty$, and v − function of bounded variation on $[-\sigma, \sigma]$. If $v(-\sigma) \ne v(-\sigma + 0)$ and $v(\sigma) \ne v(\sigma - 0)$, then finite transformation Laplace-Stiltyes (2) is a function of bounded index.

The class of entire functions of exponential type $\le \sigma$, bounded on the real axis is called class S.N. Bernstein and denote B_σ.

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SCATTERING OF SH-WAVE BY ELASTIC FIBER OF NONCANONICAL CROSS-SECTION WITH THIN INTERFACE INHOMOGENEITY OF LOW RIGIDITY

The problem of interaction of a plane time-harmonic SH-wave with an elastic fiber, when an interface thin inclusion of arbitrary thickness and low stiffness is present between the infinite elastic matrix and the fiber, is considered. The simplified asymptotically exact conditions of dynamic contact of elastic media through the inclusions of low rigidity are used [1]. The null field (T-matrix) method is applied for the solution of the scattering problem. The method was originally developed for acoustic scattering by Waterman, and has later been extended to the scattering of waves in elastic solids by elastic fiber with thin interface crack [2, 3]. The square-root behaviour of the solution at the crack edge is explicitly considered in these papers. In the present paper, the idea is realized for a thin interface inclusion of low stiffness in a fiber-reinforced composite under antiplane shear conditions, when unknown displacements and stresses at the interface are expressed as infinite series trigonometric functions. On this base, the scattering amplitudes in the far field for the different fiber shapes, inclusion/matrix materials combinations, and directions of wave incidence are studied.

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FUNCTIONAL SERIES EQUATIONS FOR THE ACOUSTIC AXIALLY-SYMMETRIC WAVE DIFFRACTION BY THE FINITE SOFT AND RIGID CONES

The problem of the acoustic wave diffraction by the finite soft (S) and rigid (R) cones $Q : \{r \in (0, c_1), \theta = \gamma, \varphi \in [0, 2\pi)\}$ is reduced to the system of the functional series equations as follows:

$$\sum_{(z_n)} \bar{x}_n P_{z_n-1/2}(\eta) + t(\eta) = \begin{cases} \sum\limits_{(\nu_p)} y_p^{(1)} P_{\nu_p-1/2}(\eta), & \chi < \eta \leq 1; \\ \sum\limits_{(\mu_k)} y_k^{(2)} P_{\mu_k-1/2}(-\eta), & -1 \leq \eta < \chi; \end{cases} \quad (1)$$
$$-1 \leq \eta \leq 1$$

$$\sum_{(z_n)} \bar{x}_n P_{z_n-1/2}(\eta) \frac{K'_{z_n}(\rho_1)}{K_{z_n}(\rho_1)} + t_1(\eta) = \begin{cases} \sum\limits_{(\nu_p)} y_p^{(1)} P_{\nu_p-1/2}(\eta) \dfrac{I'_{\nu_p}(\rho_1)}{I_{\nu_p}(\rho_1)}; \\ \quad \chi < \eta \leq 1 \\ \sum\limits_{(\mu_k)} y_k^{(2)} P_{\mu_k-1/2}(-\eta) \dfrac{I'_{\mu_k}(\rho_1)}{I_{\mu_k}(\rho_1)}. \\ \quad -1 \leq \eta < \chi \end{cases}$$
$$-1 \leq \eta \leq 1 \quad (2)$$

Here $\bar{x}_n$, $y_p^{(1)}$, $y_k^{(2)}$ are unknown coefficients; $t(\eta)$, $t_1(\eta)$ are known functions; $\rho_1 = sc_1$ ($s = -ik$, k is the wave number), c_1 is the cone generator; $\eta = \cos\theta$; $\chi = \cos\gamma$; $\{z_n\}_{n=1}^{\infty}$, $\{\xi_q\}_{q=1}^{\infty} \in \{\nu_p\}_{p=1}^{\infty} \bigcup \{\mu_k\}_{k=1}^{\infty}$ are the simple positive zeros and poles of the meromorphic functions

$$M(\nu, \gamma) = \frac{1}{\Gamma(\nu + 1/2)\Gamma(-\nu + 1/2)} \begin{cases} [P_{\nu-1/2}(\chi) P_{\nu-1/2}(-\chi)]^{-1}, & \text{for S;} \\ [-P^1_{\nu-1/2}(\chi) P^1_{\nu-1/2}(-\chi)]^{-1}, & \text{for R.} \end{cases}$$

Theorem. *The solution of equations (1), (2) belongs to the class of sequences* $b(\sigma) : \{\|x_n\| = \sup\limits_n |x_n n^{\sigma}| = 0, \lim\limits_{n\to\infty} x_n n^{\sigma} = 0\}$, *where* $x_n = \bar{x}_n P_{z_n-1/2}(\chi)$, $0 \leq \sigma < 3/2$ *for S and* $x_n = \bar{x}_n P^1_{z_n-1/2}(\chi)$, $0 \leq \sigma < 1/2$ *for R cones and satisfies the Meixner condition at the edges.*

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ON THE BOREL TYPE RELATION FOR LAPLACE-STIELTJES INTEGRALS

Let $\mathbb{R}_+ = [0, +\infty)$, ν be a nonnegative measure on $\mathbb{R}_+$ with unbounded support $\operatorname{supp}\nu$ and $f(x)$ be an arbitrary nonnegative ν-measurable function on $\mathbb{R}_+$. By $\mathcal{I}(\nu)$ we denote the class of function $F\colon \mathbb{R} \to \mathbb{R}_+$ of the form $F(x) = \int_{\mathbb{R}_+} f(u)e^{xu}\nu(du)$. Denote by L^+ the class of nonnegative continuous functions $\psi\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that $\psi(t) \nearrow +\infty$ as $t \to +\infty$. By $\mathcal{I}(\nu, \Phi)$ we denote the class of function $F \in \mathcal{I}(\nu)$ such that $(\exists c > 0)\colon \quad \ln F(x) \le \Phi(cx) \quad (x \ge x_0)$, where $\Phi \in L^+$. For $F \in \mathcal{I}(\nu)$ and $x \in \mathbb{R}$ we denote $\mu_*(x, F) = \sup\{f(u)e^{xu} \colon u \in \operatorname{supp}\,\nu\}$; $\nu_0(t) := \nu((0, t])$ $(t \ge 0)$.

Theorem ([1]). *Let $\Phi_0(x) = x\Phi(x)$, $\Phi \in L^+$, $F \in \mathcal{I}(\nu, \Phi_0)$. If conditions $(\forall \eta > 0)\colon\ \ln \nu_0(\eta\Phi(t)) = o(\Phi_0(t))$ $(t \to +\infty)$ and $(\forall\ \eta > 0)\colon$ $\varliminf\limits_{R\to+\infty} \dfrac{1}{R}\displaystyle\int_0^{\eta\Phi(R)} t^{-1}d\ln\nu_0(t) = 0$ are satisfied, then the relation $\ln F(x) \le (1 + o(1))\ln \mu_*(x, F)$ holds as $x \to +\infty$ $(x \notin E)$, where E is a some set of zero lower linear density, i.e. $\underline{D}E := \varliminf\limits_{R\to+\infty} \dfrac{1}{R}\operatorname{meas}(E \cap [0, R]) = 0$.*

The condition $(\forall\ \eta > 0)\colon$ $\varliminf\limits_{R\to+\infty} \dfrac{1}{R}\displaystyle\int_0^{\eta\Phi(R)} t^{-1}d\ln\nu_0(t) = 0$ is a necessary condition ([1], Theorem 4) of Theorem.

Conjecture 1. The assertion of Theorem is true without the condition $(\forall\eta > 0)\colon\ \ln\nu_0(\eta\Phi(t)) = o(\Phi_0(t))$ $(t \to +\infty)$.

Conjecture 2. The description of exeptional set E in our Theorem ($\underline{D}E = 0$) is the best possible.

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MODELLING AND METHODS FOR DETERMINATION OF THE THERMO-STRESSED STATE IN THE HOLLOW MULTI-LAYER THERMAL SENSITIVE SPHERES

The thermo-stressed state in the multi-layer construction elements, exploited at high or low temperatures will be determined using mathematical models which take into account the temperature dependencies of the layer materials on temperature [1].

The extension of the original method [2,3] to solve mentioned problems for the multi-layer spherical bodies under the centrally-symmetric temperature field and constant loading is proposed. The method is presented on an example of determination of the thermo-stressed state of a n-layer hollow sphere with the given classic condition of heat exchange and the constant force loading on the external surfaces and known heat generation in the layers and on the contact surfaces between the layers.

The method of solving the non-linear heat exchange problem makes it possible to take into account the heat exchange conditions on external surfaces in order to obtain the exact analytical solution of the problem or to reduce the problem to solving the one algebraic equation relative to one constant. The other constants were expressed through this one.

The thermo-elasticity problem is reduced to the $2n$-integral equations of the second kind relative to the constitutive stress components. The other components of the thermo-stressed state are expressed with analytical solutions of the integral equations.

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ASYMPTOTIC REPRESENTATIONS OF ONE CLASS OF SINGULAR SOLUTIONS OF THE SECOND-ORDER DIFFERENTIAL EQUATIONS

Consider the differential equation

$$y'' = f(t, y, y'), \tag{1}$$

where $f : [a, \omega[\times \Delta_{Y_0} \times \Delta_{Y_1} \longrightarrow \mathbf{R}$ is a continuous function, $-\infty < a < \omega \leq +\infty$, Δ_{Y_i} $(i \in \{0, 1\})$ is the one-side neighborhood of Y_i and Y_i $(i \in \{0, 1\})$ is either 0 or $\pm\infty$.

We study Eq.(1) on the singular class $P_\omega(Y_0, Y_1, \pm\infty)$-solutions defined as follows.

Definition 1. *A non-continuable to the right solution y of Eq. (1) on the interval $[t_0, t_*[\subset [a, \omega[$ $(t_* < \omega)$ is called $P_{t_*}(Y_0, Y_1, \pm\infty)$-solution, if it satisfies the conditions*

$$y^{(i)}(t) \in \Delta_{Y_i} \quad \text{for} \quad t \in [t_0, t_*[\ , \quad \lim_{t \uparrow t_*} y^{(i)}(t) = Y_i \quad (i = 0, 1),$$

$$\lim_{t \uparrow t_*} \frac{[y'(t)]^2}{y(t) y''(t)} = \pm\infty.$$

Assuming that the function f satisfies condition $(RN)^*_\infty$ (see [1], def. 3, where $\pi_\omega(t) = t_* - t$, $\lim_{t \uparrow \omega} p(t) = A_* > 0$), the function φ_0 satisfies the condition S (see [1], def. 2), using results from [1], we give necessary and sufficient conditions for existence of $P_{t_*}(Y_0, Y_1, \pm\infty)$-solutions of Eq.(1). Moreover, the asymptotic behavior of these solutions and their derivatives of the first order as $t \uparrow t_*$ are established.

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NUMERICAL ALGORITHMS AND CODES FOR RADIATIVE MAGNETOHYDRODYNAMICS

Radative magnetohydrodynamic (MHD) flow is described by the system of hyperbolic equations of form:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \mathbf{m}\\ E\\ \mathbf{B}\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho\mathbf{v}\\ \mathbf{m}\mathbf{v}+\mathbf{I}p_{\mathrm{tot}}\\ (E+p_{\mathrm{tot}})\mathbf{v}-\mathbf{B}(\mathbf{v}\cdot\mathbf{B})\\ \mathbf{v}\mathbf{B}-\mathbf{B}\mathbf{v}\end{pmatrix}^{\mathrm{T}}=\begin{pmatrix}0\\ 0\\ L\\ 0\end{pmatrix} \tag{1}$$

where ρ is the density, $\mathbf{m}=\rho\mathbf{v}$ the momentum density, $\mathbf{v}$ the flow velocity, p_{tot} the total (thermal p and magnetic p_{B}) pressure, $\mathbf{I}$ the unit vector, $\mathbf{B}$ the magnetic field strength, L describes the radiative losses, E the total energy density. The system is closed by the ideal gas equation

$$E=\frac{p}{\gamma-1}+\frac{m^2}{2\rho}+\frac{B^2}{2}. \tag{2}$$

Numerical algorithm should keeps the divergence-free condition $\nabla\cdot\mathbf{B}=0$.

MHD equations describe majority of astrophysical problems. In most cases system (1) cannot be solved analytically and various numerical techniques should be applied. The most common and well developed approach is the finite-voulme formalism. It is used to solve (1) in integral form where the averaged values in discrete cells are evolved in time. The choice of a particular computational scheme in most cases depends on a problem.

We tested performance of the computational schemes available in PLUTO [1] and AMRVAC [2] MHD codes and applied them to simulate the radiative blast wave problem.

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ANALOGUES OF THE ADJUGATE MATRIX FOR THE WEIGHTED MOORE-PENROSE INVERSE

Let the Hermitian positive definite matrices $\mathbf{M}$ and $\mathbf{N}$ of order m and n, respectively, be given. For any matrix $\mathbf{A} \in \mathbb{C}^{m\times n}$, **the weighted Moore-Penrose inverse** of $\mathbf{A}$ is the unique solution $\mathbf{X} = \mathbf{A}^{+}_{M,N}$ of the following matrix equations in $\mathbf{X}$:

$$(1)\ \mathbf{AXA} = \mathbf{A};\ (2)\ \mathbf{XAX} = \mathbf{X};$$

$$(3M)\ (\mathbf{MAX})^* = \mathbf{MAX};\ (4N)\ (\mathbf{NXA})^* = \mathbf{NXA}.$$

Theorem [1]. *If $\mathbf{A} \in \mathbb{C}^{m\times n}_r$ and $r < min\{m, n\}$, then the weighted Moore-Penrose inverse $\mathbf{A}^{+}_{M,N} = \left(\tilde{a}^{+}_{ij}\right) \in \mathbb{C}^{n\times m}$ possess the following determinantal representation:*

$$\tilde{a}^{+}_{ij} = \frac{\sum\limits_{\beta\in J_{r,\,n}\{i\}} \left|\left(\left(\mathbf{A}^{\sharp}\mathbf{A}\right)_{.\,i}\left(\mathbf{a}^{\sharp}_{.j}\right)\right){}^{\beta}_{\beta}\right|}{\sum\limits_{\beta\in J_{r,\,n}} \left|\left(\mathbf{A}^{\sharp}\mathbf{A}\right){}^{\beta}_{\beta}\right|}, \tag{1}$$

or

$$\tilde{a}^{+}_{ij} = \frac{\sum\limits_{\alpha\in I_{r,m}\{j\}} \left|\left((\mathbf{A}\mathbf{A}^{\sharp})_{j\,.}\,(\mathbf{a}^{\sharp}_{i.})\right){}^{\alpha}_{\alpha}\right|}{\sum\limits_{\alpha\in I_{r,\,m}} \left|(\mathbf{A}\mathbf{A}^{\sharp})\,{}^{\alpha}_{\alpha}\right|}, \tag{2}$$

where $\mathbf{A}^{\sharp} = \mathbf{N}^{-1}\mathbf{A}^*\mathbf{M}$ for all $i = \overline{1,n}$, $j = \overline{1,m}$.

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ON DISTRIBUTION OF COMPLEX ALGEBRAIC NUMBERS IN SMALL CIRCLES

It is well known that complex algebraic numbers of degree $n \geq 2$ constitute a countable dense set on the complex plane. We are going to find an explicit relation between the quantity of algebraic numbers in a small complex circle and their height. This relation was first analyzed in 1970 by A. Baker and W. Schmidt [1], where the notion of a regular system was introduced.

Definition 1. *Let Γ be a countable set of real numbers and $N : \Gamma \mapsto \mathbb{R}$ be a positive function. The pair (N, Γ) is called a regular system iff $\exists c_1 = c_1(N, \Gamma) > 0$ such that for any interval $I \subset \mathbb{R}$ and for all $T > T_0$, where $T_0 = T_0(N, \Gamma, I)$ is sufficiently large, there exists a sequence $\gamma_1, ..., \gamma_t \in I \bigcap \Gamma$ satisfying the following properties: $N(\gamma_i) \leq T$, $|\gamma_i - \gamma_j| > T^{-1}$, $t > c_1 |I| T$.*

We are going to construct a regular system of complex algebraic numbers with nonzero imaginary parts in circles of small radius and find an explicit expression for T_0. Let us introduce the following notation: $T(0,1) \subset \mathbb{C}$ is the unit circle; $K(z_0, r) \subset T(0,1)$ is a complex circle centered at z_0 of radius r; $c_1, c_2, ...$ are certain constants that depend only on n; $\mathcal{P}_n(Q) = \{P(z) \in \mathbb{Z}[z] : \deg P \leq n, H(P) \leq Q\}$.

Theorem 1. *Let $B_1(Q, \delta_0, K)$ be a set of complex numbers $z \in K(z_0, r)$ such that conditions $|P(z)| < c_1 Q^{-(n-1)/2}$ and $|P'(z)| < \delta_0 Q$ are satisfied for some $P(x) \in \mathcal{P}_n(Q)$. Then for a sufficiently small δ_0 and a sufficiently large c_2 we can write $\mu B_1(Q, \delta_0, K) < 1/4 \mu K$ for any circle $K(z_0, r)$ with $r > c_2 Q^{-\mu}$, $0 \leq \mu \leq 1$.*

Theorem 2. *Assume that Γ is a set of complex algebraic numbers α of degree not exceeding n and $N(\alpha) = c_3 H(\alpha)^{(n+1)/2}$. Then (N, Γ) is a regular system in the complex plane.*

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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF SOME DIFFERENTIAL EQUATIONS SYSTEMS IN A NEIGHBORHOOD OF THE POLE

Let us consider the Cauchy problem of the form

$$\begin{cases} A\,(z)\,Y^{'} = B\,(z)\,Y + f\,(z, Y, Y')\,, & (1) \\ Y\,(z) \to 0 \;\text{ as }\; z \longrightarrow 0, z \in D_0, & (2) \end{cases}$$

where matrices A, $B : D_1 \to \mathbb{C}^{p\times p}, D = \{z : |z| < R_1, R_1 > 0\} \subset \mathbb{C}$, $p \in \mathbb{N}$; matrix $A(z)$ is analytic in the domain D and $rangA = p$ in D, matrix $B(z)$ is analytic in the domain D_0, $D_0 = D\setminus\{0\}$, and has a pole of order d at the point $z = 0$; the function $f : D \times G_1 \times G_2 \to \mathbb{C}^p$, $G_k \subset \mathbb{C}^p$, $0 \in G_k$, $k = 1, 2$, is analytic in the domain $D_0\times G_{10}\times G_{20}, G_{k0} = G_k\setminus\{0\}$, $k = 1, 2$ and has removable singularity at the point $(0, 0, 0)$.

We study solutions of the Cauchy problem (1)-(2) that satisfy the additional condition

$$Y^{'}\,(z) \to 0 \;\text{ as }\; z \longrightarrow 0,\; z \in D_0. \tag{3}$$

Some theorems were proved and the sufficient conditions, when the Cauchy problem (1)-(2) has at least one analytical solution or infinitely many analytical solutions in some subdomains of the domain D, were found.

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ABOUT SYMMETRY OF ESTIMATE FOR DERIVATIVES OF THE FUNDAMENTAL MATRIX OF THE DEGENERATE PARABOLIC SYSTEMS OF THE KOLMOGOROV TYPE

For arbitrary fixed $T > 0$, $m \in \mathbb{N}$ and for a vector $\overrightarrow{2b} = (2b_1; \ldots; 2b_{n_1})$ with the even natural coordinates, we consider the system of equations

$$\left(\partial_t - \sum_{j=1}^{n_2} x_{1j} \partial_{x_{2j}} - \sum_{j=1}^{n_3} x_{2j} \partial_{x_{3j}} \right) u(t; x) = \mathbb{A}(t; \partial_{x_1}) u(t; x), \qquad (1)$$

$$(t; x) \in (0; T] \times \mathbb{R}^n,$$

where $u := \operatorname{col}(u_1; \ldots; u_m)$, $n := n_1 + n_2 + n_3 \in \mathbb{N}$, $n_1 \geq n_2 \geq n_3$, $x = (x_1; x_2; x_3) \in \mathbb{R}^n$, $x_j = (x_{j1}; \ldots; x_{jn_j}) \in \mathbb{R}^{n_j}$, and $\mathbb{A}(t; \partial_{x_1})$ is a matrix differential expression, whose coefficients are continuous complex-valued functions on $[0; T]$, such that the corresponding differential expression

$$\partial_t - \mathbb{A}(t; \partial_{x_1})$$

is $\overrightarrow{2b}$-parabolic on the set $(0; T] \times \mathbb{R}^{n_1}$. The fundamental solution matrix of the Cauchy problem (FSMCP) for the system (1) was constructed, and its main properties were studied. In particular, it was shown that the FSCMP with respect to each space variable and with fixed time variables belongs to the vector space of the type S [1]. Also, the strong differentiability of the FSMCP with respect to the time variable t and its infinite differentiability with respect to the variable x (as the abstract function of the mentioned variables) were proved in this space. We obtain estimates for its derivatives with respect to the space variables and prove the symmetry of these estimates with respect to the space and time variables.

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INVERSE COEFFICIENT PROBLEMS TO EQUATIONS WITH FRACTIONAL DERIVATIVES

We establish the unique solvability of some inverse problems for linear inhomogeneous fractional diffusion equations. We prove existence and uniqueness of a solution $(u, b) \in C_{2,\beta}(\bar{Q}_0) \times C[0, T]$ of the inverse boundary value problem

$$D_t^{\beta} u - \Delta u - b(t)u = F_0(x, t), \quad (x, t) \in \Omega_0 \times (0, T],$$
$$u(x, t) = 0, \quad (x, t) \in \Omega_1 \times [0, T],$$
$$u(x, 0) = F_1(x), \quad u_t(x, 0) = F_2(x), \quad x \in \bar{\Omega}_0,$$
$$\int\limits_{\Omega_0} u(x, t)\varphi_0(x)dx = F(t), \;\; t \in [0, T]$$

where Ω_0 is a boundary domain in $\mathbb{R}^N$, $N \geq 3$, with the boundary Ω_1 of class C^{1+s}, $s \in (0, 1)$, F_0, F_1, F_2, F, φ_0 are given functions, $D_t^{\beta} u$ is the regularized fractional derivative of the order $\beta \in (0, 2)$, $C_{2,\beta}(Q_0) = \{v \in C(Q_0) \,|\, \Delta v, D_t^{\beta} v \in C(Q_0)\}$, $C_{2,\beta}(\bar{Q}_0) = C_{2,\beta}(Q_0) \cap C(\bar{Q}_0)$ in the case $\beta \in (0, 1]$, $C_{2,\beta}(\bar{Q}_0) = \{v \in C_{2,\beta}(Q_0) \mid v, v_t \in C(\bar{Q}_0)\}$ if $\beta \in (1, 2)$. The second initial condition is absent in the case $\beta \in (0, 1]$.

