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APPROXIMATION FUNCTION FOR THE MACH NUMBER AND FLOW TURNING ANGLE

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The problem of expanding supersonic flow has been investigated for a long time. At present, the flow turning angle θ can already be expressed as a function of the Mach number. The purpose of this study is to construct an approximation function that allows us to obtain the inverse dependence, namely the Mach number as a function of the turning angle. In the problem of expanding supersonic flow, the turning angle θ is determined by Equation (1) from [1]:

$$\theta = f(M) = \sqrt{\frac{k+1}{k-1}}\arctan\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \arctan\left(\sqrt{M^2-1}\right) \quad (1)$$

However, it is impossible to express the Mach number analytically from this equation. Therefore, the problem arises of constructing an approximation $M = f(\theta)$. To address this problem, a table of turning angles θ_i and corresponding Mach numbers M_i was generated for the interval $1^\circ \le \theta \le 120^\circ$, followed by the construction of the corresponding point plot.

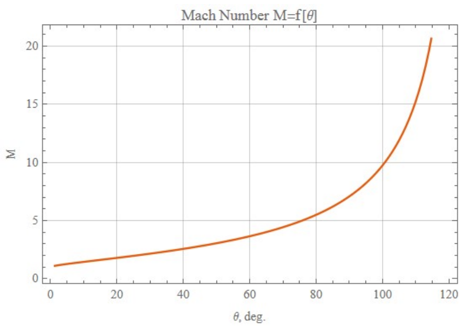


Figure 1 – Point plot of the function.

As stated previously, the objective is to construct an approximation function $M = f(\theta)$. For this purpose, a Padé rational approximant [2] was employed due to its favorable convergence properties:

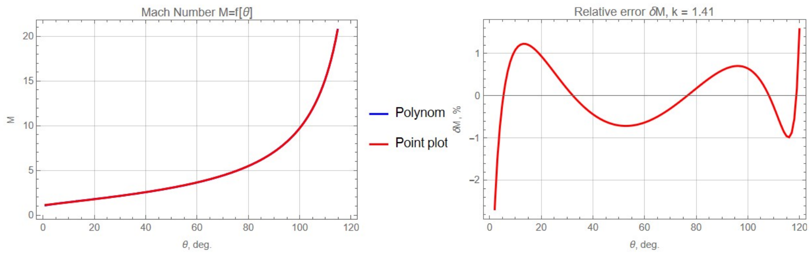
$$f(\theta) = \frac{a_0 + a_1\theta + a_2\theta^2}{1 + b_1\theta + b_2\theta^2} \tag{2}$$

Interpolation of the numerical data for $k = 1.41$ yields the following approximation:

$$f(\theta) = \frac{1.092899 + 0.035023\theta - 0.000044\theta^2}{1 + 0.001620\theta - 0.000073\theta^2} \tag{3}$$

Within this interval, the approximation demonstrates good agreement with the original function, as shown in Figures 2 and 3. The graph of relative errors was obtained from discrete points. Each point has coordinates (θ_i , δM_i) where:

$$\delta M_i = \frac{M_i(\theta) - M_{\text{appr},i}(\theta)}{M_i(\theta)} \times 100\% \tag{4}$$



Figures 2 and 3 – Comparison of the exact and approximation curves; relative error distribution of the approximation.

It should be noted that the approximation formula depends on the chosen adiabatic index and the selected angular interval. In our case, the approximation was constructed over the interval $\theta \in [1^\circ, 120^\circ]$ (using a discretization step of 1°), and $k \in [1.1, 1.44]$, with chosen $k = 1.41$. Outside these ranges, the approximation accuracy deteriorates due to the selected rational approximation form. The obtained approximation may be useful for rapid engineering estimations and numerical gas-dynamics applications.

1. L. G. Loitsyanskii, Mechanics of Liquids and Gases. Moscow: Nauka, 1987. [in Russian]

2. G. A. Baker Jr. and P. Graves-Morris, Padé Approximants, 2nd ed. Cambridge, U.K.: Cambridge University Press, 1996.

АПРОКСИМАЦІЙНА ФУНКЦІЯ ДЛЯ ЧИСЛА МАХА ТА КУТА ПОВОРОТУ ПОТОКУ

Проблема розширення надзвукового потоку досліджується вже тривалий час. На сьогодні кут повороту потоку θ вже може бути виражений як функція числа Маха. Метою даного дослідження є побудова апроксимаційного профілю, який дозволяє отримати обернену залежність, а саме — число Маха як функцію кута повороту потоку.