The conference of young scientists «Pidstryhach readings – 2026»
May 27–29, 2026, Lviv

UDC 517.95

QUALITATIVE PROPERTIES OF SOLUTIONS TO PARABOLIC ANISOTROPIC EQUATIONS

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We study non-negative solutions to anisotropic Trudinger-type operators with measurable and bounded coefficients, whose prototype is

$$u_t - \sum_{i=1}^{N} D_i\Big(u^{2-p_i}|D_i u|^{p_i-2}D_i u\Big) = 0, \quad u \geqslant 0. \tag{1}$$

More precisely, let Ω be an open bounded domain in $\mathbb{R}^N$, $T > 0$ and consider the numbers

$$p_+ := \max_{1\leqslant i\leqslant N} p_i, \qquad p_- := \min_{1\leqslant i\leqslant N} p_i, \qquad \frac{1}{p} := \frac{1}{N}\sum_{i=1}^{N}\frac{1}{p_i}\ .$$

We study non-negative solutions to the equation

$$u_t - \sum_{i=1}^{N} D_i a_i(x,t,u,Du) = 0, \quad (x,t) \in \Omega_T := \Omega \times (0,T). \tag{2}$$

assuming that the functions $a_i : \Omega_T \times \mathbb{R}_+ \times \mathbb{R}^N \to \mathbb{R}^N$ are Lebesgue measurable, for all $u \in \mathbb{R}_+$, $\xi \in \mathbb{R}^N$, and continuous for almost all $(x,t) \in \Omega_T$; and satisfy the structure conditions:

$$\begin{cases} \sum\limits_{i=1}^{N} a_i(x,t,u,Du)\, D_i u^{\frac{1}{p_+-1}} \geqslant K_1 \sum\limits_{i=1}^{N} u^{\frac{p_+-p_i}{p_+-1}} |D_i u^{\frac{1}{p_+-1}}|^{p_i}, 1 < p_- \leq p_+ < N, \\ \\ |a_i(x,t,u,Du)| \leqslant K_2\, u^{\frac{p_+-p_i}{p_+-1}} |D_i u^{\frac{1}{p_+-1}}|^{p_i-1}, \qquad \forall\, i = 1,...,N, \end{cases} \tag{3}$$

where K_1, K_2 are positive constants, and $Du = (D_1 u, ..., D_N u)$ is assumed to satisfy weakly

$$D_i u = (p_+ - 1) u^{\frac{p_+-2}{p_+-1}} D_i\big(u^{\frac{1}{p_+-1}}\big), \quad \text{for} \quad i = 1,...,N.$$

Our first main result is a Harnack inequality that holds without any restriction on the gap between p_+ and p_-. The parameters N, p_1, ..., p_N, K_1, K_2 will be referred to as *the data*.

Theorem 1. *Let u be a non-negative, local weak solution to* (2)-(3) *in* Ω_T. *There exist positive constants* C, $\bar{C}$ *depending only on the data such that, for all cylinders*

$$ {}_{8\rho}(x_0) \times (t_0 - \bar{C}(8\rho)^p, t_0 + \bar{C}(8\rho)^p) \subset \Omega_T \ , $$

there holds

$$ \frac{1}{C} \sup_{\rho(x_0)} u(\cdot, t_0 - \bar{C}\,\rho^p) \leqslant u(x_0, t_0) \leqslant C \inf_{\rho(x_0)} u(\cdot, t_0 + \bar{C}\,\rho^p), \tag{4} $$

where

$$ {}_{\rho}(x_0) := \prod_{i=1}^{N} \left\{ |x_i - x_{0i}| < \rho^{\frac{p}{p_i}} \right\}. $$

Our second main result is the local Hölder continuity of solutions. The proof is based on the expansion of positivity for anisotropic p-Laplacian under the additional assumption that the gap between p_+ and p_- is sufficiently small.

Theorem 2. *Let u be a non-negative, local weak solution to* (2)-(3). *There exists a number $\epsilon_* \in (0,1)$ depending only on the data such that, if*

$$ p_+ - p_- \leqslant \epsilon_*, \tag{5} $$

then u has a locally Hölder continuous representative.

This work was supported by the National Research Foundation of Ukraine, project No. 2025.07/0369 "Qualitative methods of nonlinear analysis of heterogeneous structures".

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ЯКІСНІ ВЛАСТИВОСТІ РОЗВ'ЯЗКІВ ПАРАБОЛІЧНИХ АНІЗОТРОПНИХ РІВНЯНЬ

Досліджуються локальні регулярні властивості слабких розв'язків анізотропних двічі нелінійних параболічних рівнянь, прототипом яких є анізотропне рівняння Трудінгера. Доведено параболічну нерівність Гарнака для невід'ємних локальних слабких розв'язків без додаткових умов на показники p_i. Крім того, для певного діапазону показників дифузії встановлено гельдерову неперервність розв'язків.