We consider the similar problem to equation

$$D_t^{\alpha} u + b(t)D_t^{\beta} u - \Delta u = F_0(x, t), \quad (x, t) \in \Omega_0 \times (0, T]$$

with regularized fractional derivatives $D_t^{\alpha} u, D_t^{\beta} u$ of the orders $\alpha \in (1, 2)$ and $\beta \in (0, 1)$.

The uniqueness solvability of the inverse Cauchy problem consisting in finding the classical, with respect to time and with values in Bessel potential spaces, solution and unknown, depending on time, continuous young coefficient is established also under some over-determination condition $< u(\cdot, t), \varphi(\cdot) >= F(t)$, $t \in [0, T]$ with given functions φ, F.

For such equations, but with Riemann-Liouville fractional derivatives in the case of generalized functions in right-hand sides of the equations and initial conditions, we find a pair of functions $(u, b) \in \mathfrak{D}'_C(\bar{Q}_0) \times C[0, T]$, where

$$\mathfrak{D}'_C(\bar{Q}_0) = \{v \in \mathfrak{D}'(\bar{Q}_0) : \; (v(\cdot, t), \varphi(\cdot)) \in C[0, T] \quad \forall \varphi \in \mathfrak{D}(\bar{\Omega}_0)\}.$$

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ON APPLICATIONS OF HÖRMANDER SPACES TO PARABOLIC PROBLEMS

We discuss applications of Hörmander inner product spaces to $2b$-parabolic partial differential equations. These spaces are parametrized with a pair of real numbers s, $s/(2b)$ and a Borel measurable function $\varphi : [1,\infty) \to (0,\infty)$ which varies slowly at infinity in the sense of Karamata. The Hörmander space $H^{s,s/(2b),\varphi}(\mathbb{R}^{n+1})$ consists of all tempered distributions w on $\mathbb{R}^{n+1}$ such that

$$\int_{\mathbb{R}^{n+1}} \bigl(1+|\xi|^2+|\eta|^{1/b}\bigr)^s \varphi^2\bigl((1+|\xi|^2+|\eta|^{1/b})^{1/2}\bigr)|\widehat{w}(\xi,\eta)|^2\, d\xi d\eta < \infty.$$

Here $\widehat{w}$ is the Fourier transform of w, whereas $\xi \in \mathbb{R}^n$ and $\eta \in \mathbb{R}$ are the frequency variables dual to the spacial and time variables, respectively. If $\varphi(\cdot) \equiv 1$, then $H^{s,s/(2b),\varphi}(\mathbb{R}^{n+1})$ becomes the anisotropic Sobolev space $H^{s,s/(2b)}(\mathbb{R}^{n+1})$, which is used generally in parabolic theory.

We consider parabolic initial-boundary value problems for $2b$-parabolic linear partial differential equations given in a bounded cylinder in $\mathbb{R}^{n+1}$. We prove that the operators corresponding to these problems establish isomorphisms between appropriate Hörmander inner product spaces built on the base of the space $H^{s,s/(2b),\varphi}(\mathbb{R}^{n+1})$. We also discuss regularity properties of the generalized solutions to these problems.

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FOURIER TRANSFORMATION IN THE ALGEBRA OF POLYNOMIAL ω–ULTRADISTRIBUTIONS OF BEURLING TYPE

We consider the multiplicative algebra $P(\mathcal{E}'_{(\omega)})$ of continuous scalar polynomials on the space $\mathcal{E}'_{(\omega)}$ ω–ultradistributions of Beurling type [1] as well as its strong dual $P'(\mathcal{E}'_{(\omega)})$. We give a description of properties of the differentiation on the algebras $P(\mathcal{E}'_{(\omega)})$ and $P'(\mathcal{E}'_{(\omega)})$ by means of their tensor representations. A connection between the differentiation of polynomials and the polynomially extended Fourier transformation in the form of operator calculus is described.

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ON THE FUNDAMENTAL SOLUTION OF A PARABOLIC EQUATION WITH FRACTIONAL DERIVATIVES

In the space $\Pi = (0, \infty) \times \mathbb{R}^n$ we considered the Cauchy problem for higher order equations with fractional derivative

$$\mathcal{D}_t^\alpha - \frac{u(0,x)}{\Gamma(1-\alpha)t^\alpha} = \sum_{|k| \leq 2b} A_k(x)\mathcal{D}_x^k u + f(t,x), \tag{1}$$

$$\lim_{t \to 0} u(t,x) = \varphi(x), \quad \alpha \in (0,1). \tag{2}$$

For equation (1) with parametric coefficients $A_k(y)$, $y \in \mathbb{R}^n$ we have constructed the Green function $\{G_1(t,x,y), G_2(t,x,y)\}$, through which the solution of the problem (1), (2) is sought in the form

$$u(t,x) = \int\limits_{\mathbb{R}^n} G_1(t, x-\xi, \xi)\varphi(\xi)d\xi + \int\limits_0^t d\tau \int\limits_{\mathbb{R}^n} G_2(t-\tau, x-\xi, \xi)\mu(\tau,\xi)d\xi. \tag{3}$$

Satisfying equation (1), to find the function $\mu(t,x)$ we obtain the integral equation of Fredholm-Voltaire with the quasiregular core. This allows us to find the solution of the Cauchy problem (1), (2) and identify core of inverse operator, i.e. construct Green's function for the problem with variable coefficients and evaluate the classical solution $u(t,x)$, if A_k, f, $\varphi \in C^{(\alpha)}$ (Holder functions).

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A CONTRIBUTION TO NO-INTERACTION THEOREM IN RELATIVISM

In addition to well-known no-interaction theorem in relativity asserting there no-existence of a relativistic direct interaction, I propose two new assertions, stating that there does not exist a relativistic third-order equation of motion even for a single particle and also that there does not exist an invariant Lagrange function.

Let equations of motion be written in the following guise of a differential one-form:

$$\boldsymbol{\epsilon} = \{\underline{\epsilon}_i\} = \{\mathsf{A}_{ij} d\mathsf{v}'^j + \mathsf{k}_i dt\} \equiv \mathbf{A} \,.\, d\mathbf{v}' + \mathbf{k} dt \,,$$

with

$$\mathbf{k} = (\mathbf{v}' \,.\, \boldsymbol{\partial}_{\mathbf{v}})\, \mathbf{A} \,.\, \mathbf{v}' + \mathbf{B} \,.\, \mathbf{v}' + \mathbf{c} \,,$$

where the symmetric matrix $\mathbf{M}$, the skew-symmetric matrix $\mathbf{A}$, and the row $\mathbf{b}$ all depend on the variables t, x^j, v^j, v'^j, and satisfy some additional system of PDEs

Let $X(\boldsymbol{\epsilon})$ denote the component-wise action of an infinitesimal generator X of a transformation group on a vector differential form $\boldsymbol{\epsilon}$. That the exterior differential system, generated by the form $\boldsymbol{\epsilon}$, possesses the symmetry of X means that there exist some matrices $\boldsymbol{\Phi}$, $\boldsymbol{\Xi}$, and $\boldsymbol{\Pi}$, depending on $\mathbf{v}$ and $\mathbf{v}'$, such that

$$X(\boldsymbol{\epsilon}) = \boldsymbol{\Phi} \,.\, \boldsymbol{\epsilon} + \boldsymbol{\Xi} \,.\, (\mathbf{x} - \mathbf{v} dt) + \boldsymbol{\Pi} \,.\, (d\mathbf{v} - \mathbf{v}' dt).$$

Proposition. *In the (pseudo)Euclidian space of dimension 4 there are no invariant variational equations of the third order.*

To produce a variational equation of the third order, the Lagrange function should be of affine type in second derivatives

Proposition. *In the space-time of dimension greater than two there does not exist a Poincaré-invariant Lagrange function which might produce a third order equation of motion by means of the variational procedure.*

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WEAKLY PERTURBED BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF DIFFERENTIAL EQUATIONS WITH MEASURES

We consider solvability conditions of the weakly perturbed linear nonhomogenous boundary value problem (BVP) $\mathcal{P}_\varepsilon$ for the system of differential equations with measures

$$y' - C_0'(x)y = f'(x) + \varepsilon C_1'(x)y,\ x \in [a,b], \tag{1}$$

$$l_0 y = \eta + \varepsilon l_1 y, \quad l_k y \equiv \int_a^b dM_k(x)y(x),\ k = 0,1, \tag{2}$$

where y, f are vectors in n-dimensional euclidean space $\mathbb{E}^n$, the elements of matrices C_0, C_1 and the components of vector f belong to the space $BV^+[a,b]$ that consists of the right-continuous functions with the bounded variation on $[a,b]$ (i.e. we understand the differentiation and the equality in (1) in the generalized sense), $\varepsilon \geqslant 0$ is a small parameter, l_k is the integral operator $BV[a,b] \to \mathbb{E}^m$, $\eta \in \mathbb{E}^m$, the elements of $(m\times n)$-matrix $M_k(x)$ belong to $BV[a,b]$. We assume that conditions A: $[\triangle C_k(x)]^2 = \triangle C_0(x)\triangle C_1(x) + \triangle C_1(x)\triangle C_0(x) = 0$, $\triangle C_k(x)\triangle f(x) = 0$ and conditions B: $\triangle M_k(x)\triangle C_k(x) = \triangle M_0(x)\triangle C_1(x) + \triangle M_1(x)\triangle C_0(x) = 0$, $\triangle M_k(x)\triangle f(x) = 0$ hold for all $x \in [a,b]$ and $k = 0,1$. These conditions ensure existence of product of distributions in (1) and existence of an integral in (2) in the sense of Riemann-Stieltjes. We look for a solution $y_\varepsilon(x)$ of (1) in the space $BV^+[a,b]$.

We treat the critical case when unperturbed BVP $\mathcal{P}_0$ doesn't admit solutions for any $f \in BV^+[a,b]$ and $\eta \in \mathbb{E}^m$ [1]. In this case, using the theory of pseudoinverse by Moore-Penrose matrices, we establish conditions for the perturbing terms $C_1'(x)$ and $l_1 y$ under which the BVP $\mathcal{P}_\varepsilon$ admits solutions and suggest an algorithm for construction of such solutions.

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ON INVESTIGATIONS OF S.D.EIDELMAN IN THE THEORY OF THE DEGENERATE PARABOLIC EQUATIONS OF KOLMOGOROV TYPE AND THEIR DEVELOPMENT

We suppose that the spatial variable $x \in \mathbb{R}^n$ consists of the three groups of variables $x_l := (x_{l1}, \ldots, x_{ln_l}) \in \mathbb{R}^{n_l}, l \in \{1,2,3\}$, such that $x := (x_1, x_2, x_3)$. Here n_1, n_2 and n_3 are given natural numbers such that $n_3 \geq n_2 \geq n_1$ and $n_1 + n_2 + n_3 = n$. Let b be a given positive integer number or the least common multiple of numbers $\{b_1, \ldots, b_{n_1}\}, m_j := b/b_j, j \in \{1, \ldots, n_1\}$. We shall write corresponding multiindices $k \in \mathbb{Z}_+^n$ in the form $k := (k_1, k_2, k_3)$, where $k_l := (k_{l1}, \ldots, k_{ln_l}) \in \mathbb{Z}_+^{n_l}, |k_l| := k_{l1} + \ldots + k_{ln_l}, l \in \{1,2,3\}, \|k_1\| := m_1 k_{11} + \ldots + m_{n_1} k_{1n_1}$.

Consider the equations

$$(S - \sum_{j,l=1}^{n_1} a_{jl}(t,x)\partial_{x_{1j}}\partial_{x_{1l}} - \sum_{j=1}^{n_1} a_j(t,x)\partial_{x_{1j}} - a_0(t,x))u(t,x) = 0,$$

$$(S - \sum_{\|k_1\| \leq 1} a_{k_1}(t,x)\partial_{x_1}^{k_1})u(t,x) = 0,\ (S - \sum_{\|k_1\| \leq 2b} a_{k_1}(t,x)\partial_{x_1}^{k_1})u(t,x) = 0.$$

Here $S := \partial_t - \sum\limits_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} - \sum\limits_{j=1}^{n_3} x_{2j}\partial_{x_{3j}}$, $(t,x) \in (0,T] \times \mathbb{R}^n$.

These equations are a natural generalization of the classical equation of diffusion with inertia. This equation appears in the study by A.Kolmogorov of models for Brownian motion.

We present main results for such classes of the degenerate parabolic equations and their generalizations which are obtained by S.Eidelman and his disciples. Some new results for ultra-parabolic equations can be found in the paper [1].

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TRAVELLING AUTO-WAVES IN A NONLINEAR SYSTEM WITH THE FRACTIONAL SPACE DERIVATIVE

In this report, we study properties of auto-waves in a nonlinear system with the fractional space derivative. Namely, the generalized FitzHugh-Nagumo model with the fractional space derivative is considered:

$$\tau_u \frac{\partial u}{\partial t} = l^2 \frac{\partial^\alpha u}{\partial x^\alpha} + u - u^3/3 - v, \quad \tau_v \frac{\partial v}{\partial t} = Bu - v + A,$$

where $u = u(x,t)$ is the fast activator variable and $v = v(x,t)$ is a slow inhibitor one [1,2], τ_u, τ_v and l are the characteristic times and diffusion length of the system, $0 \leq x \leq \mathcal{L}_x$, $\mathcal{L}_x$ is the system length, A, B are external parameters. $\partial^\alpha u/\partial x^\alpha$ represents the fractional space Riesz derivative of a real order $1 < \alpha < 2$, $\frac{\partial^\alpha u}{\partial x^\alpha} = -\frac{1}{2\cos(\pi\alpha/2)} \left(D_+^\alpha u + D_-^\alpha u\right)$, where

$$D_+^\alpha u = \frac{1}{\Gamma(2-\alpha)} \frac{d^2}{dx^2} \int_{-\infty}^{x} \frac{u}{(x-\xi)^{\alpha-1}} d\xi,$$

$$D_-^\alpha u = \frac{1}{\Gamma(2-\alpha)} \frac{d^2}{dx^2} \int_{x}^{\infty} \frac{u(\xi,t)}{(\xi-x)^{\alpha-1}} d\xi$$

are the left and right fractional Riemann-Liouville derivatives [3].

It is shown, by the linear stability analysis and computer simulation, that order of the space fractional derivative can change the speed of the travelling auto-wave and domain of their existence, but does not change the main properties – the shape, length and amplitude – of the wave.

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REFLECTIONLESS SCHRÖDINGER OPERATORS

In this talk, we will consider Schrödinger operators $H = -d^2/dx^2 + q(x)$ on $L_2(\mathbb{R})$ with real-valued potentials q from $L_{1,loc}(\mathbb{R})$ under the assumption that the limit point case holds at $\pm\infty$. For such operators, the Weyl–Titchmarsh m-functions $m_\pm$ can be defined. The operator H and its potential q will be called *reflectionless* if

$$m_+(\lambda) + \overline{m_-(\lambda)} = 0$$

for almost all $\lambda \in \mathbb{R}_+$.

Let Q stand for the set of all reflectionless potentials $q \in L_{1,loc}(\mathbb{R})$. It is known that Q can be parameterized by the set of positive Borel measures which depend on $m_\pm$. However, this parametrization is obscure since the relation between the spectral properties of Schrödinger operators and the corresponding Borel measures remains unclear.

In this our talk, we will consider another parametrization of reflectionless Schrödinger operators. Namely, let $\widetilde{Q} \subset Q$ be the set of all summable reflectionless potentials. We will show how every $q \in \widetilde{Q}$ can be described in terms of the spectral properties of H, i.e. in terms of the eigenvalues of H and the corresponding norming constants.

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HÖRMANDER SPACES AND DIFFERENTIAL EQUATIONS

The talk is a survey of our recent results [1–3] concerning the applications of Hörmander inner product spaces to the theory of interpolation between function spaces and to the theory of differential equations.

The first part of the talk is devoted to the interpolation. In terms of Hörmander spaces, we describe all Hilbert spaces that are interpolation spaces between Sobolev inner product spaces given over $\mathbb{R}^n$ or a bounded domain with Lipschitz boundary, or a smooth closed manifold. These Hörmander spaces form the extended Sobolev scale, which is obtained via the interpolation with a function parameter between Sobolev spaces and is closed with respect to this interpolation. We give examples of Hörmander spaces that are intermediate for given couples of Sobolev spaces but differ considerably on their interpolation properties. We also discuss equivalent definitions of isotropic Hörmander spaces on a closed infinitely smooth manifold.

The second part of the talk is devoted to various applications of the extended Sobolev scale to elliptic partial differential equations. For elliptic operators and elliptic boundary-value problems, we discuss the theorems on the Fredholm property and regularity of solutions in Hörmander spaces. In terms of these spaces, we give sufficient conditions under which the solutions belong to the space C^r with integer-valued $r \geq 0$. We also discuss some applications of Hörmander spaces to spectral problems.

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SPECTRAL PROPERTIES OF SINGULAR PERTURBATIONS OF OPERATORS ON THE CIRCLE

We discuss spectral properties of operators with complex-valued distributional coefficients defined on the Hilbert space $L^2(\mathbb{T})$, $\mathbb{T}=\mathbb{R}/2\mathbb{Z}$:

$$\begin{aligned}
&\mathrm{S}(V)u = \mathbb{D}^{2s}u + V(x)u, \quad s\in(1/2,\infty), \quad u\in \mathrm{Dom}(\mathrm{S}(V)),\\
&\mathbb{D}^{2s}=(\mathbb{D}^2)^s,\ \mathbb{D}^2=-\frac{d^2}{dx^2},\\
&V(x)=\sum_{k\in\mathbb{Z}}\widehat{V}(k)e^{i\,k\pi x}\in H^{-s}(\mathbb{T}),\\
&\mathrm{Dom}(\mathrm{S}(V))=\{u\in H^s(\mathbb{T})|\mathbb{D}^{2s}u+V(x)u\in L^2(\mathbb{T})\},
\end{aligned}$$

where by $H^t(\mathbb{T})$, $t\in\mathbb{R}$, it is denoted the Sobolev spaces of periodic functions/distributions on the circle.

The operators $\mathrm{S}(V)$ are well defined on the Hilbert space $L^2(\mathbb{T})$ as form sums, alternatively they can be well defined as a limit of the sequence of operators with smooth potentials in the norm resolvent sense. The operators $\mathrm{S}(V)$ are m-sectorial, they are self-adjoint iff $V(x)$ are real-valued. Their spectra are purely discrete. Their systems of root vectors are complete in the Hilbert space $L^2(\mathbb{T})$. For more details see [1].

For the case $s\in(1,\infty)$ we prove many-terms, uniform on the bounded sets of distributions $V\in H^{s\alpha}(\mathbb{T})$, $\alpha\in[-1,0]$, asymptotic formulae for the eigenvalues of operators $\mathrm{S}(V)$ [1].

We establish that the systems of root vectors of operators $\mathrm{S}(V)$ form the Bari–Markus bases in the Hilbert space $L^2(\mathbb{T})$. The case $s=1$ was treated in [2].

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ASYMPTOTIC BEHAVIOR OF A LOGARITHMIC DERIVATIVE AND AN ANGULAR DENSITY OF ZEROS OF AN ENTIRE FUNCTION

Let L be a class of slowly increasing in $[1,+\infty)$ functions, $\Gamma_m = \bigcup\limits_{j=1}^{m} l_{\theta_j}$ a finite system of rays $l_{\theta_j} = \{z\colon \arg z = \theta_j\}$, $-\pi \le \theta_1 < \ldots < \theta_m < \pi$, $H_0(\upsilon, \Gamma_m)$, $\upsilon \in L$, a class of entire functions f of zero order with zeros in Γ_m such that $\varlimsup\limits_{r\to+\infty} n(r,0,f)/\upsilon(r) < +\infty$.

Let us denote $h_j(\theta) = (\theta-\pi-\theta_j)$, $\theta_j < \theta < \theta_j + 2\pi$, $\widehat{h}_j(\theta)$ the periodic continuation of $h_j(\theta)$ from $(\theta_j, \theta_j + 2\pi)$ on $\mathbb{R}$, $j = \overline{1,m}$. For $\widetilde{\upsilon} \in L$, we set $\upsilon(r) = \int\limits_0^r \frac{\widetilde{\upsilon}(t)}{t} dt$, $n(r,\theta_j; f) = n(r,\theta_j)$ the counting function of zeros of a function $f \in H_0(\upsilon, \Gamma_m)$ on the ray l_{θ_j}. It is evident that $\upsilon \in L$ and $\widetilde{\upsilon}(r) = o(\upsilon(r))$, $r \to +\infty$.

Theorem 1. *Let $\widetilde{\upsilon} \in L$, $f \in H_0(\upsilon, \Gamma_m)$ and for each $j = \overline{1,m}$ $n(r,\theta_j) = \Delta_j \upsilon(r) + o(\widetilde{\upsilon}(r))$, $r \to +\infty$. Then, for $\theta \neq \theta_j$, we get*

$$F(re^{i\theta}) = re^{i\theta} \frac{f'(re^{i\theta})}{f(re^{i\theta})} = \Delta \upsilon(r) + iH_f(\theta)\widetilde{\upsilon}(r) + o(\widetilde{\upsilon}(r)), \quad r \to +\infty,$$

where $\Delta = \sum\limits_{j=1}^{m} \Delta_j$, $H_f(\theta) = \sum\limits_{j=1}^{m} \Delta_j \widehat{h}_j(\theta)$.

