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MFS-BASED NUMERICAL SCHEME WITH LAGUERRE TIME SEMI-DISCRETIZATION FOR 2D WAVE PROCESSES IN CHANNELS

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This paper considers the numerical solution of linear wave processes in two-dimensional models of water channels for both transverse and longitudinal sections. Let D be a section of a water channel filled with an inviscid incompressible liquid having a free surface. Denote by Γ_1 the free water boundary, by Γ_2 the wetted channel boundary and by Ω the water-filled domain bounded by those boundaries $\partial\Omega = \Gamma_1 \cup \Gamma_2$. We seek the function $u : \Gamma_1 \times [0, T) \to \mathbb{R}$, which satisfies the evolution operator equation and the initial conditions

$$\frac{\partial^2 u}{\partial t^2} + Au = f \quad \text{on } \Gamma_1 \times (0, T], \tag{1}$$

$$u\Big|_{t=0} = w_1, \qquad \frac{\partial u}{\partial t}\Big|_{t=0} = w_2 \quad \text{on } \Gamma_1, \tag{2}$$

where w_1, w_2 and f are given sufficiently smooth functions on Γ_1, f describes the force field which acts on the moving fluid. The operator A is defined as $A\psi = \frac{\partial v}{\partial \nu}$ on $\Gamma_1 \times (0, T]$, where v is the solution of the corresponding mixed Dirichlet–Neumann boundary value problem

$$\Delta v = 0 \quad \text{in } \Omega, \tag{3}$$

$$v = \psi \quad \text{on } \Gamma_1 \qquad \text{and} \qquad \frac{\partial v}{\partial \nu} = 0 \quad \text{on } \Gamma_2. \tag{4}$$

Here ν denotes the outward unit normal vector to the boundaries Γ_1 and Γ_2 and we use the notation $x = (x_1, x_2)$. The well-posedness of the hyperbolic evolution problem is discussed in [4].

In this study, we consider a widely used two-step approach. Firstly, the Laguerre transform [2] is used for time semi-discretization, reducing the problem to a sequence of elliptic boundary value problems with the recurrent operator equations. The solution is represented as a Laguerre series [1], where coefficients satisfy recurrent operator equations.

At the next step these problems are solved numerically using the method of fundamental solutions (MFS) while also taking into account the specifics of longitudinal and cross-sections of the channel. The solutions of the boundary value problems (3)-(4) are approximated by a linear combination of fundamental solutions with coefficients to be determined by the collocation method on boundaries Γ_1, Γ_2. Next we describe the rules for selecting the collocation and source points using boundary parametrizations. Having chosen the required points we can find the unknown coefficients and obtain the sequence of linear equations. We also describe the approximation error of applied MFS scheme.

Finally, we present the results of numerical examples for each type of channel section and conclusions that demonstrate the efficiency of implemented numerical approach.

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ЧИСЕЛЬНИЙ МЕТОД НА ОСНОВІ МФР ІЗ ЧАСТКОВОЮ ДИСКРЕТИЗАЦІЄЮ ЗА ЧАСОМ МЕТОДОМ ЛАГЕРРА ДЛЯ ДВОВИМІРНИХ ХВИЛЬОВИХ ПРОЦЕСІВ У КАНАЛАХ

У роботі розглядається чисельне розв'язання задачі про лінійні хвильові процеси в двовимірних моделях водних каналів для поперечних і поздовжніх перерізів за допомогою двоетапного підходу. Спочатку, використовуючи перетворення Лагерра, виконується часткова дискретизація за часом, що зводить задачу до послідовності еліптичних крайових задач. Далі ці задачі розв'язуються методом фундаментальних розв'язків із використанням колокації на межах області. Описано вибір точок джерела та колокації, а також досліджено похибку апроксимації. Наведені чисельні експерименти для різних геометрій каналів, які підтверджують ефективність запропонованого підходу.