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COMPARISON OF THE METHOD OF FUNDAMENTAL SOLUTIONS AND BOUNDARY INTEGRAL EQUATIONS FOR PLANAR SCATTERING PROBLEMS

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Let $D \subset \mathbb{R}^2$ be a bounded simply connected domain with boundary $\Gamma = \partial D$, $\Gamma \in C^1$. For a given wave number $\kappa \in \mathbb{R}$, $\kappa > 0$, find a function u satisfying the exterior Dirichlet problem for the Helmholtz equation

$$\begin{cases} \Delta u + \kappa^2 u = 0, & \text{in } \mathbb{R}^2 \setminus \overline{D}, \\ u = -u^{\text{inc}}, & \text{on } \Gamma, \\ \frac{\partial u}{\partial r} - i\kappa u = o\left(r^{-1/2}\right), & r = |x| \to \infty, \end{cases} \tag{1}$$

where $u^{\text{inc}}(x) = e^{i\kappa x \cdot d}$ is an incident acoustic plane wave propagating in the direction $d \in \mathbb{R}^2$, $|d| = 1$. The function u, also denoted by u^{sc}, represents the scattered field. The total field is given by $u^{\text{tot}} = u^{\text{inc}} + u^{\text{sc}}$. This problem describes the scattering of an acoustic wave by the obstacle D.

The problem (1) is solved numerically using the method of fundamental solutions (MFS) and the boundary integral equations method (BIE).

In the case of the MFS, the scattered field is approximated as follows

$$u^{\text{sc}}(x) \approx \sum_{j=1}^{N} \alpha_j \, \Phi(x, y_j), \qquad x \in \mathbb{R}^2 \setminus \overline{D}, \tag{2}$$

where y_j are source points located inside the obstacle D, α_j are unknown coefficients and $\Phi(x, y)$ is the fundamental solution of the Helmholtz equation in $\mathbb{R}^2$ given by

$$\Phi(x, y) = \frac{i}{4} H_0^{(1)}(\kappa |x - y|), \qquad x, y \in \mathbb{R}^2, \quad x \neq y, \tag{3}$$

where

$$H_0^{(1)}(z) = J_0(z) + iY_0(z)$$

is the Hankel function of the first kind and order zero.

The boundary condition is enforced at collocation points $x_k \in \Gamma$, leading to the linear system

$$\sum_{j=1}^{N} \alpha_j \, \Phi(x_k, y_j) = -u^{\mathrm{inc}}(x_k), \qquad k = 1, \ldots, m, \quad m \geq N. \tag{4}$$

The BIE approach is based on the representation of the solution of the problem (1) by a single-layer potential

$$u(x) = \int_{\Gamma} \varphi(y) \, \Phi(x, y) \, ds(y), \qquad x \in \mathbb{R}^2 \setminus \overline{D}, \tag{5}$$

where φ is an unknown density which is a solution of the boundary integral equation of the first kind

$$\int_{\Gamma} \varphi(y) \, \Phi(x, y) \, ds(y) = -u^{\mathrm{inc}}(x), \qquad x \in \Gamma. \tag{6}$$

The equation (6) is uniquely solvable provided that κ^2 is not an interior Dirichlet eigenvalue of the Laplacian in D.

The equation is split into real and imaginary parts. This leads to a coupled system of first-kind boundary integral equations for the real-valued densities. The boundary Γ is parameterized by a periodic mapping $x(t)$, $t \in [0, 2\pi]$, which transforms the system to integral equations on a fixed interval with periodic kernels.

The formulation contains weakly singular logarithmic kernels. To treat the singularity numerically, the kernel is split into a logarithmic term and a smooth remainder, enabling accurate quadrature rules. The resulting discrete system is solved to obtain approximations of the densities, which are then used to reconstruct the scattered field.

Numerical results show that the method of fundamental solutions is simpler and quite accurate but sensitive to source points, while the boundary integral equation method is more complex but more stable and robust.

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ПОРІВНЯННЯ МЕТОДІВ ФУНДАМЕНТАЛЬНИХ РОЗВ'ЯЗКІВ ТА ГРАНИЧНИХ ІНТЕГРАЛЬНИХ РІВНЯНЬ ДЛЯ ПЛОСКИХ ЗАДАЧ РОЗСІЮВАННЯ

Розглянуто порівняння методів фундаментальних розв'язків та граничних інтегральних рівнянь для розв'язування плоских задач розсіювання. Перший підхід базується на апроксимації шуканої функції у вигляді лінійної комбінації фундаментальних розв'язків у фіктивних точках-джерелах. У другому підході розв'язок подано у вигляді потенціалу простого шару. Результати показують, що метод фундаментальних розв'язків простіший, але чутливий до вибору джерел, тоді як метод граничних інтегральних рівнянь складніший, проте більш стійкий.