Theorem 2. *Let $G \in L^1[0,2\pi]$, $\widetilde{\upsilon} \in L$, $f \in H_0(\upsilon, \Gamma_m)$ and $F(re^{i\theta}) = \Delta \upsilon(r) + iG(\theta)\widetilde{\upsilon}(r) + o(\widetilde{\upsilon}(r))$, $r \to +\infty$. Then the sequence of zeros of f has an angular υ-density.*

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One inverse problem for fractional diffusion equation

We prove well-posedness of the inverse problem

$$D_t^\alpha u - u_{xx} = F_0(x,t), \quad (x,t) \in (0,l) \times (0,t_0] := Q_0, \tag{1}$$

$$u(0,t) = u(l,t) = 0, \quad t \in [0,t_0], \tag{2}$$

$$u(x,0) = F_1(x), \quad x \in [0,l], \tag{3}$$

$$u(x,t_0) = F(x), \quad x \in [0,l], \tag{4}$$

where $D_t^\alpha u$ is the regularized fractional derivative of u of order $\alpha \in (0,1)$ with respect to time, F_0, F are given functions, t_0 a given positive number, u, F_1 are unknown functions.

Let $C_{2,\alpha}(Q_0) = \{v \in C(Q_0) \,|\, v_{xx}, D_t^\alpha v \in C(Q_0)\}$,

$C_{2,\alpha}(\bar{Q}_0) = C_{2,\alpha}(Q_0) \cap C(\bar{Q}_0)$,

$\tilde{C}^{2s+j}(0,l)$ – a class of functions $F \in C[0,l]$ with bounded derivative of order $2s+j$ on $(0,l)$ and $F(0) = F(l) = F''(0) = F''(l) = \cdots = F^{(2s)}(0) = F^{(2s)}(l) = 0$,

$\tilde{C}^{2s+j}(Q_0) = \{v \in C(Q_0) \,|\, v(\cdot,t) \in \tilde{C}^{2s+j}(0,l) \quad \forall t \in (0,t_0]\}$, $j = 1,2$.

We prove that under the assumptions

$$F_0 \in \tilde{C}^2(Q_0),\; F \in \tilde{C}^4(0,l)$$

for all given $t_0 > 0$ there exists a unique solution

$$(u, F_1) \in C_{2,\alpha}(\bar{Q}_0) \times \tilde{C}^1(0,l)$$

of the problem (1)-(4). The estimates

$$||u||_{C(Q_0)} + ||F_1||_{C(0,l)} \leq b_1 ||F_0||_{C^1(Q_0)} + b_2 ||F_3||_{C^3(0,l)},$$

$$||u_{xx}||_{C(Q_0)} + ||D_t^\alpha u||_{C(Q_0)} + ||F_1||_{C^1(0,l)} \leq \hat{b}_1 ||F_0||_{C^2(Q_0)} + \hat{b}_2 ||F_3||_{C^4(0,l)}$$

hold with some positive constants $b_1, b_2,\; \hat{b}_1, \hat{b}_2$. The solution depends continuously on the data.

Note that in the case $\alpha = 1$ such inverse problem is not well posed.

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ON SOLVABILITY OF THE BOUNDARY-VALUE PROBLEM FOR A DIFFERENTIAL EQUATION OF THE FRACTIONAL ORDER

Let $J = [0, a], \gamma > 0$ and $n = [\gamma] + 1$, where $[\gamma]$ is the integer part of γ. For a function $f : J \to \mathbb{R}$, the following expressions $f_\gamma = I_0^\gamma f$ and $D_0^\alpha f$ are called, respectively, the Riemann-Liouville left-sided integral and left-sided derivative of the order γ.

The following boundary-value problem

$$D_0^{1+\alpha} y(x) = F(x, y(x), D_0^\alpha y(x)), \tag{1}$$

$$y(0) = y'(a) = 0, \tag{2}$$

is considered, where $0 < \alpha \leq 1$. We say that a function $f : J \to \mathbb{R}$ belongs to the class $AC^2(J)$ if $f, f' \in AC(J)$. The function $y \in C(J) \bigcap C^1((0, a])$ is called a solution of boundary-value problem (1), (2), if $y_{1-\alpha} \in AC^2(J)$, y satisfies the boundary-value condition (2) and differential equation (1) for a.a. $x \in J$.

Assume that a function $F : J \times \mathbb{R}^2 \to \mathbb{R}$ fulfils the conditions

(i) $F(\cdot, y, z)\colon J \to \mathbb{R}$ is measurable function for every y and z;

(ii) $F(x, \cdot, \cdot)\colon \mathbb{R}^2 \to \mathbb{R}$ is continuous function for every fixed $x \in J$;

(iii) $\|F(x, y, z)\| \leq M$.

Theorem 1. *Suppose that a function $F\colon J \times \mathbb{R}^2 \to \mathbb{R}$ satisfies (i), (ii), (iii). A function $y \in C(J) \bigcap C^1((0, a])$ is a solution of (1), (2), if and only if y is a solution of the integral equation*

$$y(x) = \int_0^a G(x, t) F(t, y(t), D_0^\alpha y(t))\, dt,$$

where

$$G(x, t) = \begin{cases} -\dfrac{x^\alpha (a-t)^\alpha - a^\alpha (x-t)^\alpha}{a^\alpha \Gamma(1+\alpha)}, & 0 \leq t \leq x, \\ -\dfrac{(x(a-t))^\alpha}{a^\alpha \Gamma(1+\alpha)}, & x \leq t \leq a. \end{cases}$$

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SOLUTION OF INTEGRAL EQUATIONS IN THE THEORY OF THIN SPATIAL INCLUSIONS BY THE SMALL PARAMETER METHOD

The properties of composite materials with various inclusions are determined by the elastic characteristics of all constituents as well as by the shape and distribution of strengthening particles [1]. Three-dimensional shallow spheroidal crack opening caused by the diffraction on the crack of a plane harmonic wave with using of the small parameter method was investigated in [2]. In the present work, the problem of dynamic behaviour of a thin spherical inclusion in an infinite elastic solid by falling harmonic wave is considered. Using the boundary integral equations method, the diffraction problem has been reduced to a system of two-dimensional integral equations for the functions of the displacement jumps on the inclusion. Obtained equations with the Helmholtz potential kernels are presented in hypersingular form. The analytical solution of these equations in the case of a shallow inclusion and low-frequency excitation is obtained by a small parameter method. In this case kernels, traction components, and unknown functions are given as converging double series in the frequency and a geometric content small parameter. Investigations of the dynamic stress intensity factors as function of the wave number for a spheroidal inclusion with different radius of the circle contour are done. The direction vector of wave propagation is determined by taking into account the effect of maximum of mode I and mode II stress concentration in the crack vicinity.

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MODELING THE THERMOMECHANICAL PROCESSES IN SOLIDS WITH THE SHAPE MEMORY AT ELECTRIC FIELD ACTION

The shape memory alloys, in which the solid-phase structural transformation in a certain temperature interval under the action of thermal and mechanical load occurs, are used in science and technology at present. In particular, they are successfully used in medicine, namely in fixators at the treatment of fractures. As an effective method of obtaining the required deformation caused by heating, is convenient to use the electric field. A number of new effects appear in elements of shape memory alloys developing the magnetic properties. So the actual question is modeling of phase and stress state of shape memory solids at martensitic transformation in the electric field availability.

In this work the model for quantitative description of the thermomechanical behavior of solids made of shape memory alloys in martensitic transformation area under force and temperature loading subject to electric field action is proposed. This model is built using the solid mechanics and nonequilibrium thermodynamics methods subject to electric state of relevant thermodynamic system.

The generic Gibbs equation for solid areas, where the phase transformation occurs, is written and the set of equations of state is obtained. On this base the system of differential equations of model is received. This system after adding the appropriate initial and boundary conditions is a source in the analysis of stressed-strained state of solids made at shape memory alloys under thermal and power loading at electric field action.

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ON THE SOLUTION OF THE CAUCHY PROBLEM FOR ONE CLASS OF PSEUDO-DIFFERENTIAL EQUATIONS

The fundamental solution for some class of pseudo-differential equations is constructed by the method based on the theory of perturbations. We consider a symmetric α-stable process in multidimensional Euclidean space $\mathbb{R}^d$. Its generator $\mathbf{A}$ is a pseudo-differential operator whose symbol is given by $(-c|\lambda|^\alpha)_{\lambda\in\mathbb{R}^d}$, where the constants $\alpha \in (1,2)$ and $c > 0$ are fixed. The vector-valued operator $\mathbf{B}$ has the symbol $(i|\lambda|^{\alpha-2}\lambda)_{\lambda\in\mathbb{R}^d}$.

We construct the fundamental solution of the equation

$$\frac{\partial u(t,x)}{\partial t} = (\mathbf{A} + (a(x), \mathbf{B}))_x u(t,x), \quad t > 0,\ x \in \mathbb{R}^d$$

with a continuous bounded vector-valued function $(a(x))_{x\in\mathbb{R}^d}$.

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APPROXIMATION OF FUNCTIONS OF COMPLEX VARIABLES BY CONTINUED FRACTIONS

Let a function $f(z)$ of one complex variable be defined on compact $\mathcal{C} \subset \mathbb{C}$. An interpolation problem for functions by some types of continued fractions is considered. The estimation of a remainder term of the interpolate continued fraction is obtained. Numerical examples are also considered.

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STABILITY OF GENERALIZED DIFFERENTIAL EQUATIONS

We discuss the stability problems for solutions of the quasi-differential equations

$$\sum_{i=0}^{n}\sum_{j=0}^{m}(-1)^{m-j}\left(A_{ij}(t)X^{n-i}(t)\right)^{m-j}=F(t). \tag{1}$$

The detailed conditions for matrix function $A_{ij}(t)$, $F(t)$ are considered. With the use of results [1], we have obtained the stability conditions, asymptotic stability and stability of solutions of the equation ((1)) due to permanent perturbation.

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ON THE BEST POSSIBLE DESCRIPTION OF SOME EXCEPTIONAL SET

Let $\lambda=(\lambda_n)_{n\in\mathbb{Z}^p_+}$ be a sequence such that $\lambda_n=(\lambda^{(1)}_{n_1},\dots\lambda^{(p)}_{n_p})$ $(n=(n_1,\dots,n_p)\in\mathbb{Z}^p_+)$, $0\le\lambda^{(j)}_k\uparrow+\infty$ $(0\le k\uparrow+\infty)$, $1\le j\le p$. Let H^p_a be a class of entire functions defined by the absolutely convergent multiple Dirichlet series in the whole complex space $\mathbb{C}^p$ of the form $F(z)=\sum_{||n||=0}^{+\infty}a_ne^{<z,\lambda_n>}$, $z\in\mathbb{C}^p$, $p\ge2$, where $a=(|a_n|)_{n\in\mathbb{Z}^p_+}$, $||n||=n_1+\ldots+n_p$, $<z,\lambda_n>=z_1\lambda^{(1)}_{n_1}+\ldots+z_p\lambda^{(p)}_{n_p}$. For $F\in H^p_a$ and $x=(x_1,\dots,x_p)\in\mathbb{R}^p$ we denote $M(x,F)=\sup\{|F(x+iy)|:y\in\mathbb{R}^p\}$, $\mu(x,F)=\max\{|a_n|e^{<x,\lambda_n>}:n\in\mathbb{Z}^p_+\}$, and by $K_F=\{x\in\mathbb{R}^p:\lim\limits_{t\to+\infty}\frac{1}{t}\ln\mu(tx,F)=+\infty\}$ we denote the cone of the growth of $\ln\mu(x,F)$.

Theorem 1 [1]. *Let $F\in H^p_a$ and $(\mu_k)_{k\ge0}$ be a sequence $(\ln\frac{1}{|a_n|})_{n\in\mathbb{Z}^p_+}$ arranged in ascending order. If the condition $\sum_{k=0}^{+\infty}(\mu_{k+1}-\mu_k)^{-1}<+\infty$ holds then the asymptotic relations $M(x,F)\sim\mu(x,F)\sim\inf\{|F(x+iy)|\colon y\in\mathbb{R}^p\}$ holds as $|x|\to+\infty$ $(x\in K\setminus E)$ for arbitrary cone K with vertex at the origin O such that $\overline{K}\setminus\{O\}\subset K_F$, and E is a set such that $\int_E|x|^{-p}dx_1\cdots dx_p<+\infty$.*

The following theorem shows that the description of exceptional set E in Theorem 1 is the best possible.

Theorem 2. *For every function $h:\mathbb{R}_+\to\mathbb{R}_+$ such that $h(t)\uparrow+\infty$ $(t\uparrow+\infty)$ and for any sequence (μ_k) such that the condition $\sum_{k=0}^{+\infty}(\mu_{k+1}-\mu_k)^{-1}<+\infty$ holds there exist an entire Dirichlet series $F\in H^p_a$, a set $E\subset K_F$ and a constant $\beta>0$ such that $F(x)\ge(1+\beta)\mu(x,F)$ for all $x\in E$ and $\int\limits_E|x|^{-p}h(|x|)dx_1\cdots dx_p=+\infty$.*

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INVARIANT TORI OF A FAST-SLOW SYSTEM EXHIBITING A DYNAMICAL BIFURCATION OF THE MULTI-FREQUENCY OSCILLATIONS

We consider a fast-slow differential system

$$\dot{x} = f(x, u, \varepsilon), \quad \dot{u} = \varepsilon g(x, u, \varepsilon), \tag{1}$$

where $x \in \mathbb{R}^{2n}$ and $u \in \mathbb{R}^m$ are vectors of fast and slow variables respectively, $\varepsilon \in [0, \varepsilon_0]$ is a small static parameter, and the functions f, g are smooth and bounded. The system is assumed to have a slow invariant manifold defined by the equation $x = 0$. Besides, we suppose that in a ball $B^m_{R^*} = \{u \in \mathbb{R}^m : \|u\| \leq R^*\}$, the operator $f'_x(0, u, 0)$ has eigenvalues $\pm i\omega_j(u)$, $(j = \overline{1, n})$ satisfying some non-resonance conditions, the system $\dot{u} = g(0, u, 0)$ is convergent in $B^m_{R^*}$, i. e. $\langle g(0, u, 0), u\rangle \leq -\varkappa||u||^2$ for some $\varkappa > 0$, whereas the parameters space $\mathbb{R}^m$ of the linear system

$$\dot{x} = [f'_x(0, u, 0) + \varepsilon f''_{x,\varepsilon}(0, u, 0)]x$$

contains both a zone of asymptotic stability $\mathcal{D}_s$ and a zone of complete instabilty $\mathcal{D}_u$, separated by a transition zone.

It was shown in [1] that an $O(\sqrt{\varepsilon})$-neighborhood $\mathcal{U}_\varepsilon$ of the origin in $\mathbb{R}^{2n}$ contains a set $\mathcal{B}_\varepsilon$ such that $\lim_{\varepsilon \to +0} \operatorname{mes}(\mathcal{U}_\varepsilon \setminus \mathcal{B}_\varepsilon) = 0$ and the set $\mathcal{B}_\varepsilon \times B^m_{R^*}$ belongs to the basin of a stable n-dimensional invariant torus $\mathcal{T}^n_\varepsilon$ of system (1). For this reason the passage of slow variables from the zone $\mathcal{D}_s$ to the zone $\mathcal{D}_u$ results in a dynamical bifurcation of multifrequency oscillations observed in the subsystem of fast variables.

Here we managed to show that apart from the torus $\mathcal{T}^n_\varepsilon$ system (1) also has hyperbolic invariant tori of lower dimensions with nontrivial stable and unstable manifolds. These tori attract all forward trajectories not approaching $\mathcal{T}^n_\varepsilon$. Thus, the set $\mathcal{B}_\varepsilon$ contains all points of $\mathcal{U}_\varepsilon$ except those lying on submanifolds of nonzero co-dimensions.

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KOLMOGOROV TYPE INEQUALITIES FOR NORMS OF THE HYPERSINGULAR INTEGRALS WITH HOMOGENEOUS CHARACTERISTIC

Let $L^{\nabla}_{\infty,s}(\mathbb{R}^m)$, $1 \le s \le \infty$, be the spaces of functions $f \in L_\infty(\mathbb{R}^m)$, such that $|\nabla f| \in L_s(\mathbb{R}^m)$.

New exact inequalities of Kolmogorov type that estimate uniform norm of hypersingular integrals with gomogeneous characteristic of a function $f \in L^{\nabla}_{\infty,s}(\mathbb{R}^m)$ with the help of uiform norm of f and L_s- norm of $|\nabla f|$ are obtained.

The Stechkin problem on the best approximation of unbounded hypersingular integral operator by bounded ones on the class of functions $f \in L^{\nabla}_{\infty,s}(\mathbb{R}^m)$ such that $\||\nabla f|\|_s \le 1$ and the problem about the best recovery of unbounded hypersingular integral operator on the elements of this class, given with the error δ are solved.

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ON REALIZATION OF APPROACH BY EIDELMAN-IVASYSHEN TO CHARACTERIZE SOME CLASSES OF SOLUTIONS FOR PARABOLIC EQUATIONS

In the middle of the last century, in the papers on the theory of parabolic by Petrovsky systems of equations, S.D. Eidelman [1] proposed to characterize the evolution in t of solutions for the Cauchy problem by their affiliation to a family of Banach spaces (to a certain space for every t). This approach was developed and has repeatedly realized by S.D. Ivasyshen and his disciples (such approach is after referred to as "the approach of Eidelman-Ivasyshen" [2, 3]).

A brief survey of results on a correct solvability of the Cauchy problem and on an integral representation of such solutions through their limit values on the hyperplane $\{t = 0\}$ for parabolic equations of various structures are presented in the report. These solutions are defined in domains $(0, T\,] \times \mathbb{R}^n$ and $(0, T\,] \times \Omega$, $T > 0$, Ω is unbounded domain in $\mathbb{R}^n$. They are functions of spatial variable x, increasing exponentially as $|x| \to \infty$ with the greatest possible order of growth and the type of growth dependent on t.

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APPROXIMATE METHOD FOR SOLVING THE SYSTEM OF HYPERBOLIC AND PARABOLIC EQUATIONS FOR THE MANY-LAYERS STRESSED NANOHETEROSYSTEMS

The mathematical model of linked mechanical and diffusion processes in the many-layers stressed nanoheterosystems is constructed. The base of this model is the self-assembled system of two equations: for the parameter of deformation $U_i\left(z,t\right)$ (hyperbolic equation)

$$\frac{1}{c_i^2}\frac{\partial^2 U_i}{\partial t^2}=\frac{\partial^2 U_i}{\partial z^2}-\frac{\Theta_d^{(i)}}{\rho_i c_i^2}\frac{\partial^2 N_{dl}^{(i)}}{\partial z^2}-\frac{\partial^2 \varepsilon_i\left(z\right)}{\partial z^2}, i=\overline{1,n} \tag{1}$$

where $\varepsilon_i\left(z\right)$ is function which describes the mechanical deformation that appears due to non-assembling of parameters of lattice of contacting layers of heterosystem, and for the concentration of impurities $N_{dl}^{(i)}\left(z,t\right)$ (parabolic equation)

$$\frac{\partial N_{dl}^{(i)}}{\partial t}=D_i\frac{\partial^2 N_{dl}^{(i)}}{\partial z^2}-D_i N_{do}^{(i)}\frac{\Theta_d^{(i)}}{k_B T}\frac{\partial^2 U_i}{\partial z^2}+G_d^{'(i)}-\frac{N_{dl}^{(i)}}{\tau_d^{(i)}}, \tag{2}$$

with the boundary conditions

$$\begin{aligned} &\left.\partial N_{dl}^{(1)}\left(z,t\right)/\partial z\right|_{z\to-\infty}=0, &&\left.\partial N_{dl}^{(3)}\left(z,t\right)/\partial z\right|_{z\to\infty}=0,\\ &N_{dl}^{(1)}\left(-a,t\right)=N_{dl}^{(2)}\left(-a,t\right) &&N_{dl}^{(2)}\left(a,t\right)=N_{dl}^{(3)}\left(a,t\right)\end{aligned} \tag{3}$$

and primary condition

$$N_{dl}^{i}\left(z,0\right)=0. \tag{4}$$

The solutions of the system (1), (2) are derived in the case if the parameter of deformation $U_i\left(z,t\right)$ satisfies condition

$$\left(\frac{\left(L_d^{(i)}\right)^2}{2D_i}\right)^2\frac{\partial^2 U_i\left(z,t\right)}{\partial t^2}\ll\varepsilon_i\left(0\right).$$

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CLASSICAL SOLVABILITY OF MIXED PROBLEM FOR A DEGENERATE HYPERBOLIC SYSTEM

In the domain $\Pi = \{(x,t) : 0 < t < T,\ 0 < x < l\}$, we consider a hyperbolic system

$$\begin{gathered} \frac{\partial u}{\partial t} + \Lambda(x,t)\frac{\partial u}{\partial x} = f(x,t,u,v,w), \\ \frac{\partial v}{\partial x} = g(x,t,u,v,w),\ \frac{\partial w}{\partial t} = h(x,t,u,v,w) \end{gathered}$$

with the initial and boudary conditions:

$$\begin{aligned} & u_i(x,0) = q_i(x),\ w_s(x,0) = r_s(x),\ 0 \le x \le l, \\ & u_i(0,t) = \gamma_i^0\Big(t, \big(u_j(0,t)\big)_{j\in I_l}, w(x,t)\Big), \quad i \in I_0, \\ & u_i(l,t) = \gamma_l^2\Big(t, \big(u_j(l,t)\big)_{j\in\ I_0}, w(x,t)\Big), \quad i \in I_l, \\ & v_j(0,t) = \psi_j\Big(t, \big(u_j(0,t)\big)_{j\in I_l}, w(x,t)\Big), \quad j \in \{1,\dots,m\}, \end{aligned}$$

where $I_0 = \{\ i \in \{1,\dots,l\} :\ \lambda_i(0,t) > 0\}$, $I_l = \{i \in \{1,\dots,l\} : \lambda_i(i,t) < 0\}$ are sets of indices.

Using the method of characteristics and the Banach fixed point theorem, we established the existence and uniqueness of a global classical solution to the initial-boundary problem for the hyperbolic system of the first-order equations. Moreover, globality of the solution was found thanks to a specially selected metric with weight functions.

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GENERAL ANALYTICAL ONE-WAY NULL SOLUTION FOR MAXWELL FIELD IN KERR SPACE-TIME

We consider propagation of electromagnetic waves in gravitational field, which is described by Kerr metric. This metric is provided by rotating mass M with an angular momentum per unit mass a. Electromagnetic field is assumed to be the test field.

