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PROBLEM WITH INTEGRAL CONDITIONS FOR SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS OF FOURTH ORDER

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Let K_L be a class of quasi-polynomials In the form $\phi(x) = \sum_{i=1}^{n} Q_i(x)\exp[\alpha_i x]$, where $Q_i(x)$ are given polynomials, $\alpha_i \in L \subseteq R$, $\alpha_l \neq \alpha_k$ for $l \neq k$. Each quasi-polynomial $\phi(x)$ defines a differential operator $\phi\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Big|_{\lambda=0} = \sum_{i=1}^{n} Q_i\left(\frac{\partial}{\partial\lambda}\right)\Phi(\lambda)\Big|_{\alpha_i}$ of finite order in the class of certain function $\Phi(\lambda)$.

In the strip $\Omega = \{(t,x) \in R^{n+1} : t \in ([T_1,T_2]\times[T_3,T_4]), x \in R^n\}$ we consider system of equations

$$\frac{\partial^4 U_i}{\partial t^4} + \sum_{j=1}^{4} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{4-j} U_i}{\partial t^{4-j}} = 0, \qquad (1)$$

$i = 1, \ldots, n,$

satisfies integral conditions

$$\int_{T_1}^{T_2} t^k U_i(t,x)dt + \int_{T_3}^{T_4} t^k U_i(t,x)dt = \phi_{ik}(x), \qquad k = 0,..,3, \qquad (2)$$

where $a_{ij}\left(\frac{\partial}{\partial x}\right)$ are differentia expressions, which analytical symbols $a_{ij}(\lambda)$. Let be $\eta(\lambda) = \int_{T_1}^{T_2} W^{(n-1)}(t,\lambda)dt + \int_{T_3}^{T_4} W^{(n-1)}(t,\lambda)dt$ is a certain function, $W(t,\lambda)$ is a solution of the equation $L\left(\frac{d}{dt},\lambda\right)W(t,\lambda) \equiv 0$, satisfies conditions $W^{(n-1)}(t,\lambda)\big|_{t=0} = 1$, $W^{(n-2)}(t,\lambda)\big|_{t=0} = 0, \ldots, W(t,\lambda)\big|_{t=0} = 0$.

Denote be

$$P = \{\lambda \in C : \eta(\lambda) = 0\}$$

(3)

Theorem. *Let* $\phi_i(x) \in K_M, i = 1, \ldots, n$, *then the class* $K_{M \setminus P}$ *exist and unique solution of the problem (1), (2), where* P *is set (3), can be represented in the form*

$$U_i(t,x) = \sum_{i=1}^{n} \phi_i\left(\frac{\partial}{\partial\lambda}\right)\left\{\frac{1}{\eta(\lambda)}\tilde{l}^{T}\left(\frac{d}{dt},\lambda\right)W(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}.$$

By means of the differential-symbol method [1] we construction the solution of the problem (1), (2). This problem is a continues works [2,3,4].

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ЗАДАЧА З НЕОДНОРІДНИМИ ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ ОДНОРІДНОЇ СИСТЕМИ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ІЗ ЧАСТИННИМИ ПОХІДНИМИ

За допомогою диференціально-символьного методу подано розв'язок задачі з інтегральними умовами для системи диференціальних рівнянь із частинними похідними. Цей розв'язок існує і єдиний в класі квазімногочленів.