UDC 519.6

NUMERICAL MODELING OF ATMOSPHERIC POLLUTANT DISPERSION USING THE FINITE ELEMENT METHOD

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In this work we consider a stationary diffusion–advection–reaction model together with its finite element discretization based on adaptive mesh refinement for spatially heterogeneous transport fields, which is an important problem in environmental analysis. The model is then applied to visualize the particle concentration distribution over a part of the world map.

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain. The concentration $u(x, y)$ satisfies

$$-\mu\Delta u + \boldsymbol{\beta} \cdot \nabla u + \sigma u = f \quad \text{in } \Omega, \tag{1}$$

where $\mu > 0$ is the diffusion coefficient, $\boldsymbol{\beta}(x, y)$ is the advection field, $\sigma \geq 0$ is the reaction coefficient, and $f(x, y)$ describes sources.

For simplicity, we consider the Dirichlet boundary condition

$$u = 0 \quad \text{on } \partial\Omega. \tag{2}$$

Under standard assumptions, this problem has a unique solution. In this work, we approximate it using a Galerkin discretization and the standard finite element method with linear basis functions on a triangular mesh generated using the Triangle library.

In scenarios of practical interest, this problem often involves singular perturbations. To handle them, mesh adaptation is applied.

An adaptive strategy is based on a residual-type indicator. For each element K, a cubic bubble function b_K defines an auxiliary quantity

$$\eta_K = \frac{\ell(b_K) - a(u_h, b_K)}{a(b_K, b_K)}. \tag{3}$$

The actual local error estimate is $\eta_K \|b_K\|_{H^1(K)}$. Elements with the largest values of this estimate are marked and refined, enabling resolution of sharp gradients near source regions and along flow directions.

Based on the described model and the FEM solver, an interactive GUI application was developed that allows the user to model the described problem on a small region of the Earth's surface. Given the diffusion and reaction coefficients, an advection field based on real-world wind data from the Copernicus Climate Change Service dataset, and a user-defined particle source, the proposed model can be used to predict the particle concentration in the region of interest on the world map.

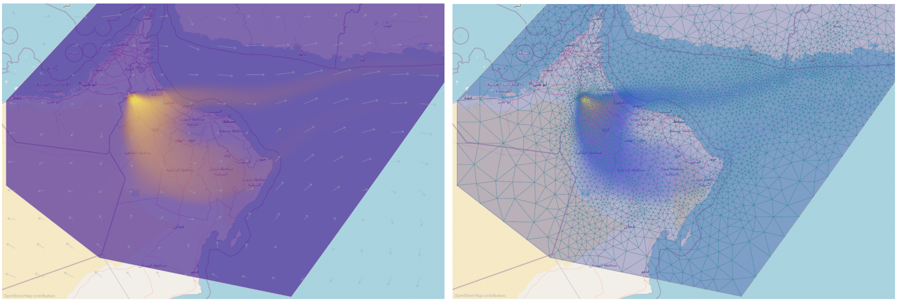

Fig. 1. *Visualization of the test problem with a source and a complex advection field: the computed concentration field with arrows indicating the wind direction on the left, and the finite element mesh on the right*

1. Brenner S. C., Scott L. R. *The Mathematical Theory of Finite Element Methods.* Springer, New York, 2008.
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ЧИСЕЛЬНЕ МОДЕЛЮВАННЯ ПОШИРЕННЯ АТМОСФЕРНИХ ЗАБРУДНЕНЬ ІЗ ВИКОРИСТАННЯМ МЕТОДУ СКІНЧЕННИХ ЕЛЕМЕНТІВ

Розглянуто стаціонарну модель дифузії–адвекції–реакції для опису просторового розподілу атмосферних забруднень та її чисельну реалізацію на основі методу скінченних елементів із адаптивним згущенням сітки. На основі моделі розроблено інтерактивний додаток для моделювання поширення концентрації домішки на деякій ділянці земної поверхні з врахуванням актуальної інформації про вітер.