Let us consider the homogeneous Maxwell equation in spinor representation

$$\nabla^{AA'}\varphi_{AB} = 0, \tag{1}$$

and assume that Maxwell spinor φ_{AB} has the form [1]

$$\varphi_{AB} = \varphi_2 o_A o_B, \tag{2}$$

which means that repeated principal spinor of Maxwell field is proportional to o_A, where o_A is repeated principal spinor of Weyl spinor. Physical meaning of this choice implies consideration of the only retarded electromagnetic wave or consideration of the wave that has "$t-r$" dependence. We also can choose orientation of Maxwell spinor along another repeated principal spinor of Weyl spinor ι_A, which allows us to consider the only advanced ("$t+r$") electromagnetic wave.

Representation (2) allows us to decouple the system (1) and to consider the first order system of partial differential equations in Boyer-Lindquist coordinates

$$\begin{cases} \dfrac{r^2+a^2}{r^2-2Mr+a^2}\dfrac{\partial\varphi_2}{\partial t} + \dfrac{\partial\varphi_2}{\partial r} + \dfrac{a}{r^2-2Mr+a^2}\dfrac{\partial\varphi_2}{\partial\phi} + \dfrac{1}{r-ia\cos\theta}\varphi_2 = 0, \\[2ex] ia\sin\theta\dfrac{\partial\varphi_2}{\partial t} + \dfrac{\partial\varphi_2}{\partial\theta} + \dfrac{i}{\sin\theta}\dfrac{\partial\varphi_2}{\partial\phi} + \left(\mathrm{ctg}\theta + \dfrac{ia\sin\theta}{r-ia\cos\theta}\right)\varphi_2 = 0. \end{cases} \tag{3}$$

We have obtained solution of the system (3) and calculated the Maxwell tensor and energy-momentum tensor in Boyer-Lindquist coordinates and locally non-rotating frame.

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ON SOLUTIONS OF STOCHASTIC BOUNDARY VALUE PROBLEMS

Let a downward flow of σ-algebras $\{F_t, t \geq 0, F_t \subset F_{t_2} \text{ at } t_1 < t_2\}$ exist on the probability space.

I. The random function $u(t, x, \omega)$ with probability 1 is a solution of the Dirichlet problem for axis

$$d_t u(t, x, \omega) = [a^2 u_{xx}(t, x, \omega)]dt + bu(t, x, \omega)dw(t, \omega), \tag{1}$$

$$u(t_0, x, \omega) = \varphi(x), x \geq 0, u(t, 0, \omega) = \mu(t), t_0 \leq t \leq T. \tag{2}$$

Here, $w(t, \omega)$ is the standard scalar Wiener process, $\varphi(0) = \mu(0) = 0$. Continuing the initial function $\varphi(x)$ by the odd manner on the whole axis, we find the Green function $G(t, x, \omega)$ of the Cauchy problem for equation (1).

Then the solution of the problem (1)-(2) with probability 1 is represented by the sum of two terms and there is the correct follow assessment (with the operation of mathematical expectation M):

$$|M\{u(t, x, \omega)\}| \leq \text{const}(|\varphi(x)|_{C(\mathbb{R}^+)} + |\mu(t)|_{KC[t_0,T]}). \tag{3}$$

II. The random function $u(t, x, y, \omega)$ with probability 1 satisfies the singular heat equation

$$u_t = a_0^2 u_{xx} + a_1 \Big(u_{yy} + \frac{2\nu + 1}{y} dy \Big), \tag{4}$$

and conditions

$$u(0, x, y, \omega) = \varphi(x, y), u_y(t, x, 0, \omega) = 0,$$

$$u(t, 0, y, \omega) = \psi_1(t) + s_1 w_1(t, \omega), u(t, l, y, \omega) = \psi_2(t) + s_2 w_2(t, \omega). \tag{5}$$

Combining the method of solving of mixed problems and elements of the theory of random processes, we have obtained the estimate on the norm of the solution of the problem (4)-(5).

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INDIVIDUAL PARTICLE APPROACH TO THE NONLINEAR SHOCK ACCELERATION

Diffusive shock acceleration of charged particles on the strong shock results in the power-law spectrum of accelerated particles, in the test-particle approximation, when particles do not modify the hydrodynamic structure of the flow. In contrast, if acceleration is effective and particles take out considerable fraction of the shock energy, then they affect the flow velocity distribution in the precursor and their spectrum $f(p)$ has the concave shape. The analytic solution of the kinetic equation which accounts for this nonlinear effect was obtained in 2002 [1]. We demonstrate that the same result may be derived in the individual-particle approach of Bell developed originally for the test-particle regime [2].

The probability for particle with momentum p and velocity v to return to the shock, being advected with the flow speed u downstream, and, thus, to continue with the ith cycle of acceleration, is $\mathcal{P} = \exp\left[-\sum_i 4u/v_i\right]$, to the leading order in u/v. The increase in momentum in one cycle is $\Delta p/p = 4(u_\mathrm{p} - u)/(3v)$ where u_p is the flow velocity upstream in the farthest point which may be reached by the particles with momentum p. In other words, particles 'see' the effective shock compression $\sigma(p) = u_\mathrm{p}/u$, different for different p. Substitution of $\mathcal{P}$ with $4/v$ from $\Delta p/p$ and use of the definition $f(p) = -n\left(4\pi p^2\right)^{-1}\partial\mathcal{P}/\partial p$ yield the particle spectrum for the modified flow velocity distribution u_p:

$$f(p) = \frac{3n}{4\pi p_\mathrm{o}^3\left[\sigma(p)-1\right]}\exp\left(-\int_{p_\mathrm{o}}^{p}\frac{3\sigma(p')}{\sigma(p')-1}\frac{dp'}{p'}\right) \qquad (1)$$

where p_o is the injection momentum, n the number density of accelerated particles. The known test-particle result follows from (2) by substitution of the uniform distribution u_p, i.e. with σ independent of p.

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A REMARK TO ESTIMATES FROM BELOW FOR AN ANALYTIC CHARACTERISTIC FUNCTIONS

A non-decreasing function F continuous on the left on $(-\infty,+\infty)$ is said to be a probability law if $\lim\limits_{x\to+\infty} F(x)=1$ and $\lim\limits_{x\to-\infty} F(x)=0$, and the function $\varphi(z)=\int_{-\infty}^{+\infty} e^{izx}dF(x)$ defined for real z is called a characteristic function of this law. If φ has an analytic continuation on the disk $\mathbb{D}_R=\{z:|z|<R\}$, $0<R\le+\infty$, then we call φ an analytic in $\mathbb{D}_R$ characteristic function of the law F.

We denote by L_{si} a class of positive continuous functions α on $(-\infty, +\infty)$ such that $\alpha(x)=\alpha(x_0)$ for $x\le x_0$, $0<\alpha(x)\uparrow+\infty$ and $\alpha(cx)=(1+o(1))\alpha(x)$ as $x_0\le x\uparrow+\infty$ for each $c\in(0,+\infty)$.

Theorem. *Let $\alpha\in L_{si}$ be a continuously differentiable function and φ be an analytic in $\mathbb{D}_R$ characteristic function of a probability law F. Suppose that is executed one of conditions:*

1) $\varrho>1$, $\overline{\lim\limits_{x\to+\infty}} \frac{d\ln\alpha^{-1}(x)}{d\ln\alpha^{-1}(\varrho x)}=q(\varrho)<1$, $\alpha\left(\frac{x}{\alpha(x)}\right)=(1+o(1))\alpha(x)$ as $x\to+\infty$ and

$$\alpha\left(\frac{x_k}{\ln\left(W_F(x_k)e^{Rx_k}\right)}\right)\le\frac{\alpha(x_k)}{\varrho} \tag{1}$$

for some increasing to $+\infty$ sequence (x_k) of positive numbers such that $\alpha^{-1}(\alpha(x_{k+1})/\varrho)=O(\alpha^{-1}(\alpha(x_k)/\varrho))$ as $k\to\infty$;

2) $\varrho<1$, $\overline{\lim\limits_{x\to+\infty}} \frac{d\ln\alpha^{-1}(\varrho x)}{d\ln\alpha^{-1}(x)}=q(\varrho)<1$, $\frac{d\alpha^{-1}(\varrho\alpha(x))}{dx}=\frac{1}{f(x)}\downarrow 0$, $\alpha^{-1}(\varrho\alpha(f(x)))=O(\alpha^{-1}(\varrho\alpha(x))$ as $x\to+\infty$ and

$$\alpha\left(\ln\left(W_F(x_k)e^{Rx_k}\right)\right)\ge\varrho\alpha(x_k) \tag{2}$$

for some increasing to $+\infty$ sequence (x_k) of positive numbers such that $\overline{\lim\limits_{k\to\infty}}\frac{f(x_{k+1})}{f(x_k)}<2$.

Then

$$\alpha(\ln\mu(r,f))\ge(1+o(1))\rho\alpha\left(\frac{1}{R-r}\right),\quad r\uparrow R. \tag{3}$$

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SOLUTIONS OF MATHISSON-PAPAPETROU EQUATIONS FOR FAST PARTICLES

The Mathisson-Papapetrou (MP) equations [1] are some generalization of the geodesic equations. It is important that the MP equations can be used for the investigation of spinning particle motions with any velocity relative to the source of the gravitational field (for example, Schwarzschild's or Kerr's black hole), up to the speed of light, similarly as the geodesic equations are used for a fast moving spinless particle.

The priority of the Mathisson-Pirani supplementary condition for the MP equations is argued. At this condition a new representation of the MP equations by using constants of the particle's motions, energy, and angular momentum is elaborated [2]. A procedure of separation of the MP equations solutions which describe motions of the particle proper center of mass in Schwarzschild's metric is considered. Different solutions of the MP equations in the Schwarzschild and Kerr metric both in the linear and nonlinear spin approximation are investigated, computed and illustrated by the typical figures.

Some connections between the MP equations and the general relativistic Dirac equation are discussed. It is stressed that for correct description of a highly relativistic fermion in the gravitational field it is necessary to take into account the nonlinear spin terms.

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THE PROBLEM WITH MOVABLE BOUNDARIES FOR THE SINGULAR HYPERBOLIC SYSTEM OF QUASI-LINEAR EQUATIONS

On the curvilinear domain $R_T = \{(x,t) \in \mathbb{R}^2 | s_1(t) < x < s_2(t),\ s_1(0) = s_2(0) = 0\ , 0 < t < T\}$, where functions $s_1, s_2 : [0,T] \to \mathbb{R}, s_1, s_2 \in C^1[0,T]$, we consider the following singular hyperbolic system of quasi-linear equations

$$\sum_{j=1}^{n} g_{ij}(x,t,\mathrm{w}) \left(\frac{\partial u_j}{\partial t} + \lambda_i(x,t,\mathrm{w}) \frac{\partial u_j}{\partial x} \right) = F_i(x,t,\mathrm{w})\ ,\ i \in \{1,...,n\}, \quad (1)$$

$$\frac{\partial \upsilon_j}{\partial t} = G_j(x,t,\mathrm{w}), \frac{\partial \omega_k}{\partial t} = R_k(x,t,\mathrm{w}), j \in \{1,...,m\}, k \in \{1,...,r\}, \quad (2)$$

where $\mathrm{w} = (u, \upsilon, \omega)$ is unknown function.

The initial and boundary conditions for the system (1)–(2) are

$$\mathrm{w}(0,0) = \mathrm{w}^0, \quad (3)$$

$$u_i(s_1(t),t) = k_i(t, u_1(s_2(t),t), u_2(s_1(t),t)), \quad (4)$$

$$u_p(s_2(t),t) = k_p(t, u_1(s_2(t),t), u_2(s_1(t),t)), \quad (5)$$

$$\upsilon_j(s_1(t),t) = k_j^1(t, u_1(s_2(t),t), u_2(s_1(t),t)), \quad (6)$$

$$\upsilon_j(s_2(t),t) = k_j^2(t, u_1(s_2(t),t), u_2(s_1(t),t)), \quad (7)$$

$$\omega_k(s_1(t),t) = k_k(t, u_1(s_2(t),t), u_2(s_1(t),t)). \quad (8)$$

Applying the method of characteristics and the Banach fixed point theorem for system (1)–(8), the conditions of existence and uniqueness of a local solution in a curvilinear sector are established.

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ON COMMON EIGENVECTORS OF TWO MATRICES OVER A PRINCIPAL IDEAL DOMAIN

Let R be a principal ideal domain with an identity element $e \neq 0$ and let $M_{m,n}(\mathrm{R})$ denote the set of $m \times n$ matrices over R. The symbol $\big[A, B\big]$ is the standard notation for the commutator $AB - BA$ of the matrices $A, B \in M_{n,n}(\mathrm{R})$.

It is said that the matrices $A, B \in M_{n,n}(\mathrm{R})$ have a common left eigenvector, if there exists a nonzero vector $\bar{u} \in M_{1,n}(\mathrm{R})$ such that

$$\bar{u}A = \bar{u}\alpha \qquad \text{and} \qquad \bar{u}B = \bar{u}\beta, \quad \text{where} \quad \alpha, \beta \in \mathrm{R}.$$

Matrices $A, B \in M_{n,n}(\mathrm{R})$ are said to be simultaneously triangularizable if there exists a matrix $U \in GL(n, \mathrm{R})$, such that

$$UAU^{-1} \qquad \text{and} \qquad UBU^{-1}$$

are lower triangular matrices (see [1]).

Theorem. *Let $A, B \in M_{n,n}(\mathrm{R})$ be the matrices with minimal polynomials $m_A(\lambda) = (\lambda - \alpha_1)(\lambda - \alpha_2)$ and $m_B(\lambda) = (\lambda - \beta_1)(\lambda - \beta_2)$ respectively, where $\alpha_i, \beta_i \in \mathrm{R}$, $\alpha_1 \neq \alpha_2$ and $\beta_1 \neq \beta_2$. The pair of matrices $A, B \in M_{n,n}(\mathrm{R})$ have a common left eigenvector over R if and only if the commutator $\big[A, B\big]$ is a singular matrix.*

Let $A \in M_{n,n}(\mathrm{R})$ be an involutory matrix, that is $A^2 = I_n$. If $A \neq \pm I_n$, then $m(\lambda) = (\lambda - e)(\lambda + e)$ is the minimal polynomial of an involutory matrix A. The most important application of the Theorem are the following corollaries.

Corollary 1. *The involutory matrices $A, B \in M_{n,n}(\mathrm{R})$ have a common eigenvector if and only if the commutator $\big[A, B\big]$ is a singular matrix.*

Corollary 2. *The involutory matrices $A, B \in M_{n,n}(\mathrm{R})$ are simultaneously triangularizable if and only if the commutator $\big[A, B\big]$ is a nilpotent matrix.*

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PERTURBATION OF A SINGULAR BOUNDARY VALUE PROBLEM FOR A SECOND ORDER DIFFERENTIAL EQUATION

Consider the following BVP on the half-line

$$x^2y'' + xp(x)y' + q(x)y = \varepsilon f(x,y), \quad y(+0) = y(+\infty) = 0. \tag{1}$$

Here $p(\cdot) \in \mathrm{C}^1(\mathbb{R}_+;\mathbb{R})$, $q(\cdot) \in \mathrm{C}^2(\mathbb{R}_+;\mathbb{R})$, $f(\cdot,\cdot) \in \mathrm{C}^2(\mathbb{R}_+ \times \mathbb{R};\mathbb{R})$, and ε is a small parameter. We study a complicated critical case when the unperturbed equation ($\varepsilon = 0$) is dichotomic on $[1,\infty)$ and has a fundamental system of solutions $y^0(\cdot)$, $y^*(\cdot)$ such that $y^0(\cdot)$ is a solution of the unperturbed BVP while $y^*(\cdot)$ is non-differentiable at $x = 0$ and unbounded on $[1,\infty)$.

Under certain additional conditions (in particular, $q(0) \neq 0$, $p(0) + q(0) \neq 0$, $f(0,y) \equiv 0$, $\lim_{x\to+\infty} x^{-2}f(x,y) = 0$), applying the results [1,2], we prove that the problem (1) is soluble for all sufficiently small $\varepsilon > 0$ once the determining equation

$$\Phi(c) := \int\limits_0^\infty \exp\left(\int\limits_1^x \frac{p(s)}{s}\mathrm{d}s\right) \frac{y^0(x)f\left(x, cy^0(x)\right)}{x}\mathrm{d}x = 0$$

has a simple solution.

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ON THE INVERSE PROBLEM FOR THE SEMILINEAR SECOND-ORDER ULTRAPARABOLIC EQUATION WITH THREE UNKNOWN FUNCTIONS IN THE RIGHT-HAND SIDE

Let $\Omega \subset \mathbb{R}^n$ and $D \subset \mathbb{R}^l$ be bounded domains with the boundaries $\partial\Omega \in C^1$ and $\partial D \in C^1$; $x \in \Omega,\ y \in D,\ t \in (0,T)$, where $T > 0$ is a fixed number.

Denote: $G = \Omega \times D$, $Q_T = G \times (0,T)$, $\Sigma_T = \partial\Omega \times D \times (0,T)$, $S_T = \Omega \times \partial D \times (0,T)$, $\Pi_1 = D \times (0,T)$, $\Pi_2 = \Omega \times (0,T)$.

The conditions of the existence and uniqueness of a set $(u(x,y,t), q_1(x), q_2(t), q_3(y))$ from Sobolev spaces that is a weak solution for the inverse problem

$$u_t + \sum_{i=1}^{l} \lambda_i(y) u_{y_i} - \sum_{i,j=1}^{n} (a_{ij}(x) u_{x_i})_{x_j} + c(x,y,t)u + g(x,y,t,u) =$$

$$= f_1(x,y,t)q_1(x) + f_2(x,y,t)q_2(t) + f_3(x,y,t)q_3(y) + f_0(x,y,t) \qquad (1)$$

$$u(x,y,0) = u_0(x,y), \quad (x,y) \in G, \quad u|_{\Sigma_T} = 0, \qquad u|_{S_T^1} = 0, \qquad (2)$$

$$\int\limits_{\Pi_1} K_1(y,t) u(x,y,t)\, dydt = E_1(x), \qquad x \in \Omega, \qquad (3)$$

$$\int\limits_{G} K_2(x,y) u(x,y,t)\, dxdy = E_2(t), \qquad t \in [0,T], \qquad (4)$$

$$\int\limits_{\Pi_2} K_3(x,t) u(x,y,t)\, dxdt = E_3(y), \qquad y \in D \qquad (5)$$

are obtained in the domain Q_T. Here $S_T^1 = \{(x,y,t) \in S_T : \sum\limits_{i=1}^{l} \lambda_i(x,y,t) \times \times \cos(\nu, y_i) < 0\}$, ν is an outward unit normal vector to the surface S_T, function $g(x,y,t,u)$ satisfies a Lipschitz condition on the variable u.

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PROBLEM WITH MULTIPLE NODES FOR A LINEAR PARTIAL DIFFERENTIAL EQUATION

We considered the problem

$$L\left(\frac{\partial}{\partial t}, D_x\right) u(t,x) \equiv \frac{\partial^n u}{\partial t^n} + \sum_{j=1}^{n} A_j(t, D_x) \frac{\partial^{n-j} u}{\partial t^{n-j}} = 0, \ (t,x) \in Q_T^p, \quad (1)$$

$$\begin{cases} \dfrac{\partial^{q_j-1} u(t,x)}{\partial t^{q_j-1}}\Big|_{t=t_j} = \varphi_{j,r_j}(x), \ q_j = \overline{1,r_j}, \ j = \overline{1,l}, \ x \in \Omega^p, \\ 2 \le l \le n, \quad r_1 + \ldots + r_l = n, \quad 0 \le t_1 < \ldots < t_l \le T, \end{cases} \quad (2)$$

where $D_x = (-i\partial/\partial x_1, \ldots, -i\partial/\partial x_p)$, $Q_T^p = (0;T) \times \Omega^p$, $\Omega^p = (\mathbb{R}/2\pi\mathbb{Z})^p$, $T > 0$ and $A_j(t, D_x)$, $j = \overline{1,n}$, are differential expressions of the form

$$A_j(t, D_x) = \sum_{|s| \le N_j} A_j^s(t)(-i\partial/\partial x_1)^{s_1} \ldots (-i\partial/\partial x_p)^{s_p}, \ A_j^s \in \mathbb{C}, \ N_j \in \mathbb{N},$$

and $A_j^s(t)$, $s = (s_1, \ldots, s_p) \in \mathbb{Z}_+^p$, $j = \overline{1,n}$, are smooth functions on $[0;T]$.

We have obtained the conditions of existence and uniqueness of solution of the problem (1), (2). We have used the metric approach [1, 2] to solve the problem of small denominators arising in the solution construction of this problem.

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INVESTIGATION OF SOME MATHEMATICAL MODEL OF DYNAMICS OF THE NONLINEAR FLEXURAL VIBRATIONS OF A DRILL COLUMN

As a mathematical model of the flexural vibrations of the well drilling column, rotating with an angular velocity Ω and with incompressible fluid moving along it with relative velocity V, we consider the equation

$$(\rho_1+\rho_2)\frac{\partial^2 u(x,t)}{\partial t^2}+\rho_2 V\frac{\partial^2 u(x,t)}{\partial t\partial x}-\left(S(x)-\rho_2 V^2\right)\frac{\partial^2 u(x,t)}{\partial x^2}-$$
$$-\frac{\partial S(x)}{\partial x}\frac{\partial u(x,t)}{\partial x}+EI\frac{\partial^4 u(x,t)}{\partial t^4}-(\rho_1+\rho_2)\Omega^2 u(x,t)=$$
$$=k_1 EI\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^3-k_2\frac{\partial u(x,t)}{\partial x},$$

where $u(x,t)$ is a transverse deviation of the column section with x coordinate at arbitrary instant of time t, ρ_1, ρ_2 are respectively masses of length unit of the column and the fluid, moving inside, $S(x)$ is an axial thrust in any column section made by special loads for the pressure on the drill, and the force of column weight, EI is a flexural rigidity of the column, k_1 and k_2 are the coefficients that define deviation of elastic properties of the drilling column material from a linear law and the resistance force respectively.

We investigate the influence of the motion of fluid flushing the cutter of a well drilling column, and the angular rotational velocity upon dynamic characteristics of its flexural vibrations. We take into account the nonlinear elastic features of column material. As a base of the research we took the Galerkin method and the Van der Pol method. Combining those two methods made possible to obtain the relations describing the main parameters of the dynamical process in both nonresonance and resonance case.

