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# On the one-dimensional boundary-value problems with parameter in Slobodetsky spaces

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We arbitrarily choose a compact interval $[a,b]\subset\mathbb{R}$ and numbers

$$p\in(1,\infty),\quad m\geq 1,\quad r\geq 2,\quad s\in\mathbb{R}_+\setminus\mathbb{Z}_+,\quad s:=[s]+\{s\},$$

where $[s]\in\mathbb{Z}_+$ is the integer part of a number and $\{s\}\in(0,1)$ is it fractional part. We use the complex Slobodetsky spaces $(W_p^s)^m:=W_p^s([a,b],\mathbb{C}^m)$ and $(W_p^s)^{m\times m}:=W_p^s([a,b],\mathbb{C}^{m\times m})$, formed by functions, vector functions and matrix functions, respectively.

Let a real number $\varepsilon_0>0$ be fixed, and let a real parameter $\varepsilon$ range over the interval $[0,\varepsilon_0)$. We investigate a parameter-dependent linear boundary-value problem of the form

$$L(\varepsilon)y(t,\varepsilon)\equiv y^{(r)}(t,\varepsilon)+\sum_{j=1}^{r}A_{r-j}(t,\varepsilon)y^{(r-j)}(t,\varepsilon)=f(t,\varepsilon),\quad a\leq t\leq b,\quad (1)$$

$$B(\varepsilon)y(\cdot,\varepsilon)=c(\varepsilon).\quad (2)$$

For every fixed $\varepsilon\in[0,\varepsilon_0)$, the solution $y(\cdot,\varepsilon)$ to the problem is considered in the class $(W_p^{s+r})^m$. We suppose that $A_{r-j}(\cdot,\varepsilon)\in(W_p^s)^{m\times m}$ for each $j\in\{1,\dots,r\}$ and that $f(\cdot,\varepsilon)\in(W_p^s)^m$. Thus, (1) is a system of $m$ scalar linear $r$-th order differential equations given on $[a,b]$. Note we do not assume $A_{r-j}(\cdot,\varepsilon)$ to have any regularity in $\varepsilon$. As to the boundary condition (2), we suppose that $B(\varepsilon)$ is an arbitrary continuous linear operator

$$B(\varepsilon):(W_p^{s+r})^m\to\mathbb{C}^{rm}$$

and that $c(\varepsilon)\in\mathbb{C}^{rm}$.

For the boundary-value problem (1), (2) we introduce the following limit conditions as $\varepsilon\to 0+$:

(0) The homogeneous boundary-value problem as $\varepsilon=0$ has only the trivial solution.

(I) $A_{r-j}(\cdot,\varepsilon)\to A_{r-j}(\cdot,0)$ in $(W_p^s)^{m\times m}$ for each $j\in\{1,\dots,r\}$;

(II) $B(\varepsilon)y \to B(0)y$ in $\mathbb{C}^{rm}$ for every $y \in (W_p^{s+r})^m$;

(III) $f(\cdot,\varepsilon) \to f(\cdot,0)$ in $(W_p^s)^m$;

(IV) $c(\varepsilon) \to c(0)$ in $\mathbb{C}^{rm}$.

**Definition.** We say that the solution to the boundary-value problem (1), (2) depends continuously on the parameter $\varepsilon$ at $\varepsilon = 0$ in space $(W_p^{s+r})^m$ if the following two conditions are satisfied:

($*$) There exists a positive number $\varepsilon_1 < \varepsilon_0$ that this problem has a unique solution $y(\cdot,\varepsilon) \in (W_p^{s+r})^m$ for arbitrarily chosen $\varepsilon \in [0,\varepsilon_1)$, $f(\cdot,\varepsilon) \in (W_p^s)^m$, and $c(\varepsilon) \in \mathbb{C}^{rm}$.

($**$) It follows from Limit Conditions (III) and (IV) that

$$y(\cdot,\varepsilon) \to y(\cdot,0) \ \text{ in } \ (W_p^{s+r})^m \ \text{ as } \ \varepsilon \to 0+.$$

In paper [1], we proved problems (1), (2) are Fredholm, and obtained conditions that are sufficient for their well-posedness and continuity in the parameter of their solutions in Slobodetsky spaces.

Now, we proved that earlier found constructive sufficient conditions are also necessary.

**Theorem.** *The solution to the boundary-value problem* (1), (2) *depends continuously on the parameter* $\varepsilon$ *at* $\varepsilon = 0$ *in space* $(W_p^{s+r})^m$ *if and only if this problem satisfies Condition* (0) *and Limit Conditions* (I) *and* (II).

Also a two-sided estimate for the degree of convergence of these solutions was obtained.

The case $r = 1$ was investigated earlier in [2].

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## ПРО ОДНОВИМІРНІ КРАЙОВІ ЗАДАЧІ З ПАРАМЕТРОМ У ПРОСТОРАХ СЛОБОДЕЦЬКОГО

*Досліджено найбільш широкий клас лінійних крайових задач для систем звичайних диференціальних рівнянь, порядку* $r \geq 2$, *розв'язки яких належать комплексному простору Слободецкого* $W_p^{s+r}$, *де* $s \in \mathbb{R}_+ \setminus \mathbb{Z}_+$ *і* $p \in (1,\infty)$. *Для таких задач встановлено конструктивний критерій неперервності за параметром розв'язків у нормованому просторі* $W_p^{s+r}$. *Також отримана двостороння оцінка для швидкості збіжності цих розв'язків.*