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ON THE CONDITIONS FOR CONVERGENCE OF SOLUTIONS OF BOUNDARY-VALUE PROBLEMS

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The question of finding the conditions for the convergence of solutions of systems of ordinary differential equations arises in many problems of modern analysis and its applications. It were deeply investigated in the case of the solutions of Cauchy's problems for the system of first-order differential equations. More complicated case of linear boundary-value problems was studied by I.T. Kiguradze [1] and his followers [2, 3].

On a finite interval $(a, b) \subset \mathbb{R}$, we consider the systems of linear differential equations of the higher order of the form

$$y^{(r)}(t,n) + A_{r-1}(t,n)y^{(r-1)}(t,n) + \ldots + A_0(t,n)y(t,n) = f(t,n) \qquad (1)$$

with inhomogeneous boundary conditions

$$B_j(n)y(\cdot,n) = c_j(n), \quad j \in \{1,\ldots,r\} := [r], \quad n \in \mathbb{N} \cup \{0\}, \qquad (2)$$

where $B_j(n) : C^{(r-1)}([a,b];\mathbb{C}^m) \to \mathbb{C}^m$ are linear continuous operators, the matrix-valued functions $A_{j-1}(\cdot,n) \in L_1([a,b];\mathbb{C}^{m\times m})$, the vector-valued functions $f(\cdot,n) \in L_1([a,b];\mathbb{C}^m)$, and the vectors $c_j(n) \in \mathbb{C}^m$.

A solution of the system of differential equations (1), (2) is understood as a vector-valued function $y(\cdot,n) \in W_1^r([a,b];\mathbb{C}^m)$ satisfying the equation almost everywhere. The inhomogeneous boundary conditions (2) are correctly defined on the solutions of system (1) and cover all classical types of boundary conditions.

Under what conditions imposed on the left-hand sides of problems (1), (2) their solutions $y(\cdot,n)$ exist and are unique for sufficiently large $n \in \mathbb{N}$? What additional conditions imposed on the left- and right-hand sides of problems (1), (2) guarantee the limit equality

$$\left\| y^{(j-1)}(\cdot,0) - y^{(j-1)}(\cdot,n) \right\|_\infty \to 0, \quad n \to \infty, \qquad (3)$$

where $\|\cdot\|_\infty$ – sup-norm on the compact interval $[a,b]$.

For the first time, these problems were investigated by Kiguradze [2] in the case $r = 1$. For applied to these problems, Kiguradze Theorem takes the following form.

Theorem 1. *Suppose that the solutions of problem (1), (2) are uniquely defined and*

(I) $\|R^{\vee}_{A_{j-1}}(\cdot, n)\|_{\infty} \to 0;$

(II) $\|R_{A_{j-1}}(\cdot, n)\|_{1} = O(1);$

(III) $B_j(n)y \to B_j(0)y, y \in C^{(r-1)}([a, b]; \mathbb{C}^m).$

Then, for sufficiently large n *problems* (6), (7) *possesses the unique solutions. Moreover, if*

(IV) $c_j(n) \to c_j(0),$

(V) $\|R^{\vee}_{F}(\cdot, n)\|_{\infty} \to 0, n \to \infty$

then the unique solutions of problems (1), (2) *satisfy the limit equality* (3).

All conditions of the Kiguradze Theorem are essential and none of them can be omitted. However, some conditions can be weakened.

Theorem 2. *In Theorem 1, condition* (II) *can be replaced by the condition*

(II**) $\|R_{A_{r-1}}(\cdot, n)R^{\vee}_{A_{j-1}}(\cdot, n)\|_{1} \to 0,$

if the additional condition are fulfilled

(VI**) $\|R_{A_{r-1}}(\cdot, n)R^{\vee}_{F}(\cdot, n)\|_{1} \to 0, n \to \infty.$

The condition (VI^{**}) *is fulfilled if conditions* (II), (V) *hold.*

These and other results are presented in more detail in [2, 3].

1. Kiguradze I.T *Some singular boundary value problems for ordinary differential equations.* Tbilisi: Izdat. Tbilis. Univ., 1975.

2. Mikhailets V.A., Pelekhata O.B., Reva N.V.*On the Kiguradze theorem for linear boundary-value problems.* Dopov. Nats. Acad. Nauk Ukr., 2017,(12), 8–13.

3. Pelekhata O.B., Reva N.V. *Limit theorems for the solutions of linear boundary-value problems for systems of differential equations.* Ukrainian Math. J. 2019,**71** (7), 1061–1070.

ПРО УМОВИ ЗБІЖНОСТІ РОЗВ'ЯЗКІВ ЗАГАЛЬНИХ КРАЙОВИХ ЗАДАЧ

Знайдено достатні умови збіжності розв'язків послідовності загальних крайових задач для систем лінійних звичайних диференціальних рівнянь довільного порядку у рівномірній нормі на скінчкнному інтервалі.