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SOLUTION OF THE BOUNDARY VALUE PROBLEM FOR A CYLINDRICAL ORTHOTROPIC SHELL WITH A BIG RECTANGULAR HOLE BY THE METHOD OF FINITE BODIES

The scientific report concerns the stress state of the orthotropic cylindrical shell with a considerable rectangular hole with sides $2a$, $2b$ ($ab > R^2$). We use a system of equations that exactly satisfies all the equations of the equilibrium shell:

$$V_1\Phi = \varepsilon E_1 \frac{\partial^2 w}{\partial z^2}, \;\; V_2 w = -(1-\nu_1\nu_2)\frac{12\varepsilon}{h^4 E_2}\frac{\partial^2 \Phi}{\partial z^2} - \frac{\varepsilon^2}{h^2 R^2}\frac{\partial^2 w}{\partial \varphi^2}, \qquad (1)$$

where $\Phi(z,\varphi)$ is the stress function; $w(z,\varphi)$ is the function of displacement of the middle surface; $V_1, \; V_2$ are differential operators of the fourth order.

The system of equations (1) differs from known equations of the theory of shells constructed using Kirchhoff-Love hypotheses with only the term $\frac{\varepsilon^2}{h^2}\frac{\partial^2 w}{\partial \gamma^2}$. Forces and moments are to be defined.

An analytical-numerical method of finite bodies for solving the boundary value problem in a triply connected domain is proposed. The shell is divided into circumferences $z = \pm a$ on three simply-connected parts. Countable number of resolving functions that accurately satisfy the equations of the shell was found.

The algorithm of analytical and numerical solution of boundary value problems is based on an approximation of the stress state of separate parts the shell by finite sums of resolving functions. All the boundary conditions and the conditions of the contact of separate parts shell are satisfied. The method is based also on minimization of the square-law form which characterizes integral of a square-law deviation of the found solution from the set boundary conditions. The criteria under which the construction of approximate solutions coincides with the exact ones was established. The distribution of stresses in a cylindrical shell was determined.

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ON THE DIVISIBILITY OF THE SECOND-ORDER MATRICES

Let R be a commutative domain elementary divisor [1] with $1 \neq 0$ and A be a 2×2 matrix over R. For the matrix A there exist invertible matrices P_A , Q_A , such that

$$P_A A Q_A = \mathrm{E} = diag(\varepsilon_1, \varepsilon_2), \text{ where } \varepsilon_1 | \varepsilon_2 .$$

The matrix E called the canonical diagonal form for the matrix A.

If $A = BC$, then the matrix B is a left divisor of the matrix A and the matrix A is a right multiples of the matrix B.

Let's denote the greatest common divisor of the elements a and b by (a, b).

The conditions under which the matrix B is a left divisor of the matrix A over commutative domain elementary divisor was proposed by V.Shchedryk in 2009 [2].

Theorem. *Let*

$$A \sim E = diag(\varepsilon_1, \varepsilon_2),\ \varepsilon_1 \mid \varepsilon_2,\ B \sim \Delta = diag(\delta_1, \delta_2),\ \delta_1 \mid \delta_2$$

be matrices over R and

$$D \sim \Phi = diag(\varphi_1, \varphi_2),\ \varphi_1 \mid \varphi_2,\ T \sim \Gamma = diag(\gamma_1, \gamma_2),\ \gamma_1 \mid \gamma_2,$$

moreover $A = DA_1$, $B = DB_1$ and $A = TA_2$, $B = TB_2$. If

$$\gamma_1 | \varphi_1 = (\varepsilon_1, \delta_1) \text{ and } \gamma_2 | \varphi_2, \text{ then } D = TN.$$

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BIRKHOFF SPLINE INTERPOLATION IN AN INITIAL VALUE PROBLEM FOR AN ORDINARY DIFFERENTIAL EQUATION

Consider the initial value problem on $[-1, 1]$:

$$\begin{cases} y' = f(x, y) \\ y(-1) = y_0. \end{cases} \tag{1}$$

In [1] and [2], a new method was proposed for the function interpolation using 3-th and 4-th order polynomial splines on a regular mesh with the following conditions: function and its first derivative can be interpolated in integer points and the interpolation points for the subsequent higher order derivatives are placed twice as frequent as for the previous order. Such interpolation is usually called a Birkhoff interpolation. The basis splines with a compact support based on perfect splines $\sigma_3(x)$ and $\sigma_4(x)$ were used for the interpolation.

We propose to seek a solution $y(x)$ of (1) in a form of its cubic spline interpolant, which needs its values and the values of its first and second derivatives at the points -1, 0, 1, and besides, the values of y'' at -1/2 and 1/2. After interpolating $f(x, y(x))$ using the identity

$$y(a) - y(b) = \int_a^b f(x, y(x))dx$$

and knowing the values of integrals of the basis splines, we obtain the iterative formulae to find $y(0)$ and $y(1)$, and then to find $y(-1/2)$ and $y(1/2)$.

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ON $h-$MEASURE AND ENTIRE DIRICHLET SERIES

Let $D(\Lambda)$ be the class of entire Dirichlet series of the form $F(z) = \sum\limits_{n=0}^{+\infty} a_n e^{z\lambda_n}$, where $\Lambda = (\lambda_n)$, $0 = \lambda_0 < \lambda_n \uparrow +\infty$ $(1 \leq n \to +\infty)$. For $F \in D(\Lambda)$ and $x \in \mathbb{R}$ we denote $\mu(x, F) = \max\{|a_n|e^{x\lambda_n} \colon n \geq 0\}$, $\nu(x, F) = \max\{n \colon |a_n|e^{x\lambda_n} = \mu(x, F)\}$. It is known [1] that for every entire function $F \in D(\Lambda)$ the relation

$$F(x + iy) = (1 + o(1))a_{\nu(x,F)}e^{(x+iy)\lambda_{\nu(x,F)}} \tag{1}$$

holds as $x \to +\infty$ outside some exceptional set E of finite Lebesgue measure uniformly in $y \in \mathbb{R}$, if and only if $\sum\limits_{n=0}^{+\infty} \dfrac{1}{\lambda_{n+1} - \lambda_n} < +\infty$. In [2] it is proved that the finiteness of Lebesgue measure of an exceptional set E is the sharp estimate in the class $D(\Lambda)$.

Let h, Φ be positive continuous functions increasing to $+\infty$ on $[0; +\infty)$ and φ the inverse function to function Φ.
Denote $D(\Lambda, \Phi) = \{F \in D(\Lambda) : \ \ln \mu(x, F) \geq x\Phi(x) \ (x > x_0)\}$.

Theorem. *If a differentiable function h such that $h'(x)$ is non-decreasing to $+\infty$ on $[0; +\infty)$ and*

$$(\forall b > 0) : \quad \sum_{k=0}^{+\infty} \frac{1}{\lambda_{k+1} - \lambda_k} h'\Big(\varphi(\lambda_k) + \frac{b}{\lambda_{k+1} - \lambda_k}\Big) < +\infty,$$

then for every function $F \in D(\Lambda, \Phi)$ relation (1) holds as $x \to +\infty$ outside some set E of finite h-measure ($\int_E dh(x) < +\infty$) uniformly in $y \in \mathbb{R}$.

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BRINGING THE SYSTEM OF EQUATIONS, THAT ARE NOT SOLVED IN RESPECT TO DERIVATIVES, TO THE SYSTEM OF EQUATIONS, THAT ARE PARTIALLY SOLVED IN RESPECT TO DERIVATIVES

Let's consider the Cauchy problem of the form

$$\begin{cases} F\left(z, W, W'\right) = 0, \\ W\left(z\right) \to 0 \ \text{ as } \ z \longrightarrow 0 \end{cases} \tag{1}$$

where the function $F : D \times G \times \widetilde{G} \to \mathbb{C}^n$, $D = \{z : |z| < r, r > 0\} \subset \mathbb{C}$, $G, \widetilde{G} \subset \mathbb{C}^m$, $n, m \in \mathbb{N}$, $(0, 0, 0) \in \partial(D \times G \times \widetilde{G})$ is analytic in the domain $D \times G \times \widetilde{G}$.

We study the solutions of the Cauchy problem (1) that satisfy the additional condition

$$W^{'}\left(z\right) \to 0 \ \text{ as } \ z \longrightarrow 0 \tag{2}$$

It is proved that, if the conditions of the theorem about the implicit holomorphic function are met, the system (1) with some change of variables may be written in the form

$$S\left(z, Y\right) Y^{'} = K\left(z, Y, Y'\right), \tag{3}$$

where $S : U \to \mathbb{C}^{m\times n}$, $K : J \to \mathbb{C}^m$, $U \subset D \times G, J \subset D \times G \times \widetilde{G}$.

In particular, we consider the transformation of the Cauchy problem (1) to the Cauchy problem of the form

$$\begin{cases} A\left(z\right) Y^{'} = B\left(z\right) Y + f\left(z, Y, Y'\right), \\ Y\left(z\right) \to 0 \ \text{ as } \ z \longrightarrow 0, \end{cases} \tag{4}$$

where matrices A, $B : D_1 \to \mathbb{C}^{m\times n}, D_1 = \{z : |z| < R_1, 0 < R_1 < r\} \subset \mathbb{C}$, the function $f : D_1 \times G_1 \times G_2 \to \mathbb{C}^m$, $G_k \subset \mathbb{C}^n$, $0 \in G_k$, $k = 1, 2$.

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BOUNDEDNESS OF FABER OPERATORS

Let $\Omega \subset \mathbb{C}$ be a compact set, not a single point, and suppose that $\widehat{\mathbb{C}} \setminus \Omega$ is a simply connected domain in the extended complex plane $\widehat{\mathbb{C}}$. Than Riemann mapping $\Psi : \widehat{\mathbb{C}} \setminus \overline{\mathbb{D}} \to \widehat{\mathbb{C}} \setminus \Omega$ exists such that $\Psi(\infty) = \infty$ and $\Psi'(\infty) > 0$.

We denote by $A(\Omega)$ the Banach algebra of functions f, analytic in the interior of Ω and continuous on Ω with the supremum norm.

Let $\mathcal{P}_n$ denote the set of all polynomials of degree at most n and set $\mathcal{P} = \cup_{n=1}^{\infty} \mathcal{P}_n$. When we consider $\mathcal{P}$ as a subspace of $A(\Omega)$ or $A(\overline{\mathbb{D}})$, $\overline{\mathbb{D}} := \{w : |w| \le 1\}$, we denote it by $\mathcal{P}(\Omega)$ or $\mathcal{P}(\overline{\mathbb{D}})$ respectively.

We define the Faber operator $T : \mathcal{P}(\overline{\mathbb{D}}) \to \mathcal{P}(\Omega)$ by

$$T(f)(z) = \frac{1}{2\pi i} \int_{|w|=1} \frac{f(w)\Psi'(w)}{\Psi(w) - z} dw.$$

Ω is called a Faber set if $T : \mathcal{P}(\overline{\mathbb{D}}) \to \mathcal{P}(\Omega)$ is bounded. In this case T admits a unique extension to a continuous operator from $A(\overline{\mathbb{D}})$ to $A(\Omega)$, also denoted by T.

In this talk, we describe the set of all compacts Ω for which $\|T\| := \sup\{\|T(f)\|_{A(\Omega)} : \|f\|_{A(\overline{\mathbb{D}})} \le 1\} = 1$.

Theorem. *Let Ω and T be as above. Then Ω is s Faber set and $\|T\| = 1$ if and only if*

$$\inf_{w:|w|>1} \operatorname{Re} \frac{\Psi'(w)w}{\Psi(w) - z} \ge \frac{1}{2} \quad \forall\ z \in \Omega.$$

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METRIC ESTIMATES OF SMALL DENOMINATORS FOR ONE NONLOCAL CONJUGATION PROBLEM

Let $\lambda_1, \ldots, \lambda_n$, $\mu_1, \ldots, \mu_n$, $n \in \mathbb{N}$, be different sets of pairs of real numbers, $\{\lambda_1, \ldots, \lambda_n\} \cap \{\mu_1, \ldots, \mu_n\} = \emptyset$. Let $\Lambda = \|\lambda_j^{q-1}\|_{j,q=1}^n$, $M = \|\mu_j^{q-1}\|_{j,q=1}^n$ be the Vandermonde matrices. We denote

$$A(k) = \|\lambda_j^{q-1} \exp(-\alpha\lambda_j P(k))\|_{j,q=1}^n, \quad B(k) = \|\mu_j^{q-1} \exp(\beta\mu_j P(k))\|_{j,q=1}^n,$$

$$\Delta(k) = \det \begin{Vmatrix} \Lambda & M \\ A(k) & \nu B(k) \end{Vmatrix}, \quad k = (k_1, \ldots, k_p) \in \mathbb{Z}^p,$$

where $\alpha, \beta > 0$, $\nu \in \mathbb{C}$, and $P(D_x)$, $D_x = (-i\partial/\partial x_1, \ldots, -i\partial/\partial x_p)$, is a differential expression of the degree N such that

$$\inf_{k \in \mathbb{Z}^p} |P(k)|(1 + |k|)^{-N} > 0, \quad |k| = |k_1| + \ldots + |k_p|. \tag{1}$$

A set of numbers $\lambda_1, \ldots, \lambda_n$ will be called a set of general type if, for any r, $1 \leq r \leq n-1$ and for arbitrary sets $(i_1, \ldots, i_r)$, $(j_1, \ldots, j_r)$, $1 \leq i_1 < \ldots < i_r \leq n$, $1 \leq j_1 < \ldots < j_r \leq n$, $\{i_1, \ldots, i_r\} \cap \{j_1, \ldots, j_r\} = \emptyset$, the inequalities $\lambda_{i_1} + \ldots + \lambda_{i_r} \neq \lambda_{j_1} + \ldots + \lambda_{j_r}$ are valid.

Theorem. *Let the condition* (1) *hold and let sets of positive numbers* $\lambda_1, \ldots, \lambda_n$, $\mu_1, \ldots, \mu_n$ *be sets of a general type. Then, for almost all (with respect to Lebesgue measure in the space* $\mathbb{R}^2$*) pairs* $(\alpha, \beta) \in (0, \alpha_0] \times (0, \beta_0]$, $\alpha_0, \beta_0 > 0$, *the estimates*

$$|\Delta(k)| \geq \begin{cases} |k|^{-\omega} \exp\left(-\beta(\mu_1 + \ldots + \mu_n) Re\, P(k)\right), & Re\ P(k) \geq 0, \\ |k|^{-\omega} \exp\left(\alpha(\lambda_1 + \ldots + \lambda_n) Re\, P(k)\right), & Re\ P(k) < 0, \end{cases}$$

are satisfied for all (except of a finite number) vectors $k \in \mathbb{Z}^p$ *if* $\omega > p2^n$.

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CONSTRUCTION OF THE SOLUTIONS FOR THE SINGULAR BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF DIFFERENTIAL EQUATIONS IN CRITICAL CASE

We consider the singular system of differential equations

$$B_0(t)\frac{dz}{dt} = A_0(t)z(t) + f(t), \quad t \in [a,b], \tag{1}$$

under the linear functional boundary condition

$$lz(\cdot) = d, \tag{2}$$

where $rank\ B_0(t) = n + m - 1\ \forall t \in [a,b]$, $A_0(t)$, $B_0(t)$ are the $((n+m) \times (n+m))$-dimensional matrices, $f(t)$ is the $(n+m)$-dimensional vector-function, of the following structure:

$$B_0(t)=\begin{bmatrix} B_1(t) & 0 \\ 0 & J \end{bmatrix}, \quad A_0(t)=\begin{bmatrix} A_1(t) & 0 \\ 0 & A_2(t) \end{bmatrix}, \quad f(t)=\begin{bmatrix} f^{(1)}(t) \\ f^{(2)}(t) \end{bmatrix},$$

$A_2(t)$ is the $(m\times m)$-dimensional, $A_1(t), B_1(t)$ are the $(n\times n)$-dimensional matrices, $detB_1(t) \neq 0$; $f^{(1)}(t)$ is the n-dimensional, $f^{(2)}(t)$ is the m-dimensional vector-functions, J is the $(m\times m)$-dimensional Jordan block with 0 on the diagonal; l is the $(n+m)$-dimensional linear vector functional, $d \in \mathbb{R}^{(n+m)}$ the vector column of constants.

We obtain the necessary and sufficient conditions for existence of solutions of the singular boundary value problem (1), (2) in a critical case, when the corresponding homogeneous singular boundary value problem has k linear independent solutions. Moreover we show, that solutions of the nonhomogeneous singular boundary value problem (1), (2) form k-parametric family solutions.

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A PARTICULAR CASE OF CONSTRUCTING GRACEFUL GRAPHS FROM GRACEFUL TREES

Labeling f of graph G is a mapping from the set of elements of G to the set of positive integers. If the mapping domain is the vertex set or the edge set then labeling f is called vertex labeling or edge labeling, respectively. An injection $f : V \to \{0, 1, 2, ..., q\}$ is called the graceful labeling of graph $G = (V, E)$ of size q, if it induces labeling $f^* : E \to \{1, 2, ..., q\}$ such that f^* is a bijection and $f^*(uv) = |f(u) - f(v)|$ for any two adjacent vertices $u, v \in V(G)$. Graph G is graceful if it admits a graceful labeling f.

Let S, T be trees, and let u, v be vertices of S, T, respectively. Consider attaching one copy of T to each vertex of S other than u, by identifying each vertex of S other than u with the vertex corresponding to v in a distinct copy of T. We denote the resulting tree by $S\Delta_{+1}T$, and we call the construction the Δ_{+1}-construction. Consider constructing graceful graphs from graceful trees obtained by Δ_{+1}-construction. A particular case is presented in the following theorem.

Theorem. *For a graceful tree $S\Delta_{+1}T$ of order p there exists a vertex $w \in V(S\Delta_{+1}T)$, $w \neq u$, such that unicyclic graph $G = S\Delta_{+1}T + uw$ is graceful.*

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ESTIMATES OF THE BEST ORTHOGONAL TRIGONOMETRIC APPROXIMATIONS FOR THE CLASSES OF CONVOLUTIONS OF PERIODIC FUNCTIONS WITH NOT HIGH SMOOTHNESS IN A UNIFORM METRIC

Let $C^{\psi}_{\beta,p}$ be the class of 2π–periodic functions f, represented by the convolutions $f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \Psi_\beta(x-t)\varphi(t)dt$, $\varphi \perp 1$, $\|\varphi\|_p \leq 1$, $\beta \in \mathbb{R}$, $a_0 \in \mathbb{R}$, where $\Psi_\beta(t) = \sum\limits_{k=1}^{\infty} \psi(k)\cos\left(kt - \frac{\beta\pi}{2}\right)$, $\psi(k) > 0$, $\Psi_\beta \in L_{p'}$, $1 < p < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$.

We consider the problem of finding the exact–order estimates of quantities $e^{\perp}_m(C^{\psi}_{\beta,p})_C = \sup\limits_{f\in C^{\psi}_{\beta,p}} \inf\limits_{\gamma_m} \|f(x) - \sum\limits_{k\in\gamma_m} \hat{f}(k)e^{ikx}\|_C$, $1 < p < \infty$, where γ_m, $m \in \mathbb{N}$, is an arbitrary set of m integer numbers, and $\hat{f}(k)$, $k \in \mathbb{Z}$, are Fourier coefficients of the function f.

By $\mathfrak{M}$ we denote the set of continuous, convex downward, positive functions $\psi(t)$, $t \geq 1$, that vanish at infinity. For every function $\psi \in \mathfrak{M}$ we introduce the characteristic $\alpha(\psi;t) = \frac{\psi(t)}{t|\psi'(t)|}$, $\psi'(t) := \psi'(t+0)$ and we denote $\mathfrak{M}_0 = \{\psi \in \mathfrak{M} : \exists K > 0\ \forall t \geq 1\ \ \alpha(\psi;t) \geq K\}$.

Theorem. *Let* $1 < p < \infty$, $\sum\limits_{k=1}^{\infty} \psi^{p'}(k)k^{p'-2} < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$, *and the function* $g_p(t) = \psi(t)t^{1/p}$ *such that* $g_p \in \mathfrak{M}_0$ *and* $\underline{\alpha}_1(g_p) = \inf\limits_{t\geq 1} \alpha(g_p;t) > p'$. *Then for arbitrary* $\beta \in \mathbb{R}$ *the correlation is true:*

$$e^{\perp}_n(C^{\psi}_{\beta,p})_C \asymp \Big(\sum_{k=n}^{\infty} \psi^{p'}(k)k^{p'-2}\Big)^{\frac{1}{p'}}.$$

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ON OPEN AND DISCRETE MAPPINGS WITH A MODULUS CONDITION

Everywhere further D is a domain in $\mathbb{R}^n$, $n \geq 2$, m is the Lebesgue measure in $\mathbb{R}^n$. Given a family Γ of curves γ in $\mathbb{R}^n$, a Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, abbr. $\rho \in \operatorname{adm}\Gamma$, if $\int\limits_{\gamma} \rho(x)|dx| \geqslant 1$ for each (locally rectifiable) $\gamma \in \Gamma$. Given $p \geqslant 1$, the *p-modulus* of Γ is defined as the quantity $M_p(\Gamma) := \inf\limits_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x) dm(x)$. Denote $\Gamma(E, F, D)$ a family of all paths $\gamma : [a, b] \to \overline{\mathbb{R}^n}$, which join E and F in D. Given $y_0 \in f(D)$ and numbers $0 < r_1 < r_2 < \infty$, we denote $A(r_1, r_2, y_0) = \{y \in \mathbb{R}^n : r_1 < |y - y_0| < r_2\}$. Set $S(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| = r\}$, $B(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| < r\}$. Let $\Gamma(y_0, r_1, r_2)$ be the family of all paths γ in D such that $f(\gamma) \in \Gamma(S(y_0, r_1), S(y_0, r_2), A(r_1, r_2, y_0))$. We write $\varphi \in FMO(x_0)$, if $\overline{\lim\limits_{\varepsilon \to 0}} \frac{1}{\Omega_n \varepsilon^n} \int\limits_{B(x_0,\, \varepsilon)} |\varphi(x) - \varphi_\varepsilon| dm(x) < \infty$, $\Omega_n := m(B(0, 1))$, $\varphi_\varepsilon := \frac{1}{\Omega_n \varepsilon^n} \int\limits_{B(x_0,\, \varepsilon)} \varphi(x)\ dm(x)$. Set $\omega_{n-1} = \mathcal{H}^{n-1}(S(0, 1))$, $q_{x_0}(r) := \frac{1}{\omega_{n-1} r^{n-1}} \int\limits_{|x - x_0| = r} Q(x)\, d\mathcal{H}^{n-1}$.

