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PARASTROPHY ORBIT OF (R,S,T) – INVERSE QUASIGROUPS

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An algebra $(Q, \circ, \overset{l}{\circ}, \overset{r}{\circ})$ with identities

$$(x \circ y) \overset{l}{\circ} y = x, \quad (x \overset{l}{\circ} y) \circ y = x, \quad x \overset{r}{\circ} (x \circ y) = y, \quad x \circ (x \overset{r}{\circ} y) = y$$

is called a *quasigroup*; the operation $(\circ)$ is *main*, $(\overset{l}{\circ})$, $(\overset{r}{\circ})$ are called *left* and *right divisions* of $(\circ)$.

Each inverse of an invertible operation is also invertible. All such operations are called *parastrophes* of $(\circ)$ and they are defined by $x_{1\sigma} \overset{\sigma}{\circ} x_{2\sigma} = x_{3\sigma} :\Leftrightarrow x_1 \circ x_2 = x_3$, where $\sigma \in S_3 := \{\iota, s, l, r, sl, sr\}$, $l := (13)$, $r := (23)$, $s := (12)$. In particular, the left and right divisions of $(\circ)$ are its parastrophes. It is easy to verify that ${}^{\sigma}(\overset{\sigma}{\circ}) = (\overset{\sigma\tau}{\circ})$ holds for all $\sigma, \tau \in S_3$.

Let P be an arbitrary proposition in a class of quasigroup $\mathfrak{A}$. A proposition ${}^{\sigma}P$ is said to be a σ-parastrophe of P, if it can be obtained from P by replacing the main operation with its σ^{-1}-parastrophe.

Let ${}^{\sigma}\mathfrak{A}$ denote the class of all σ-parastrophes of quasigroups from $\mathfrak{A}$. A set of all pairwise parastrophic classes is called a *parastrophy orbit* of $\mathfrak{A}$:

$$Po(\mathfrak{A}) = \{{}^{\sigma}\mathfrak{A} \mid \sigma \in S_3\}.$$

A parastrophy orbit of varieties is uniquely defined by one of its varieties. Parastrophy orbits were studied by Alla Lutsenko and Fedir Sokhatsky [1], [2].

A quasigroup $(Q, \circ)$ is called an *(r,s,t)– inverse quasigroup* if there exists a permutation J of the elements such that, for all $x, y \in Q$ the following identity holds:

$$J^r(x \circ y) \circ J^s(x) = J^t(y),$$

where r, s, t are integers [3].

Theorem 1. *The parastrophy orbit of (r, s, t)-inverse quasigroups comprises six varieties.*

$$Po(\mathfrak{A}) = \{\mathfrak{A},\ {}^{s}\mathfrak{A},\ {}^{l}\mathfrak{A},\ {}^{r}\mathfrak{A},\ {}^{sl}\mathfrak{A},\ {}^{sr}\mathfrak{A}\}$$

Variety	Identity
$\mathfrak{A}$	$J^r(x \circ y) \circ J^s(x) = J^t(y)$
${}^{s}\mathfrak{A}$	$J^s(x) \circ J^r(y \circ x) = J^t(y)$
${}^{l}\mathfrak{A}$	$J^t(x) \circ J^s(y \circ x) = J^r(y)$
${}^{r}\mathfrak{A}$	$J^r(x) \circ J^t(y \circ x) = J^s(y)$
${}^{sl}\mathfrak{A}$	$J^t(x \circ y) \circ J^r(x) = J^s(y)$
${}^{sr}\mathfrak{A}$	$J^s(x \circ y) \circ J^t(x) = J^r(y)$

Example 1. The quasigroup $(Z_{11}, \circ)$ defined by $x \circ y = (c - 5x - 4y)\ mod 11$ is a $(6, 2, 2)$-inverse quasigroup (see [3, p.191]). That is, in this quasigroup, the identity of the variety $\mathfrak{A}$ holds for all $x, y, c \in Z_{11}$.

In a (6,2,2)–quasigroup $(Z_{11}, \circ)$, only the identity of the variety $\mathfrak{A}$ holds for all $x, y, c \in Z_{11}$.

1. Sokhatsky F., Lutsenko A. *Classification of quasigroups according to directions of translations II.* Commentationes Mathematicae Universitatis Carolinae 2021, **62** (3), 309–323.

2. Sokhatsky F.M., Lutsenko A.V., Fryz I.V. *Construction of Quasigroups with Invertibility Properties.* Journal of Mathematical Sciences 2024, **279**, 115–132.

3. Shcherbacov V. Elements of Quasigroup Theory and Applications. Chapman and Hall/CRC, Boca Raton, 2017.

ПАРАСТРОФНА ОРБІТА (R,S,T)-ОБЕРНЕНИХ КВАЗІГРУП

Знайдено парастрофну орбіту (r, s, t)-обернених квазігруп, яка складається з шести многовидів. Також знайдено приклад, який демонструє виконання рівності $J^r(x \circ y) \circ J^s(x) = J^t(y)$ для (r, s, t)-обернених квазігруп.