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EIGENVALUE ANALYSIS OF VISCOUS HYDROELASTIC SYSTEMS IN DISPLACEMENT FORMULATION: FEniCSx IMPLEMENTATION

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Introduction. Among the formulations of acoustic fluid-structure interaction, the displacement-based (**u-u**) one occupies a distinguished position [3]: solid and fluid share a single primary unknown, matrices $(\mathbf{K}, \mathbf{M}, \mathbf{C})$ are symmetric, and a natural-viscous regularisation lifts the variational problem from $H(\nabla\cdot, \Omega)^n$ into $H^1(\Omega)^n$ while reflecting the physics of dissipation. The price is a $\sim 2N/3$-dimensional solenoidal null space whose discrete counterpart populates the spectrum with non-physical modes. Two classifiers shape the literature: the energy ratio r_E [1, 2] and the modal damping ratio ζ_k [3] — complementary, not redundant. Each illuminates one region of the spectrum and leaves another in shadow. We argue that the question is inherently multi-dimensional, and demonstrate that a four-indicator composition accounts for every mode of the discrete spectrum on the canonical container of [1].

Formulation and implementation. The damped problem $(\alpha^2\mathbf{M}+\alpha\mathbf{C}+\mathbf{K})\boldsymbol{\varphi} = \mathbf{0}$ inherits from [3] the natural-viscous form $c_F = \int_{\Omega^F}[\lambda_v(\nabla \cdot u)(\nabla \cdot v) + 2\eta\,\boldsymbol{\varepsilon}(u){:}\boldsymbol{\varepsilon}(v)]dx$, $\lambda_v = \xi - \frac{2}{3}\eta$. Each eigenvalue $\alpha_k = -\zeta_k\omega_k + i\omega_k\sqrt{1-\zeta_k^2}$ encodes a damped frequency and a decay rate; under M-orthonormalisation the modal damping is the Rayleigh quotient [6] $\zeta_k = \boldsymbol{\varphi}_k^T\mathbf{C}\boldsymbol{\varphi}_k/(2\omega_k)$ to $O(\zeta^2)$. Building on the eigenvalue framework for 3D elastic bodies [4], we discretise by Q_2 Serendipity hexahedra in DOLFINx [7] and solve the QEP by multi-target ARPACK shift-invert on a real $2N \times 2N$ linearisation [5], recovering the complex spectrum without complex-arithmetic builds.

A four-indicator classifier. Each indicator captures one signature of an eigenmode: r_S — solid share of kinetic energy; r_E — fluid compressibility; p_{RMS} — magnitude of the recovered pressure field; ζ_k — energy loss under viscous regularisation. Together they fix four axes along which physical and spurious modes occupy distinct regions. The composite labels each mode as *structural*, *acoustic*, *hybrid-coupled*, *spurious-overdamped* or *spurious-solenoidal*; each branch catches a class that at least one single-indicator classifier misidentifies — most notably structurally-dominated modes ($r_S \to 1$, hence

$r_E \to 0$) and pre-asymptotic discrete coupled modes (small r_S, r_E but non-trivial p_{RMS}).

Verification. The implementation is verified at two levels: by formulation-level identities that hold for any viscous fluid in [3], and by demonstrating that the four-indicator composition resolves every mode of the canonical container of [1] into one of the five classes above.

Conclusions. Mode classification in the displacement formulation is inherently multi-dimensional. A four-indicator composition closes the gap left by single-criterion approaches and accounts for every mode of the discrete spectrum on the canonical benchmark. The accompanying FEniCSx pipeline opens a path to systematic mode analysis of three-dimensional hydroelastic systems.

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АНАЛІЗ ВЛАСНИХ КОЛИВАНЬ В'ЯЗКИХ ГІДРОПРУЖНИХ СИСТЕМ У ФОРМУЛЮВАННІ ПЕРЕМІЩЕНЬ: РЕАЛІЗАЦІЯ В FEniCSx

У формулюванні переміщень [3] акустичної взаємодії пружного тіла з рідиною дискретний спектр містить як фізичні моди, так і соленоїдальні залишки чисельного походження. Жоден з відомих критеріїв — ані енергетичний r_E *[1,2], ані модальне демпфування* ζ_k *[3] — не охоплює весь спектр самостійно: вони взаємно доповнюються. Запропоновано чотирикомпонентний класифікатор* $\{r_S, r_E, p_{\mathrm{RMS}}, \zeta_k\}$*, який пояснює всі моди еталонного контейнера [1] без додаткового відсіювання. Реалізовано у відкритому пакеті FEniCSx [7].*