Theorem. *Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, $p \in (n-1, n]$, $Q : \mathbb{R}^n \to (0, \infty)$ be a Lebesgue measurable function, $f : D \to \mathbb{R}^n$ be a sense-preserving mapping obeying $M_p(\Gamma(y_0, r_1, r_2)) \leqslant \int\limits_{f(D)} Q(y) \cdot \eta^p(y) dm(y)$ for every $y_0 \in f(D)$, every $0 < r_1 < r_2 < \infty$, and every nonnegative Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ with $\int\limits_{r_1}^{r_2} \eta(r) dr \geqslant 1$. Then f is discrete and open whenever the function Q satisfies at least one of the following conditions: 1) $Q \in FMO(y_0)$ for every $y_0 \in f(D)$; 2) for every $y_0 \in f(D)$ there exists $\delta(y_0) > 0$ such that for every small enough $\varepsilon > 0$ $\int\limits_{\varepsilon}^{\delta(y_0)} \frac{dt}{t^{\frac{n-1}{p-1}} q_{y_0}^{\frac{1}{p-1}}(t)} < \infty$ and $\int\limits_{0}^{\delta(y_0)} \frac{dt}{t^{\frac{n-1}{p-1}} q_{y_0}^{\frac{1}{p-1}}(t)} = \infty$.*

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ABOUT SMOOTH VARIABLE FUNCTIONS OF PROXIMATE ORDER IN THE SENSE OF BOUTROUX

Let p be an integer non-negative number, $1 < p < \eta_1 \leq \eta_2 < p+1$. The positive continuously differentiable on $(a;+\infty)$, $a \geq 0$, function $h(r)$ is called the smooth variable function of proximate order in the sense of Boutroux if

$$\begin{gathered} \varliminf_{r\to+\infty} (\ln h(e^r))' = \eta_1, \varlimsup_{r\to+\infty} (\ln h(e^r))' = \eta_2, \\ \lim_{r\to+\infty} (\ln h(e^r))^{(n)} = 0, \ \ \forall n \in \mathbb{N} \setminus \{1\}. \end{gathered} \tag{1}$$

Theorem A. *If positive continuously differentiable on* $(a;+\infty)$*,* $a \geq 0$*, function* $h(r)$ *satisfies the conditions* (1)*, then for any* $n \in \mathbb{N}$ *holds*

$$\varliminf_{t\to+\infty} \frac{t^n h^{(n)}(t)}{h(t)} = \eta_1(\eta_1 - 1)...(\eta_1 - n + 1),$$

$$\varlimsup_{t\to+\infty} \frac{t^n h^{(n)}(t)}{h(t)} = \eta_2(\eta_2 - 1)...(\eta_2 - n + 1).$$

Continuously differentiable in some neighborhood of $+\infty$ *function* $l(r)$ *is called perfect proximate order in the sense of Bouroux if for any* $n \in \mathbb{N}$ *holds* $\lim\limits_{r\to+\infty} r^n \ln r l^n(r) = 0$.

Theorem B. *Positive continuously differentiable on* $(a;+\infty)$*,* $a \geq 0$*, function* $h(r)$ *is smooth variable function of proximate order in the sense of Boutroux then and only then, when function* $l(r) = \frac{\ln h(r)}{\ln r}$ *is the perfect proximate order, for which the next equalities are hold:*

$$\varliminf_{r\to+\infty} l(r) = \eta_1, \quad \varlimsup_{r\to+\infty} l(r) = \eta_2.$$

Theorems A and B are the generalizations of corresponding results from [1].

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SOME MIXED PROBLEMS OF CRACKS CLOSURE IN SHALLOW SHELLS

The problem of the cracks edges contact in a shallow shell in combined tension and bending loading is considered in the two-dimensional statement. The model of a contact along a line [1] for describing of the crack closure phenomenon has been used. The mixed problem of the shallow shells theory with the coupled boundary conditions on a cut has been formulated:

$$\Delta\Delta\varphi - \frac{B}{R}\Delta_k w = 0, \quad \Delta\Delta w + \frac{1}{DR}\Delta_k \varphi = 0, \quad (x,\ y) \in \mathbf{R}^2 \backslash L,$$

$$N_y = 0, \quad M_y = -m(x), \quad [u_y] > h|[\theta_y]|, \quad x \in L_1,$$

$$[u_y] = h|[\theta_y]| > 0, \quad M_y = hN_y sgn[\theta_y] - m(x), \quad N_y \leq 0, \quad x \in L_2,$$

$$[u_y] = 0, \quad [\theta_y] = 0, \quad hN_y \pm (M_y + m(x)) \leq 0, \quad x \in L_3,$$

$$N_{xy} = 0, \quad Q_y^* = 0, \quad x \in L = L_1 \cup L_2 \cup L_3,$$

$$N_x = N_{xy} = N_y = 0, \quad M_x = M_{xy} = M_y = 0, \quad (x, y) \to \infty. \qquad (1)$$

Questions of existence, uniqueness and smoothness of solutions of such problems in Sobolev spaces have been investigated by A. Khludnev by means of the theory of variational inequalities [2].

In this report, the results of asymptotic and numerical solution of the boundary problem (1) by method of singular integral equations are presented. The asymptotic algorithms of small parameter for the shells of small curvature are built. The numerical results are found with the quadrature method.

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ABSOLUTE EXTENSORS IN ASYMPTOTIC CATEGORY

In this report, the question of an absolute extension in the asymptotic category is considered. The condition is provided, when the metric space is the absolute extensor in this category.

Definition 1 [1] *An object* Y *of a category* $\mathcal{A}$ *is its* **absolute extensor,** $Y \in AE(\mathcal{A})$*, if for any object* X *in* $\mathcal{A}$*, its subobject* (A, i) *and morphism* $f : A \to Y$ *there exists a proper asymptotically Lipschitz map* $\overline{f} : X \to Y$ *such that* $\overline{f} \circ i = f$*.*

The following theorem help us to prove the main result of the paper.

Theorem 1 [1]. $\mathbb{R}^n{}_+ \in AE(\mathcal{A})$ *and* $\mathbb{R}^n_+ \in AE(\widetilde{\mathcal{A}})$ *for all* n*.*

The condition of the absolute extension in an asymptotic category is following.

Theorem 2. *Let us have the metric space* $(\mathbf{T}^{n+1}, \widehat{d})$,

$$\mathbf{T}^{n+1} = \left\{ (x_1, \ldots, x, y) |\ y \geq \sum_{i=1}^{n} t(x_i) \right\},$$

where function t *is continuous, positive definite, monotonic, and even.*

It is the absolute extensor in the category $\mathcal{A}$ *if for all* $x, x' \in \mathbb{R}^{n+1}_+$ *the following condition holds:*
there is a constant s *such that*

$$\max_{i=\overline{1,n}} \left\{ |\mathrm{sgn}(x_i) t^{-1}(|x_i|) - \mathrm{sgn}(x'_i) t^{-1}(|x'_i|), \max_{i=\overline{1,n}} \{|x_i - x'_i|\} \right\} \leq$$
$$\leq \max_{i=\overline{1,n}} \{|x_i - x'_i|, s\}. \qquad (1)$$

Some examples of the absolute extensors of the asymptotic categories are presented.

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ON THE SEQUENCES OF ZEROS AND CRITICAL POINTS OF ENTIRE SOLUTIONS OF THE EQUATION $f'' + Af = 0$

We consider the question, when the equation

$$f'' + Af = 0 \tag{1}$$

has entire solution with prescribed sequences of zeros and critical points.

Theorem. *For any divisors* $\{\lambda_n; p_n\}$, $\{\mu_n; q_n\}$, $p_n > 1$, $n \in \mathbb{N}$, *where* λ_n *and* μ_n *are an arbitrary sequence of complex numbers,* $\lambda_n \neq \mu_k$, $n, k \in \mathbb{N}$, *with no finite accumulation points there exists a meromorphic function* A *with poles of second order at the points* λ_n *such that the equation (1) has at least one meromorphic solution* f *with zeros at the points* λ_n *of order* p_n, *for which* f' *except zeros at the points* λ_n *has zeros at the points* μ_n *of order* q_n.

The case, when $p_n = q_n = 1$ is considered in paper [1].

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ON THE CANONICAL FORM FOR A CERTAIN CLASS OF MATRICES OF THE SECOND ORDER WITH RESPECT TO THE SEMISCALAR EQUIVALENCE

The notion of semiscalar equivalence of polynomial matrices is introduced and considered first in [1]. In this report the semiscalar equivalence for one class of polynomial matrices of second order is investigated. Without loss of generality, we can assume that first invariant multiplier of considered matrix is identity and this matrix can be considered in the form

$$A(x) = \left\| \begin{array}{cc} 1 & 0 \\ a(x) & \Delta(x) \end{array} \right\|, \quad \deg a(x) < \deg \Delta(x). \qquad (1)$$

Proposition. *Let there be given a matrix $A(x)$ (1) and a partition*

$$M = M_1 \cup \ \ldots \ \cup M_w, \quad M_u \cap M_v = \emptyset, \quad u \neq v, \qquad (2)$$

of the set M of characteristic roots of matrix $A(x)$ into subsets M_u such that α, $\beta \in M_u$ if $a(\alpha) = a(\beta)$. Subsets M_u are uniquely defined by a class of semiscalarly equivalent matrices $\{CA(x)Q(x)\}$.

Theorem. *Let in the partition (2) for matrix $A(x)$ (1) we have $w = 1$; n_i and m_i be the multiplicities of some root $\alpha_i \in M$ in the characteristic polynomial $\Delta(x)$ and in polynomial $a(x)$ of matrix, $A(x)$ respectively, moreover $2m_i < n_i$. Then in the class of semiscalarly equivalent matrices $\{CA(x)Q(x)\}$ there exists a matrix $B(x)$ of the form*

$$B(x) = \left\| \begin{array}{cc} 1 & 0 \\ b(x) & \Delta(x) \end{array} \right\|, \deg b(x) < \deg \Delta(x),$$

where entry $b(x)$ satisfies the following conditions: $b(\alpha_i) = 0$, $b^{(m_i)}(\alpha_i) = m_i!$, $b^{(2m_i)}(\alpha_i) = 0$. For a fixed root α_i the matrix $B(x)$ is defined uniquely.

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ON EXISTENCE OF A SOLUTION OF THE MULTIPLE INTERPOLATION PROBLEM IN SOME CLASS OF THE ENTIRE FUNCTIONS

Let g be entire function, $M_g(r) = \max\{|g(z)| : |z| = r\}$ and let $|q| > 1$. A. Hel'fond and Yu. Kaz'min (see [1]) investigated the existence of the unique solution of the interpolation problem

$$g(q^{n-1}) = b_n, \tag{1}$$

where (b_n) is some sequence of complex numbers. It follows from the Kaz'min results that, for every sequence (b_k), such that

$$\varlimsup_{n\to\infty} |q|^{-\frac{n-1}{2}} |b_n|^{1/n} \leqslant r_1, \quad r_1 \in (1/|q|; 1)$$

interpolation problem (1) has a unique solution in the class of entire functions g, that satisfies the condition

$$\ln M_g(r) \leqslant \frac{\ln^2 \rho_1 r}{2\ln|q|} + \frac{\ln r}{2} + c_1 \quad \text{for each} \quad \rho_1 > r_1.$$

We consider the following interpolation problem:

$$g(q^{n-1}) = b_{n,1}, \quad g'(q^{n-1}) = b_{n,2}. \tag{2}$$

Theorem 1. *Let* $|q| > 1$; $R \in (1/|q|; 1]$ *be a fixed number,* $R < R_1 < 1$, $c_1 > 0$. *Then for every sequences* $(b_{n,1})$ *i* $(b_{n,2})$ *such that*

$$\varlimsup_{n\to\infty} |q|^{-(n-1)} |b_{n,1}|^{2/n} \leqslant R^2, \quad \varlimsup_{n\to\infty} |q|^{-(n-1)(n-2)/n} |b_{n,2}|^{2/n} \leqslant R^2$$

the interpolation problem (2) has a unique solution in the class of entire functions g*, for which*

$$\ln M_g(r) \leq \frac{\ln^2 R_1 r}{\ln|q|} + \ln r + c_1.$$

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APPROXIMATIVE CHARACTERISTICS OF THE CLASSES $L^{\psi}_{\beta,p}$ OF PERIODIC FUNCTIONS IN THE SPACE L_1

The paper is devoted to the study of the approximation of periodic functions of one variable of the classes $L^{\psi}_{\beta,p},\ 1<p<\infty$ in the space L_1 [1].

Let B be the set of functions ψ satisfying the following conditions: 1) ψ are positive and nonincreasing; 2) exists a constant $C>0$ such that $\frac{\psi(t)}{\psi(2t)}\leq C,\ t\in\mathbb{N}$.

Let L_1 be the space of 2π-periodic functions f with the usual norm. We denote by

$$e_m(L^{\psi}_{\beta,p})_1=\sup_{f\in L^{\psi}_{\beta,p}}\inf_{\Theta_m}\inf_{T(\Theta_m,\cdot)}\|f(\cdot)-T(\Theta_m,\cdot)\|_1,$$

$$e^{\perp}_m(L^{\psi}_{\beta,p})_1=\sup_{f\in L^{\psi}_{\beta,p}}\inf_{\Theta_m}\|f(\cdot)-S_{\Theta_m}(f,\cdot)\|_1$$

the best m-term and orthogonal trigonometric approximations of the classes $L^{\psi}_{\beta,p}$, where $T(\Theta_m,x)=\sum\limits_{k=1}^{m}c_ke^{in_kx}$, $S_{\Theta_m}(f,x)=\sum\limits_{k=1}^{m}\hat{f}(n_k)e^{in_kx}$, $\Theta_m\subset\mathbb{N}$ is a finite set containing m elements, c_k are complex numbers and $\hat{f}(k)=\frac{1}{2\pi}\int\limits_{-\pi}^{\pi}f(t)e^{-ikt}dt$ the Fourier coefficients of f.

We prove the following

Theorem. *Let $1<p<\infty$, $\psi\in B$, $\beta\in\mathbb{R}$. Then the following estimate is true:*

$$e_m(L^{\psi}_{\beta,p})_1\asymp e^{\perp}_m(L^{\psi}_{\beta,p})_1\asymp\psi(m).$$

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THE NUMERICALLY-ANALYTICAL ANALYSIS OF THE DISPERSIVE RELATIONS FOR THE GUIDED ELASTIC WAVES IN AN ANISOTROPIC LAYER WITH THE GENERALISED MIXED BOUNDARY CONDITIONS

At present, the spectrum of the normal elastic waves in the anisotropic layer of the orthorombical class is deeply investigated for various types of homogeneous boundary conditions of the first and second kinds with reference to cases of symmetry or antisymmetry of a wave field on a thickness of a layer. There are works devoted to analysis of wave motions during propagation of elastic waves in an anisotropic layer, which lies on the fixed basis and has a free top side, as well as with other types of non-classical boundary conditions. The structure of the full dispersive spectra in an anisotropic layer of the orthorombical systems with the generalized mixed boundary conditions remains however an actual problem of the wave mechanics of deformable environments.

The purpose of the present research consists in developing and realization of the complex numerically-analytical technique for qualitative and quantitative analysis of the full dispersive spectra, as well as of the kinematic and energetic properties of normal waves with arbitrary direction of propagation in the plane of the anisotropic elastic layer of the orthorombical system, with the same generalized mixed boundary conditions (with the variable factor of proportionality between stress and displacement) on the opposite flat sides of the leyer.

The analytical form of the transcendental dispersive equations describing the spectra for cases of propagation of the three-partial normal waves with different symmetry is derived. The analytical form of equations for special cases of the wave propagation along the elastic-equivalent directions of the layer is also presented. Qualitative analysis of roots of the dispersive equations is conducted in the parameter space of the frequency and wave number, on the basis of analysis of a characteristic polynomial for the system of equations which describes the stationary dynamic deformation of an anisotropic material of a waveguide.

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ON SOLUTIONS OF HIGHER-ORDER ANISOTROPIC ELLIPTIC-PARABOLIC EQUATIONS WITH VARIABLE EXPONENTS OF NONLINEARITY

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with the piecewise smooth boundary $\partial\Omega$, $\nu = (\nu_1, \ldots, \nu_n)$ be a unit outward pointing normal vector on the $\partial\Omega$. Put $Q := \Omega \times \mathbb{R}$, $\Sigma := \partial\Omega \times \mathbb{R}$.

A partial case of considered problem is to find a function $u : \overline{Q} \longrightarrow \mathbb{R}$, which satisfies (in some sense) the following equation and boundary conditions

$$(b(x)u)_t + \sum_{|\alpha|\in M} (-1)^{|\alpha|} D^\alpha \Big(a_\alpha(x,t) |D^\alpha u|^{p_\alpha(x)-2} D^\alpha u \Big) = \sum_{|\alpha|\in M} D^\alpha f(x,t),$$

$$\frac{\partial^j u}{\partial \nu^j}\Big|_\Sigma = 0\,, \quad j = \overline{0, m-1},$$

$(x,t) \in Q$, where $m \in \mathbb{N}$, M is a subset of set $\{0, 1, \ldots, m\}$ such that $\{0, m\} \subset M$, $a_\alpha \in L_\infty(Q)\,(|\alpha| \in M)$ are positive, $b \in L_\infty(\Omega)$ is nonnegative, while there exists open set $\Omega_0 \subset \Omega$ such that $b(x) > 0$ for a.e. $x \in \Omega_0$, and $b(x) = 0$ for a.e. $x \in \Omega \setminus \Omega_0$, $p_\alpha \in L_\infty(\Omega)$, $\operatorname{ess\,inf}_{x\in\Omega} p_\alpha(x) > 1\,(|\alpha| \in M)$, and $f_\alpha\,(|\alpha| \in M)$ is an integrable function. It is clear, that the equation is parabolic in $\Omega_0 \times \mathbb{R}$ and elliptic in $(\Omega \setminus \Omega_0) \times \mathbb{R}$. We study weak solutions of considered problem from generalized Sobolev spaces (see, for example, [1]).

The question of well-posedness of such type problem for the higher-order anisotropic elliptic-parabolic equations with variable exponents of nonlinearity is studied. The existence of periodic and almost periodic solutions is investigated.

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GENERATORS OF LINEARLY GROWING C_0-GROUPS WITH SIMPLE PURELY IMAGINARY EIGENVALUES

Given a Hilbert space H with a Riesz basis $\{e_n\}_{n=1}^{\infty}$. We introduce the space $H_k(\{e_n\})$, $k \in \mathbb{N}$, to present the construction of the generator A_k of a C_0-group possessing the following properties. A_k has simple purely imaginary eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$, which essentially cluster at infinity, i.e. $\lim\limits_{n\to\infty} |\lambda_n - \lambda_{n+1}| = 0$. Moreover, the corresponding family of eigenvectors is dense but do not constitute a Schauder basis, see [3]. This construction is very close to the recent remarkable results of G. Q. Xu, S. P. Yung [1] and H. Zwart [2] in the spectral theory of C_0-semigroups. Our approach is based on the application of the discrete Hardy inequality for $p = 2$.

The construction is given by the following theorem.

Theorem. *Let $f : [1, +\infty) \mapsto \mathbb{R}$ be a real function such that*

$$\lim_{x\to\infty} f(x) = +\infty; \qquad \lim_{n\to\infty} n|f(n-1) - f(n)| < \infty.$$

Then the operator $A_k : H_k(\{e_n\}) \supset D(A_k) \mapsto H_k(\{e_n\})$ defined by the formula $A_k x = A_k(\mathfrak{f}) \sum\limits_{n=1}^{\infty} c_n e_n = (\mathfrak{f}) \sum\limits_{n=1}^{\infty} i f(n) \cdot c_n e_n$, with domain

$$D(A_k) = \left\{ x = (\mathfrak{f}) \sum_{n=1}^{\infty} c_n e_n \in H_k(\{e_n\}) : \ \{f(n) \cdot c_n\}_{n=1}^{\infty} \in \ell_2(\Delta^k) \right\},$$

generates a C_0-group $\{e^{A_k t}\}_{t\in\mathbb{R}}$ of linear growth at $\pm\infty$ on $H_k(\{e_n\})$.

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HARNACK INEQUALITY AND CONTINUITY OF SOLUTIONS FOR THE DOUBLE PHASE EQUATIONS WITH LOWER ORDER TERMS

We prove the Harnack inequality and continuity of solutions for a class of the divergence-type elliptic equations with a nonstandard growth and lower order terms from the corresponding Kato-type-classes.

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ON ONE METHOD OF IMPROVEMENT OF THE FOURIER SERIES CONVERGENCE

When the problems related to investigation of the localized actions are considered, the quick-changing solutions are obtained. The respective Fourier series converge slowly. The Fourier series method is used to solve complex problems together with other approaches (in particular, the boundary elements or the finite elements methods), that demands large amount of calculations. The report presents the method of improvement the Fourier series convergence for the functions which can be approximated with adequate high accuracy by the least-squares method applied to the piecewise-continuous polynomials of the first degree on the whole interval of the series assignment or on the whole interval except of the regions of small sizes and the known location of the latter.

Let's consider the function $f(x)$ continuous on the interval $-l \leqslant x \leqslant l$ $f(x)$, which is given by the Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{i\lambda_n x}, \quad \lambda_n = \frac{\pi n}{l}$$

and approximate it by linear continuous functions. We divide the interval by the nodes $x_j = jh + a$, $j = 0, ..., N$, $h = (b-a)/N$, and obtain

$$aA_{j-1} + bA_j + aA_{j+1} = \beta_j, \;\; j = 0, ..., N,$$

where $\beta_j = h \sum\limits_{n=-\infty}^{\infty} a_n \gamma(0.5\lambda_n h) e^{i\lambda_n x_j}$, $\gamma(z) = \frac{1}{2}\sin^2 z$. This results in the function

$$\widetilde{f}(x) = \sum_{n=-\infty}^{\infty} a_n \Gamma_n \exp(i\lambda_n x), \; \Gamma_n = g(\lambda_n h), \; g(t) = \frac{3}{2+\cos t}\left(\frac{\sin(t/2)}{t/2}\right)^2.$$

The efficiency of obtained formulas is illustrated on example of the numerical inversion of the Laplace transform, based on the Fourier series.

