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SHRINKAGE-BASED ESTIMATION OF THE MAXIMUM SHARPE RATIO PORTFOLIO

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The Sharpe ratio is a key measure in portfolio theory, quantifying the trade-off between risk and return. In high-dimensional settings—where the number of assets k is comparable to or exceeds the sample size n—traditional estimators of the maximum Sharpe ratio portfolio become unreliable. Shrinkage-based methods address this by combining sample estimators with structured targets, reducing estimation error.

Portfolio optimization in high dimensions is increasingly relevant, as investors consider large asset universes. The ratio $c = k/n$ fundamentally alters estimator properties and requires new theoretical tools, such as random matrix theory.

The following results provide guarantees for the proposed shrinkage estimator in both $c < 1$ and $c > 1$ regimes.

Theorem 1. *Let α^* be defined as the optimal shrinkage intensity for $c < 1$. Then:*

$$|\alpha_n^* - \alpha^*| \xrightarrow{a.s.} 0 \quad \text{as } n \to \infty,\ k/n \to c < 1. \tag{1}$$

This theorem ensures that, as the data set grows, the estimated shrinkage intensity converges to its optimal value, improving portfolio performance.

Proof. The proof uses random matrix theory to show that, under suitable conditions, the difference between α_n^* and α^* converges to zero almost surely. ■

For $c > 1$, an analogous result is obtained, important for very large portfolios.

Theorem 2. *Let α^+ be defined as the optimal shrinkage intensity for $c > 1$. Then:*

$$\left|\alpha_n^* - \alpha^+\right| \xrightarrow{a.s.} 0 \quad \text{as } n \to \infty,\ k/n \to c > 1. \tag{2}$$

This result extends the consistency guarantee to the ultra-high-dimensional regime, where the sample covariance matrix is singular and shrinkage is essential.

Simulation results show that the shrinkage estimator (measured by the relative deviation of its variance from the true value) is superior to the sample estimator for all c except when c is close to 0. For moderate and large c, the shrinkage estimator provides substantial risk reduction and more stable out-of-sample performance.

The shrinkage estimator is relevant for institutional investors and analysts managing large portfolios, especially when traditional methods fail due to high estimation error. It is useful for equity, bond, or multi-asset portfolios with limited historical data, and helps achieve better risk-adjusted returns.

Conclusion

This paper addresses the estimation of the maximum Sharpe ratio portfolio in high-dimensional settings, proposing a shrinkage estimator with optimal intensity derived by minimizing out-of-sample variance. Consistent estimators are constructed using random matrix theory. Simulation results confirm the superiority of the shrinkage estimator over the sample estimator for most values of c. The approach is broadly applicable for modern portfolio management.

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СТИСНЕНА ОЦІНКА ПОРТФЕЛЯ ІЗ МАКСИМАЛЬНИМ ВІДНОШЕННЯМ ШАРПА НА ОСНОВІ МІНІМІЗАЦІЇ ДИСПЕРСІЇ

Розглянуто оцінювання портфеля з максимальним відношенням Шарпа у високорозмірному випадку. Запропоновано стиснену оцінку з оптимальною інтенсивністю стиснення, визначеною через мінімізацію позавибіркової дисперсії. Побудовано консистентну оцінку цієї інтенсивності та практичну стиснену оцінку ваг портфеля.