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ON THE GROWTH OF THE ENTIRE DIRICHLET SERIES OF SEVERAL VARIABLES

Let $\Lambda^p = (\lambda_n)_{n\in\mathbb{Z}^p_+}$ be a sequence such that $\lambda_n = (\lambda^{(1)}_{n_1}, \dots \lambda^{(p)}_{n_p})$ $(n = (n_1, \dots, n_p) \in \mathbb{Z}^p_+)$, $0 \le \lambda^{(j)}_k \uparrow +\infty$ $(0 \le k \uparrow +\infty)$, $1 \le j \le p$. For $a \in \mathbb{R}^p$ we put $\|a\| := a_1 + \dots + a_p$. Let $H^p(\Lambda^p)$ be a class of entire functions defined by the absolutely convergent multiple Dirichlet series in the whole complex space $\mathbb{C}^p$ of the form $F(z) = \sum_{||n||=0}^{+\infty} a_n e^{<z,\lambda_n>}$, $z \in \mathbb{C}^p$, $p \ge 2$, where $< z, \lambda_n >= z_1\lambda^{(1)}_{n_1} + \dots + z_p\lambda^{(p)}_{n_p}$. For $F \in H^p(\Lambda^p)$ and $x = (x_1, \dots, x_p) \in \mathbb{R}^p$ we denote $M(x, F) = \sup\{|F(x + iy)| : y \in \mathbb{R}^p\}$, $\mu(x, F) = \max\{|a_n|e^{<x,\lambda_n>} : n \in \mathbb{Z}^p_+\}$, and
$\rho_F = \varlimsup\limits_{|x|\to+\infty} \frac{\ln\ln M(x,F)}{\ln\|e^x\|}$, $\rho_\mu = \varlimsup\limits_{|x|\to+\infty} \frac{\ln\ln \mu(x,F)}{\ln\|e^x\|}$, $k_F = \varlimsup\limits_{\|n\|\to+\infty} \frac{\|\lambda_n \ln\lambda_n\|}{-\ln|a_n|}$,
where $\lambda_n \ln\lambda_n = (\lambda^{(1)}_{n_1} \ln\lambda^{(1)}_{n_1}, \dots, \lambda^{(p)}_{n_p} \ln\lambda^{(p)}_{n_p})$.

1. It is easy to see that $\rho_\mu = k_F$ for every function $F \in H^p(\Lambda^p)$.

2. (B.V.Vynnyts'kyi, 1975, p=2) $k_F \le \rho_F \le k_F/(1-h)$ for every function $F \in H^p(\Lambda^p)$ such that $\varlimsup\limits_{\|n\|\to+\infty} \frac{\ln\|n\|}{-\ln|a_n|} \le h < 1$.

3. $F \in H^p(\Lambda^p)$ and $h = 0 \Longrightarrow \rho_F = \rho_\mu = k_F$.
The proposition **3.** follows immediately from **1.** and **2.**

4. $F \in H^p(\Lambda^p)$ and $\delta(\Lambda) := \varlimsup\limits_{\|n\|\to+\infty} \frac{\|\lambda_n \ln\lambda_n\|}{-\ln|a_n|} = 0 \Longrightarrow \rho_F = \rho_\mu = k_F$.
It is easy to see that the proposition **4.** follows from **2.** and **3.**

5. For every sequence Λ^p such that $\delta(\Lambda) > 0$ there exists a Dirichlet series $F \in H^p(\Lambda^p)$ such that $\rho_F > \rho_\mu$ $(= k_F)$.

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ON THE R-ORDER AND LOWER R-ORDER OF DIRICHLET SERIES ABSOLUTELY CONVERGENT IN THE HALFPLANE

Let

$$F(s) = \sum_{n=1}^{\infty} a_n \exp\{s\lambda_n\}, \quad s = \sigma + it, \tag{1}$$

be the Dirichlet series with arbitrary abscissa of absolute convergence $\sigma_a = A \in (-\infty, +\infty]$, where $0 = \lambda_0 < \lambda_n \uparrow +\infty$ $(n \to \infty)$. For $\sigma < A$, let $\mu(\sigma, F) = \max\{|a_n| \exp\{\sigma\lambda_n\} : n \geq 0\}$ be the maximal term of (1) and $M(\sigma, F) = \sup\{|F(\sigma + it)| : t \in \mathbb{R}\}$. If $A = +\infty$ then the R-order ϱ_R and lower R-order λ_R are introduced [1] by formulas $\varrho_R = \varlimsup\limits_{\sigma\to\infty} \dfrac{\ln\ln M(\sigma, F)}{\sigma}$ and $\lambda_R = \varliminf\limits_{\sigma\to\infty} \dfrac{\ln\ln M(\sigma, F)}{\sigma}$. Then, under the condition $\ln n = o\big(\lambda_n \ln \lambda_n\big)$, $n \to \infty$, the following inequality $\lambda_R \leq \varrho_R \beta$, where $\beta = \varliminf\limits_{n\to\infty} \dfrac{\ln \lambda_n}{\ln \lambda_{n+1}}$, is true. The order and lower order of the Dirichlet series (1) with null abscissa of absolute convergence are respectively $\varrho^0 = \varlimsup\limits_{\sigma\uparrow 0} \dfrac{\ln\ln M(\sigma, F)}{-\ln|\sigma|}$ and $\lambda^0 = \varliminf\limits_{\sigma\uparrow 0} \dfrac{\ln\ln M(\sigma, F)}{-\ln|\sigma|}$, and if $\ln\ln n = o(\ln \lambda_n)$, $n \to \infty$, then $\lambda^0 \leq \varrho^0 \beta$. For the characterization of the growth of the Dirichlet series (1) with null abscissa of absolute convergence the R-order $\varrho_R^0 = \varlimsup\limits_{\sigma\uparrow 0} |\sigma| \ln\ln M(\sigma, F)$ and the formula to find it are introduced [2]. The lower R-order for its Dirichlet series will be denoted by the value $\lambda_R^0 = \varliminf\limits_{\sigma\uparrow 0} |\sigma| \ln\ln M(\sigma, F)$. The following theorem is true.

Theorem. *If* $\ln n = o\left(\dfrac{\lambda_n}{\ln \lambda_n}\right)$ *as* $n \to \infty$ *then* $\lambda_R^0 \leq \varrho_R^0 \beta$.

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COMBINING EXPLICIT AND IMPLICIT EXPONENTIAL INTEGRATORS FOR SOLVING THE SEMILINEAR STIFF EVOLUTION EQUATIONS

Exponential integrators are a class of powerful methods specifically designed for the stiff semilinear evolution equations

$$\partial_t u(t) = Au(t) + g(t, u(t)), \quad u(0) \text{ given},$$

arising from a spatial discretization of the time-dependent partial differential equations, where the problem splits into a linear (stiff) and a nonlinear (nonstiff or mildly stiff) parts. The linear term is exactly treated by computation of the action of the matrix exponential, and a time-stepping technique is applied to the nonlinear term.

We discuss multistep versions of the exponential integrators, in particular, the combination of explicit and implicit schemes which are generalizations of the classical Adams schemes. The basic integrator is explicit, and a more accurate implicit scheme is used to control the integration. This is accomplished by a technique of predictor-corrector type, which can be interpreted as a local discrete defect-correction procedure.

We also consider realizations of a posteriori error estimators based on the principle of the defect correction. The idea is to combine the defect of a given numerical approximation with a 'reconstruction' (backsolving) procedure based on a simple auxiliary scheme, for example exponential Euler scheme, with the goal to produce an asymptotically correct global error estimate in an efficient and stable way.

Numerical examples are given, and rational versions of these schemes are also considered.

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SOME NECESSARY CONVERGENCE CONDITIONS FOR ONE CLASS OF THE TWO-DIMENSIONAL CONTINUED FRACTIONS WITH COMPLEX ELEMENTS

We deal with analysis of convergence for a two-dimensinal continued fraction (TDCF) of the form:

$$b_0 + \mathop{\mathrm{D}}_{j=1}^{\infty} \frac{1}{b_{j,0}} + \mathop{\mathrm{D}}_{j=1}^{\infty} \frac{1}{b_{0,j}} + \mathop{\mathrm{D}}_{k=1}^{\infty} \frac{1}{b_{k,k} + \mathop{\mathrm{D}}\limits_{j=1}^{\infty} \frac{1}{b_{j+k,k}} + \mathop{\mathrm{D}}\limits_{j=1}^{\infty} \frac{1}{b_{k,j+k}}}, \qquad (1)$$

where all partial denominators $b_{k,j}$, $k,\ j = 0,\ 1,\ \ldots,\ k+j \neq 0,$ are complex numbers,

$$b_{k,j} \in G_\beta = \left\{ z \in \mathbb{C}:\ z \neq 0,\ |\arg z| \leq \beta < \frac{\pi}{2} \right\}.$$

The ordinary approximants of TDCF (1) are defined in that way

$$f_n = b_0 + \mathop{\mathrm{D}}_{j=1}^{n} \frac{1}{b_{j,0}} + \mathop{\mathrm{D}}_{j=1}^{n} \frac{1}{b_{0,j}} + \mathop{\mathrm{D}}_{k=1}^{n} \frac{1}{b_{k,k} + \mathop{\mathrm{D}}\limits_{j=1}^{n-k} \frac{1}{b_{j+k,k}} + \mathop{\mathrm{D}}\limits_{j=1}^{n-k} \frac{1}{b_{k,j+k}}},$$

the figured approximants of TDCF (1) are defined as follows

$$\hat{f}_n = b_0 + \mathop{\mathrm{D}}_{j=1}^{[\sqrt{n}]} \frac{1}{b_{j,0}} + \mathop{\mathrm{D}}_{j=1}^{[\sqrt{n-1}]} \frac{1}{b_{0,j}} +$$

$$\mathop{\mathrm{D}}_{k=1}^{[\sqrt{n+1}]-1} \frac{1}{b_{k,k} + \mathop{\mathrm{D}}\limits_{j=1}^{[\sqrt{n-2k}]-k} \frac{1}{b_{j+k,k}} + \mathop{\mathrm{D}}\limits_{j=1}^{[\sqrt{n-2k-1}]-k} \frac{1}{b_{k,j+k}}},\ n = 1,\ 2,\ \ldots .$$

In our communication, we plan to consider some necessary conditions for convergence of sequences $\{f_n\}$ and $\{\hat{f}_n\}$.

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ON SMOOTHNESS OF INVARIANT TORI FOR EVOLUTIONARY EQUATIONS IN INFINITE-DIMENSIONAL SPACES

Let us start by considering a system of nonlinear equations

$$\varphi_{n+1} = \varphi_n + a(\varphi_n, x_n), \quad x_{n+1} = P(\varphi_n, x_n)x_n + c(\varphi_n), \quad n \in Z, \quad (1)$$

where Z is the set of integers, $\varphi \in \mathbf{W}$, $x = (x^1, x^2, x^3, \dots) \in \mathfrak{M}$, $\mathbf{W}$ is an arbitrary Banach space, $\mathfrak{M}$ is the space of bounded number sequences, the function $c(\varphi) = (c_1(\varphi), c_2(\varphi), c_3(\varphi), \dots)$ is defined on $\mathbf{W}$ with values in the space $\mathfrak{M}$, the function $a(\varphi, x)$ and the infinite matrix $P(\varphi, x) = [p_{i,j}(\varphi, x)]_{i,j=1}^{\infty}$ are defined on the set $\mathbf{W} \times \{x \in \mathfrak{M} | \|x\| \le d = const > 0\}$.

Next, let us write down a linear in x system of equations

$$\frac{d\varphi}{dt} = a(\varphi), \quad \frac{dx(t)}{dt} = B(\varphi, t)x(t + \Delta) + c(\varphi, t). \quad (2)$$

Here $\varphi = (\varphi_1, \varphi_2, \dots, \varphi_n) \in R^n$, $x \subset \mathfrak{M}$; $B(\varphi, t) = [b_{ij}(\varphi, t)]_{i,j=1}^{\infty}$ is an infinite matrix; $a(\varphi)$ and $c(\varphi, t)$ are vector functions of appropriate dimensions; the elements $b_{ij}(y_1(\varphi, t), y_2(\varphi, t), \dots, y_n(\varphi, t))$ of the matrix $B(\varphi, t)$ and the coordinates $c_i(\varphi, t) = c_i(z_1(\varphi, t), z_2(\varphi, t), \dots, z_n(\varphi, t))$ of the vector function $c(\varphi, t)$ are determined by the equalities $y_s(\varphi, t) = \varphi_{s_{t+\Gamma_s}}(\varphi)$ and $z_s(\varphi, t) = \varphi_{s_{t+\delta_s}}(\varphi)$ respectively; $x(t + \Delta) = (x_1(t + \Delta_1), x_2(t + \Delta_2), \dots)$; Γ_i, δ_i and Δ_i are arbitrary fixed real numbers, $i = 1, 2, \dots$; $s = 1, 2, \dots, n$; $\varphi = \varphi_t(\varphi)$, $\varphi_0(\varphi) = \varphi$, is a solution of the first equation of the system (2).

We establish sufficient conditions for existence of a smooth semi-invariant manifold for the system of equations (1) and a smooth invariant torus for the system of equations (2), where the smoothness of the latter one is understood in the sense of coordinate-wise differentiability. If the space $\mathbf{W}$ is replaced with $\mathfrak{M}$, then, under additional standard conditions of periodicity, the semi-invariant manifold for the system (1) becomes its semi-invariant torus.

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MEAN VALUE THEOREMS FOR SOME CLASSES OF POLYNOMIALS

Polyanalytic functions are very important subject since they have many different applications. For example, theory of several complex variables, theory of polygarmonic functions, analytic functions, investigation of the different equations with Cauchy-Rieman's operator and others.

The classic Gauss theorem that characterizes the class of harmonic functions using the mean-value formula was developed and modified in many works. For example, works of M.O. Reade, Ramsey and Weit, Volchkov V.V.

The main direction of present investigations is the description of classes of functions that satisfy given integral equations that have a certain geometric sense. For example, the solutions of equation with mean value can be determined by the next equation

$$\left(\frac{\partial}{\partial z}\right)^{m-s}\left(\frac{\partial}{\partial \bar{z}}\right)^{m} f=0,$$

where $m \in \mathbb{N}$ and $s \in 0, ..., m-1$ are fixed.

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ON PROPERTIES OF SOLUTIONS OF THE WEBER EQUATION

Let $f(z)$ be an entire function, l be a positive continuous on $[0, +\infty)$ function. Function f is said to be a function of bounded l-index [1], if there exists $N \in \mathbb{Z}_+$ such that for all $n \in \mathbb{Z}_+$ and $z \in \mathbb{C}$

$$\frac{|f^{(n)}(z)|}{n! l^n(|z|)} \le \max\left\{\frac{|f^{(k)}(z)|}{k! l^k(|z|)} : 0 \le k \le N\right\}.$$

The least such integer N is called l-index and is denoted by $N(f, l)$.

An analytic univalent in $\mathbb{D} = \{z : |z| < 1\}$ function f is said to be convex if $f(\mathbb{D})$ is a convex domain.

Differential equation $w'' - \left(\frac{z^2}{4} - \nu - \frac{1}{2}\right) w = 0$ is said to be the Weber equation. Properties of solutions of the Weber equation if $\nu \neq -\frac{1}{2}$ are investigated in [2]. If $\nu = -\frac{1}{2}$ then we have

$$w'' - \frac{z^2}{4} w = 0. \tag{1}$$

Theorem. *Common solution of the equation* (1) *can be written in the form* $w(z) = C_1\alpha(z^4) + C_2 z\beta(z^4)$, *where entire functions* $\alpha(z)$ *and* $\beta(z)$ *are convex in* $\mathbb{D}$, $N(\alpha, l) \le 1$ *with* $l(|z|) \equiv \dfrac{31 + 2\sqrt{238}}{336}$ *and* $N(\beta, l) \le 1$ *with* $l(|z|) \equiv \dfrac{41 + 2\sqrt{414}}{720}$, *also* $\ln M_\alpha(r) = (1 + o(1))\dfrac{\sqrt{r}}{4}$ *and* $\ln M_\beta(r) = (1 + o(1))\dfrac{\sqrt{r}}{4}$ *as* $r \to \infty$, *where* $M_f(r) = \max\{|f(z)| : |z| = r\}$.

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NONLINEAR OPTIMAL CONTROL IN PARABOLIC PROBLEM WITHOUT INITIAL CONDITIONS

Let $\Omega \subset \mathbb{R}^n$ $(n \in \mathbb{N})$ be a bounded domain with a regular boundary Γ, $S := (-\infty, 0]$, $Q := \Omega \times S$, $\Sigma := \Gamma \times S$. For arbitrary $\omega \in \mathbb{R}$, $\alpha, \gamma \in L^\infty_{\mathrm{loc}}(S)$ ($\alpha(t) > 0$ and $\gamma(t) > 0$ for almost all $t \in S$), and a Hilbert space X, we denote by $L^2_{\omega,\gamma}(S; X)$ the space of functions $f : S \to X$ such that $\int_S \gamma(t) e^{2\omega \int_0^t \alpha(s)ds} \|f(t)\|^2_X dt < \infty$.

Let $U := \{v \in L^\infty(Q) \big| v \geq 0 \quad \text{a. e.} \quad Q\}$, U_∂ is a convex and closed set in U. Given control $v \in U$, the state $y(v) = y(\cdot, \cdot; v) \in L^2_{\omega,\alpha}(S; H^1_0(\Omega))$ $\cap C(S; L^2(\Omega))$ of evolution system is defined as a weak solution of the problem

$$y_t(v) - \sum_{i,j=1}^{n} \big(a_{ij} y_{x_i}(v)\big)_{x_j} + a_0 y(v) + v y(v) = f \quad \text{in } Q,$$

$$\lim_{t \to -\infty} e^{2\omega \int_0^t \alpha(s)ds} \int\limits_\Omega |y(x,t;v)|^2 \, dx = 0.$$

Here $f \in L^2_{\omega,1/\alpha}(S; L^2(\Omega))$, $a_0, a_{ij} \in L^\infty_{\mathrm{loc}}(\overline{Q})$, $\quad a_{ij} = a_{ji}$ $(i, j = \overline{1,n})$, $a_0(x,t) \geq \alpha(t)$, $\sum\limits_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \geq \alpha(t)\|\xi\|^2 \quad (\xi \in \mathbb{R}^n)$ a. e. Q.

The cost function has the form

$$J(v) = \|y(\cdot, 0; v) - z_0(\cdot)\|^2_{L^2(\Omega)} + \mu \|v\|^2_{L^\infty(Q)}, \quad v \in U,$$

for given $z_0 \in L^2(\Omega)$ and $\mu = \text{const} > 0$.

Problem: To find $u \in U_\partial$ such that

$$J(u) = \inf_{v \in U_\partial} J(v) \, .$$

Under some conditions on the data-in, we prove the existence of a solution of this problem.

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ABOUT MIXED PROBLEMS FOR SOME PARABOLIC EQUATIONS WITH INCREASING COEFFICIENTS

Let $x := (x', x_n),\ x' := (x_1, ..., x_{n-1}); T > 0;\ \Pi_T^+ := \{(t, x', x_n)|t \in (0, T], x' \in \mathbb{R}^{n-1}, x_n > 0\},\ \Pi_0^+ := \{(t, x', x_n)|t = 0, x' \in \mathbb{R}^{n-1}, x_n > 0\}$, $\Pi_T^0 := \{(t, x', x_n)|t \in (0, T], x' \in \mathbb{R}^{n-1}, x_n = 0\}$;

$$(Lu)(t,x) := \left(\partial_t - \sum_{j,k=1}^{n} a_{jk}\partial_{x_j}\partial_{x_k} - \sum_{j=1}^{n} a_j\partial_{x_j} - a_0\right) u(t,x) -$$

$$-b\sum_{j=1}^{n} \partial_{x_j}\left(x_j u(t,x)\right),$$

where a_{jk}, a_j, a_0, b are real constants, and matrix $A := (a_{jk})_{j,k=1}^n$ is symmetric and positively determined; $\partial_{\vec{\nu}_A} u := -\sum_{j=1}^n a_{nj}\partial_{x_j} u$ is derivative of function u along conormal to hyperplane $\{(x', x_n)|x' \in \mathbb{R}^{n-1}, x_n = 0\}$.

Such problems are considered in the report:

$$\begin{cases} Lu = 0 \text{ in } \Pi_T^+, \\ u = 0 \text{ on } \Pi_T^0, \\ u = \phi \text{ on } \Pi_0^+; \end{cases} \tag{1}$$

$$\begin{cases} Lu = 0 \text{ in } \Pi_T^+, \\ \partial_{\vec{\nu}_A} u = 0 \text{ on } \Pi_T^0, \\ u = \phi \text{ on } \Pi_0^+. \end{cases} \tag{2}$$

Such results are obtained for problems (1) and (2):

1) homogeneous Green's functions $G_0^{(l)}, l \in \{1, 2\}$ are constructed and studied;

2) the properties of the Poisson's integrals, generated by functions $G_0^{(l)}, l \in \{1, 2\}$, are established;

3) the theorems about correct solvability of problems in spaces of functions,rapidly increasing as $|x| \to \infty$, are proved.

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PROBLEM WITH MULTIPOINT CONDITIONS FOR SYSTEM OF PARABOLIC EQUATIONS OF HIGH ORDER

Let $L = -\sum_{i,j=1}^{p} \partial_{x_i} \left(p_{ij}(x)\partial_{x_j}\right) + q(x)$, where $p_{ij}(x) = p_{ji}(x) > 0$, $q(x) \geq 0$, $x \in Q$, $\{\lambda_k,\ k \in \mathbb{N}\}$ and $\{X_k(x), k \in \mathbb{N}\}$ are the set of eigenvalues and the system of responsible eigenfunctions of the problem $LX = \lambda X,\ X|_{\partial Q} = 0$, $E_{\alpha,\beta}$, $\alpha, \beta \in \mathbb{R}$ is a space of the vector functions $\vec{\varphi} = \sum \vec{\varphi}_k X_k(x)$, $\vec{\varphi}_k = \mathrm{col}(\varphi_k^1, \ldots, \varphi_k^m) \in \mathbb{C}^m$, $k \in \mathbb{N}$, with finite norm $\|\vec{\varphi}; E_{\alpha,\beta}\|^2 = \sum_{k=1}^{\infty} \|\vec{\varphi}_k\|^2 \lambda_k^{2\alpha} \exp(2\beta\lambda_k^b)$, $\|\vec{\varphi}_k\|^2 = |\varphi_k^1|^2 + \ldots + |\varphi_k^m|^2$.

In the spase $C^n([0,T], E_{\alpha,\beta})$, we consider the problem

$$W\left(\partial_t, L\right)\vec{u} \equiv \partial_t^n \vec{u}(t,x) + \sum_{r=0}^{n-1} A_r(L)\partial_t^r \vec{u}(t,x) = \vec{0}, (t,x) \in (0,T) \times Q, \qquad (1)$$

$$\sum_{r=0}^{N_q} B_r^q(L)\partial_t^r \vec{u}(t,x)\big|_{t=t_q} = \vec{\varphi}_q(x),\ 0 \leq N_q \leq n-1,\ q = 1, \ldots, n, \qquad (2)$$

where $A_r(L) = \|a_{i,j}^r(L)\|_{i,j=1}^m$, $a_{i,j}^r(L) = \sum_{q=0}^{(n-r)b} a_{i,j}^{r,q} L^q$, $a_{i,j}^{r,q} \in \mathbb{C}$, $b \in \mathbb{N}$, $B_r^q(L) = \|b_{i,j}^{r,q}(L)\|_{i,j=1}^m$, $b_{i,j}^{r,q}(L) = \sum_{s=0}^{M} b_{i,j}^{r,q,s} L^s$, $b_{i,j}^{r,q,s} \in \mathbb{C}$, $M \in \mathbb{N}$, $0 \leq t_1 < t_2 < \ldots < t_n \leq T$. Let $\mu_1(k), \ldots, \mu_{mn}(k)$ be the roots of equation $\det\|W(\mu, \lambda_k)\| = 0$, the following inequalities $\mathrm{Re}\mu_l(k) \leq -\delta\lambda_k^b$, $\delta > 0$, are hold, $\vec{h}_l(k) = \mathrm{col}(h_l^1(k), \ldots, h_l^m(k))$ is the first column of matrix $W^*(\mu_l(k), \lambda_k)$ whitch is the adjugate matrix of the matrix $W(\mu_l(k), \lambda_k)$, $\Delta(k) = \det\left\|\sum_{r=0}^{N_q} \mu_l^r(k) \sum_{j=1}^{m} b_{i,j}^{r,q}(k) h_{l,k}^j \exp(\mu_l(k)t_q)\right\|_{q=1,\ldots,n,\, i=1,\ldots,m,}^{l=1,\ldots,mn}$.

Theorem. *Suppose that for all $k \in \mathbb{N}$ the inequalities $|\Delta(k)| \neq 0$ are satisfied, and suppose that constants γ, ν exist such that for all (except of a finite number) $k \in \mathbb{N}$, the following inequalities $|\Delta(k)| \geq \lambda_k^{-\gamma} \exp(-\nu\lambda_k^b)$ are hold. If $\vec{\varphi}_q \in E_{\alpha_0,\beta_0}$, $q = 1, \ldots, n$, where $\alpha_0 = \alpha + \gamma + bn(nm^2 - nm + 1) + bm(N_1 + \ldots + N_n) - b\min_{1 \leq q \leq n}\{N_q\} + M(mn-1)$, $\beta_0 = \beta + \nu - m\delta(t_1 + \ldots + t_n) + \delta t_n$, then a unique solution of the problem (1), (2) from the space $C^n([0,T], E_{\alpha,\beta})$ exists, which depends continuously on the functions $\vec{\varphi}_q$, $q = 1, \ldots, n$.*

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FRACTAL ANALYSIS OF SOLUTIONS OF REACTION-DIFFUSION SYSTEMS

In order to study the behavior of solutions of the dissipative system

$$\tau_\theta \frac{\partial \theta}{\partial t} = l^2 \frac{\partial^2 \theta}{\partial x^2} - Q(\theta, \eta), \quad \tau_\eta \frac{\partial \eta}{\partial t} = L^2 \frac{\partial^2 \eta}{\partial x^2} - q(\theta, \eta), \tag{1}$$

where $\tau_\theta, \tau_\eta, l^2, L^2$ are characteristic times and lengths of the system, and $Q(\theta, \eta), q(\theta, \eta)$ are nonlinear sources, it is important to investigate the chaotic behavior of solutions and their classification. For this purpose, the idea of fractal (Hausdorff) dimension is used

$$D = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln(1/\varepsilon)}, \tag{2}$$

where $N(\varepsilon)$ is the smallest number of hypercube with edge ε which is required to meet a set of points in p - dimensional space [1,2]. Therefore, the metric dimension, information dimension and correlation dimension are calculated.

Calculations are made for a system with the cubic nonlinearity:

$$\tau_\theta \frac{\partial \theta}{\partial t} = l^2 \frac{\partial^2 \theta}{\partial x^2} - \theta^2 - \eta + 1, \quad \tau_\eta \frac{\partial \eta}{\partial t} = L^2 \frac{\partial^2 \eta}{\partial x^2} - \eta(\eta - (\theta - A)^3). \tag{3}$$

The results of calculations are following. For regular oscillations, the fractal dimension of the projection solutions (3) is equal to unity with accuracy of 0.01. In the case of the quasi-periodic regime, the fractal dimension is close to unity. In the case of chaotic regime, the fractal dimension has the value 1,859, which corresponds to the fractal set and indicates the presence of chaos. Therefore, the value of the fractal-dimension projection clearly defines the nature of fluctuations in the system.

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THE MULTIDIMENSIONAL MOVING BOUNDARY PROBLEM GOVERNED BY ANOMALOUS DIFFUSION: ANALYTICAL AND NUMERICAL STUDY

We study the anomalous diffusion version of the quasistationary Stefan problem (the fractional Hele-Shaw problem) in the multidimensional case $\Omega(t) \subset R^n$, $n \geq 2$. This free boundary problem is a mathematical model of a solute drug release from a polymer matrix ($n = \overline{1,3}$). We prove the existence and uniqueness of the classical solution to the moving boundary problem locally in time. In the two-dimensional case, we construct numerical solutions.

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ON THE BOUNDEDNESS OF SOLUTIONS OF NONLINEAR HIGH-ORDER ELLIPTIC EQUATIONS WITH DATA IN THE ORLICZ-ZYGMUND SPACE

Let $n, m \in \mathbb{N}$, $n \geqslant 3$, $m \geqslant 2$, and let Ω be a bounded open set of $\mathbb{R}^n$ and $f \in L^1(\Omega)$. We consider the equation

$$\sum_{|\alpha|=m} (-1)^m D^\alpha A_\alpha(x, D^m u) = f \quad \text{in } \Omega, \tag{1}$$

where $D^m u = \{D^\alpha u : |\alpha| = m\}$. The coefficients $A_\alpha(x,\xi)$ satisfy the Carathéodory conditions and the following inequalities:

$$\sum_{|\alpha|=m} |A_\alpha(x,\xi)|^{p/(p-1)} \leqslant c_1 \sum_{|\alpha|=m} |\xi_\alpha|^p + g_1(x),$$

$$\sum_{|\alpha|=m} A_\alpha(x,\xi)\xi_\alpha \geqslant c_2 \sum_{|\alpha|=m} |\xi_\alpha|^p - g_2(x),$$

where c_1, $c_2 > 0$, $p > 1$, $g_{1,2} \in L^1(\Omega)$ and $g_{1,2} \geqslant 0$ in Ω.

Definition. A generalized solution of equation (1) is a function $u \in W^{m,p}(\Omega)$ such that $\forall\, v \in C_0^\infty(\Omega)$, $\sum\limits_{|\alpha|=m} \int_\Omega A_\alpha(x, D^m u) D^\alpha v dx = \int_\Omega f v dx$.

By $L(\log L)^{n-1}(\Omega)$ we denote the Orlicz-Zygmund space generated by the function $\varphi(t) = |t|[\ln(1+|t|)]^{n-1}$ and equipped by the Luxemburg norm $\|w\|_\varphi = \inf\big\{\lambda > 0 : \int_\Omega \varphi(w(x)/\lambda)dx \leqslant 1\big\}$, $w \in L(\log L)^{n-1}(\Omega)$.

Theorem. *Let $n = mp$, f, g_1, $g_2 \in L(\log L)^{n-1}(\Omega)$ and let M be a majorant for the norms $\|g_1\|_\varphi$, $\|g_2\|_\varphi$ and $\|f\|_\varphi$. Let $u \in W_0^{m,p}(\Omega)$ be a generalized solution of equation (1). Then $u \in L^\infty(\Omega)$ and $\|u\|_\infty \leqslant C$, where the positive constant C depends only on $n, m, c_1, c_2, meas\,\Omega$ and M.*

Remark. In the case $n = mp$ J. Frehse [1] established the L^∞-regularity of arbitrary generalized solution $u \in W_0^{m,p}(\Omega)$ of equation (1) under the stronger assumption that f, g_1, $g_2 \in L^r(\Omega)$, $r > 1$.

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THE LINEAR SECOND-ORDER EQUATION IN ONE SOLUTION OF THE DYNAMIC PROBLEM FOR A NONAUTONOMOUS GYROSTAT

The equations of motion about a fixed point for a gyrostat with gyrostatic momentum $\boldsymbol{\lambda} = \lambda(t)\boldsymbol{\alpha}$ ($\boldsymbol{\alpha}$ is a constant vector) have been studied. The solution characterized by invariant relations linear in angular velocity and linear in total angular momentum has been obtained. It coincides with the Hess solution in the case of $\boldsymbol{\lambda} = 0$ and preserves the main analytic properties of it's classical analogue in a general case. The computation of the time dependence for the state variables is reduced to integration of

$$U'' + \frac{P_9(\rho)U'}{\rho f(\rho) r_2(\rho)} + \frac{r_1(\rho) r_2(\rho) U}{\rho^2 f(\rho)} = 0, \tag{1}$$

where $U = U(\rho)$, ρ varies over a bounded interval, $f(\rho)$ is the even polynomial of the 6-th order, $r_1(\rho), r_2(\rho)$ are the cubic polynomials and

$$P_9(\rho) = [r_2(\rho) - \rho r_2'(\rho)] f(\rho) + \frac{\rho}{2} r_2(\rho) f'(\rho).$$

It is proved that equation (1) with admissible values of coefficients of $r_1(\rho), r_2(\rho), f(\rho)$ is Fuchsian and has no more then 11 singularities. The infinite point, unlike $\rho = 0$, is always singular.

The cases when (1) can be significantly simplified are singled out: this equation, for example, can have eight, five or even only four singular points. In the latter case (1) can be turned into the nondegenerate Heun equation with the singularities, forming the harmonic quadruple, and parameters, satisfying conditions of the Theorem C.2 [1]. Thus, there exists a transformation between the Heun and the Gauss equations, therefore the solution of (1) in the neighborhood of $\rho = 0$ is expressible in terms of the hypergeometric functions.

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THREE-POINT PROBLEM FOR A HOMOGENEOUS PARTIAL DIFFERENTIAL EQUATION WITH ONE SPATIAL VARIABLE

The work is devoted to investigation of the three-point problem for the linear partial differential equation

$$\frac{\partial^3 u}{\partial t^3} + \left(a_{21}\frac{\partial}{\partial x} + a_{20}\right)\frac{\partial^2 u}{\partial t^2} + \left(a_{12}\frac{\partial^2}{\partial x^2} + a_{11}\frac{\partial}{\partial x} + a_{10}\right)\frac{\partial u}{\partial t} + \\ + \left(a_{03}\frac{\partial^3}{\partial x^3} + a_{02}\frac{\partial^2}{\partial x^2} + a_{01}\frac{\partial}{\partial x} + a_{00}\right)u = 0, \quad (1)$$

$$u(0,x) = \varphi_1(x), \quad u(\tau,x) = \varphi_2(x), \quad u(T,x) = \varphi_3(x), \qquad (2)$$

in cylinder $\mathcal{D} = [0;T] \times \Omega$, where $T > 0$, Ω is one-dimensional torus $\mathbb{R}/2\pi\mathbb{Z}$, where $\vec{a} = (a_{21}, a_{12}, a_{03}, a_{20}, a_{11}, a_{02}, a_{10}, a_{01}, a_{00})$ is a vector of complex coefficients a_{ij}, $i,j = 0,1,2,3$ of equation (1), $\varphi_1 = \varphi_1(x)$, $\varphi_2 = \varphi_2(x), \varphi_3 = \varphi_3(x)$ are given functions, $u = u(t,x)$ is unknown function.

The existence and uniqueness conditions of the solution of this problem in the Sobolev spaces (Abel spaces) are establish. For multiple spatial variables $x_1, x_2, \ldots, x_p$ similar problem is incorrect in the Hadamard sense and its solvability depends on the small denominators, arising in the construction of the solution [1]. For single spatial variable the corresponding denominators are estimated from below by some small constants. That's why in this case the problem (1), (2) is correct after Hadamard and is solved in Abel spaces.

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ON FUNDAMENTAL SOLUTION OF THE ULTRA-PARABOLIC KOLMOGOROV EQUATION WITH DEGENERATION ON THE INITIAL HYPERPLANE

Let n, n_1, n_2 be given natural numbers such that $n_1 \geq n_2 \geq 1$ and $n = n_1 + n_2$. The spatial variable $x \in \mathbb{R}^n$ consist of two groups of variables: the main group $x_1 \in \mathbb{R}^{n_1}$ and the group of degeneration's variables $x_2 \in \mathbb{R}^{n_2}$, where $x_j := (x_{j1}, \ldots, x_{jn_j}) \in \mathbb{R}^{n_j}$, $j \in \{1, 2\}$, and $x := (x_1, x_2)$.

Consider the equation

$$\Big(\alpha(t)\partial_t - \beta(t)\left[\sum_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} + \sum_{j,l=1}^{n_1} a_{jl}(t,x_1)\partial_{x_{1j}}\partial_{x_{1l}} + \sum_{j=1}^{n_1} a_j(t,x_1)\partial_{x_{1j}}\right] -$$

$$-a_0(t,x_1)\Big)u(t,x) = 0,\ (t,x) \in (0,T] \times \mathbb{R}^n.$$

Assume that the coefficients $a_{jl}, a_j, a_0 : [0,T] \times \mathbb{R}^n \to \mathbb{C}$ satisfy the conditions **1,2** from paper [1]. The functions α and β are continuous on $[0,T]$ which satisfy the conditions: $\alpha(t) > 0$, $\beta(t) > 0$ for each $t \in (0,T]$, $\alpha(0)\beta(0) = 0$, β is monotonically nondecreasing function.

Under these assumptions, the classical fundamental solution for the ultra-parabolic Kolmogorov equation with degeneration on the initial hyperplane is constructed and investigated by using Levi method [2].

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ANGULAR BOUNDARY VALUES OF HOLOMORPHIC IN THE HALF-PLANE FUNCTIONS OF EXPONENTIAL GROWTH

We construct the holomorphic in the half-plane $\mathbb{C}_+ = \{z : \mathrm{Re} z > 0\}$ function $F \not\equiv 0$ satisfying the conditions

$$|F(iy)| = \exp\left(\gamma\left(|y|\right)\right) \quad \text{for almost all} \quad y \in \mathbb{R},$$

$$\gamma(r) - \frac{2}{\pi} x\psi(r) - c_1(1+x) \leq \log|F(z)| \leq \gamma(r) - \frac{2}{\pi} x\psi(r) + c_1(1+x),$$

where $c_1 > 0$ is some constant, $z = x + iy = re^{i\varphi} \in \mathbb{C}_+$, $\psi(r) = \int\limits_1^r \frac{\gamma(t)}{t^2} dt$ for $r \in [1; +\infty)$, and $\psi(r) = 0$ for $r \notin [1; +\infty)$. Here $\gamma : [0; +\infty) \to \mathbb{R}$ is a measurable function such that $|\gamma(t)| \leq c_0\left(|t| + 1\right)$ as $t \in [0; +\infty)$, and $|\gamma(t) - \gamma(r) - \alpha(r)(t - r)| \leq c_1 + c_1\left|t - r\right|^2/r$ for all $t \in [r/2; 2r]$ and for a measurable bounded function $\alpha : [r_0; +\infty) \to \mathbb{R}$, $r_0 \geq 0$.

We generalize the results of the papers [1-3].

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PARALLELIZATION OF THE CONVOLUTION TYPE COMPUTATIONS

We consider fragments of computations being similar to the convolution by their internal structure. Above all, we investigate the aggregation procedures [1, 2] in order to obtain estimates with different degrees of generality for the condition and quality of functioning of complex dynamical systems. The result of such procedures is a generalized conclusion, which is obtained by means of the lower degree of generality and weighting factors. The later represents the importance of the individual objects in the structure of the system and priority functions being performed. In addition, we consider the one-dimensional problem of digital filtering with adaptive smoothing [3]. In this case, the recalculation of values of the variable x(j) is executed via the floating window of the size M(j) for all j=1, 2, ..., N. These problems are solved in real time. There is a need therefore to develop and explore the parallel computation algorithms.

We propose the efficient algorithmic constructions for the parallel execution of the aggregation procedures [1]. The parallel-pipelined algorithm [3], being optimal by speed and usage of memory, for solving the filtering problem is also constructed. The optimality is proved in a certain class of algorithms which are equivalent by the information graph.

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FUNDAMENTAL SOLUTION OF THE CAUCHY PROBLEM FOR SOME PARABOLIC EQUATIONS WITH INCREASING COEFFICIENTS

We consider the parabolic equation of the arbitrary $2b$ order that generalizes some Fokker-Plank-Kolmohorov equations for normal Marcov processes and takes the form

$$\left(\partial_t - \sum_{|k|\leqslant 2b} a_k(t,x)\partial_x^k - S_a\right) u(t,x) = 0, \quad (t,x) \in \Pi_{(0,T]}, \qquad (1)$$

where $\Pi_{(0,T]} := \{(t,x) \in R^{n+1} | t \in (0,T], x := (x_1 \dots x_n) \in R^n\}$, T is a positive number; a_k, $k \leqslant 2b$, – complex-valued functions that are defined in $\Pi_{(0,T]}$, $S_a u := a\sum_{i=1}^{n} \partial_{x_j}(x_j u(t,x))$ – differential expression with increasing coefficients at infinity, $a \in R$.

On the basis of Levi's method there has been constructed the FSCP for the equation (1), its estimates and estimates of its derivatives have been found.

The estimates of the FSCP for the equation (1) are different in the cases of the second and the arbitrary orders. In the first case the FSCP is estimated as exponent with constant type of decreasing at infinity. In the second case the FSPS's estimate has the form of a series the terms of which contain of exponent with type of decreasing becomes vanishingly small.

Let us remark that in the previous papers of S.Ivasyshen and the author the FSCP has been constructed when coefficients of the equation (1) are constant or depend only on a time variable t and it has been specified the resolvent's estimates for integral equations originated from using Levy's method for the equation (1).

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ASYMPTOTIC BEHAVIOR OF THE MAIN CHARACTERISTICS OF ENTIRE FUNCTIONS OF THE ZERO ORDER

We denote with $H_0(\upsilon)$ a class of entire functions f of the zero order such that the counting function $n(r, 0, f)$ of zeros of f satisfies the condition

$$\overline{\lim_{r\to+\infty}} n(r, 0, f)/\upsilon(r) < +\infty,$$

where υ is a slowly increasing function.

For functions $f \in H_0(\upsilon)$, the relationships between:

1) the strongly regular growth of f ([1], p. 198),
2) the existence of υ-angular density zeros of f,
3) the regular growth in $L^p[0, 2\pi]$-metrics, $p \geq 0$, $ln|f|$ and $arg\ f$ [2],
4) the asymptotic behavior of the logarithmic derivative of function f,
5) the regular growth of the Fourier coefficients of function lnf and the logarithmic derivative $F(z) = zf'(z)/f(z)$

are studied.

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QUALITATIVE ANALYSIS OF THE IMPLICIT SINGULAR INITIAL VALUE PROBLEM: SOLVABILITY, ASYMPTOTIC, NUMBER OF SOLUTIONS

The following initial value problem is considered

$$\alpha_1(t)x_1'(t) = a_1 t + b_1 x_1(t) + \varphi_1(t, x_1(t), x_2(t), x_1'(t), x_2'(t)),$$

$$\alpha_2(t)x_2'(t) = a_2 t + b_2 x_2(t) + \varphi_2(t, x_1(t), x_2(t), x_1'(t), x_2'(t)),$$

$$x_1(0) = 0, x_2(0) = 0,$$

where $x_i : (0,\tau) \to \mathbb{R}$, are unknown functions, a_i, b_j are constants, $\varphi_i : D \to \mathbb{R}$ and $\alpha_i : (0,\tau) \to (0,+\infty)$ are continuous functions, $D \subset (0,\tau) \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, $\alpha_i(t) \to 0$ as $t \to +0$, $i, j \in \{1,2\}$.

Existence of continuously differentiable solutions $x_i : (0,\sigma) \to \mathbb{R}$, $i \in \{1,2\}$ is proved (σ is small enough). Asymptotic properties for each of these solutions are obtained. If some conditions are fulfilled, then the number of such solutions are also determinated.

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FAST CONVERGENT GREEN FUNCTION FOR INTEGRAL EQUATIONS IN THE 3D PERIODICAL DYNAMIC CRACK PROBLEMS

Investigation of different kinds of periodic structures such as phononic crystals, especially the elastic wave propagation in such structures, is of great importance because of their potential engineering applications [1]. In many cases periodic systems of cracks can act as the wave scatterers in periodic structures with generation of specific wave patterns due to their sharp edges. Previous known works, which take into account of the presence of multiple cracks in 3D elastic wave field, are related to the situations with a few defects [2]. The reason lies in the computational difficulties of the corresponding large-scale problems. The models with periodically distributed cracks can simplify the analysis, especially by the boundary integral equations (BIEs) method and introducing effective Green's functions to consider properly the dynamic interactions between the infinite number of cracks. There are usually two ways to deal with the periodic structures by the BIEs method: one is that the BIEs are formulated in the unit cell according to the wave equations and general Green's functions, then the Bloch conditions of periodicity are forced on the boundaries of the unit cell; the other is that the Bloch conditions are first substituted into the wave equations, then the BIEs are formulated based on the periodic Green's functions. In the present work, the second approach, which does not demand the BIEs formulation on the boundary of unit cell, is used for the analysis of normally incident plane longitudinal elastic waves on doubly periodic array of coplanar penny-shaped cracks.

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