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ADAPTIVE FINITE ELEMENT METHOD WITH DOUBLE-SIDED A POSTERIORI ERROR ESTIMATIONS

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We present the technique for obtaining double-sided a posteriori error estimations of finite element (FE) approximations for singular perturbed and/or nonlinear boundary value problems of the following kind

$$-\nabla.(\mu\nabla u)+\beta.\nabla u+\sigma u=f[u] \;\; in \;\; \Omega\subset\mathbb{R}^2, \;\; u=g \;\; on \;\; \partial\Omega. \qquad (1)$$

After obtaining the linear finite element approximation $u_h \in V_h \subset V$ of the solution of the problem (1) we formulate a posteriori error estimator (AEE) problem in subspace $E_h \subset E$, $V = E \oplus V_h$ with orthogonal basis $\{\phi_K\}_{K\in\mathfrak{I}_h}$, see [1]. On each finite element $K \in \mathcal{T}_h$ we define both Dirichlet estimator $e_K^{Dir} = \lambda_K^{Dir} 27 L_1 L_2 L_3$ and Neumann estimator $e_K^{Dir} = \lambda_K^{Dir} 3(L_1 L_2 + L_2 L_3 + L_3 L_1)$ with unknown $\lambda_K^{Dir}, \lambda_K^{Neu} \in \mathbb{R}$. Dirichlet and Neumann AEEs provide double-sided a posteriori error estimations for a solution of the problem (1).

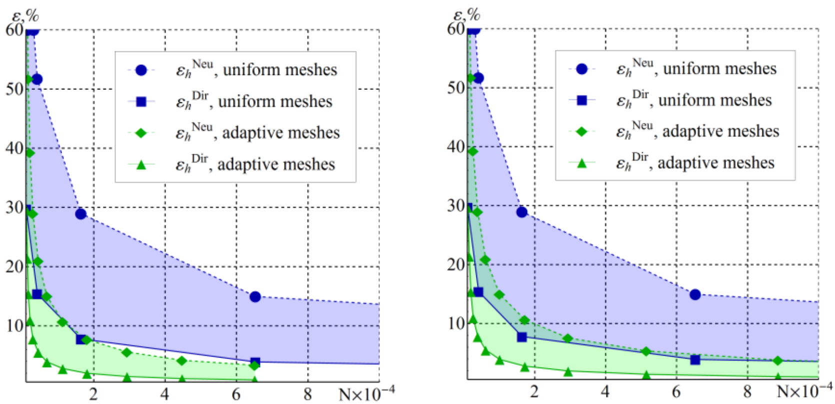

Fig. 1. The convergence of Dirichlet and Neumann error estimators on uniform and adaptive meshes for 1% accuracy. For the adaptation we use the bisection method and the refinement criterion (for det. see [1]) that based on Dirichlet (left) or Neumann (right) estimators.

For example we solve the problem (1) with $\mu = 10^{-2}$, $\beta = (1,-1)$, $f = 1.0$, $\sigma = g = 0$ over the L-shaped domain $\Omega = [0,0.5]\times[-1,0]\bigcup[0.5,1]\times[-0.5,0]$ by using linear approximations on uniform and adaptive triangular meshes. We observe the following characteristics: $N = Nod\,\mathcal{T}_h$ is the number of nodes in mesh; $Card\,\mathcal{T}_h$ the number of finite elements; $\varepsilon_h := \| e_h \|_{1,\Omega} \| u_h \|_{1,\Omega}^{-1} 100\%$ the relative error of estimation; $p := 2\ln(\| e_h^k \| \| e_h^{k+1} \|^{-1})(\ln(N_{k+1} N_k^{-1}))^{-1}$ the convergence rate of the estimator, $\| e_h \|_{1,\Omega} = \sum_{K\in\mathcal{T}_h} \| e_K \|_{1,\Omega}$ is the appropriate AEE, see [1].

Fig. 1 shows that the adaptive meshes have a great advantage in accuracy over uniform meshes. Using Neumann estimator in refinement criterion requires more finite elements then Dirichlet one because provides an upper error bound.

Table 1. Convergence of Dirichlet and Neumann estimators on adaptive meshes by using Dirichlet estimator in refinement criterion (k is the adaptation step)

k	$Nod\,\mathcal{T}_h$	$Card\,\mathcal{T}_h$	ε_h^{Dir}	ε_h^{Neu}	p^{Dir}	p^{Neu}
1	1 067	2 028	29.736	76.983	-	-
2	1 365	2 568	21.421	64.735	3.0	3.0
4	2 118	3 966	10.965	39.289	2.9	2.9
6	4 427	8 374	5.524	20.950	1.7	1.7
8	11 304	21 720	2.784	10.723	1.4	1.4
10	29 372	57 084	1.442	5.575	1.4	1.4
12	65 092	127 210	0.864	3.344	1.2	1.2

Table 1 and Fig. 1 confirm that Neumann and Dirichlet estimators provide the double-sided error estimates, no matter which one is used in the refinement criterion. The convergence rate for both Dirichlet and Neumann estimators tends to the theoretical expected value, which is equal 1.0.

1. *Ostapov O. Yu., Shynkarenko H. A. and Vovk O. V.* A posteriori error estimator and h-adaptive finite element method for diffusion-advection-reaction problems // Recent Advances in Comput. Mech., Taylor & Francis Group, London. – 2014. – P. 329–337.

АДАПТИВНИЙ МЕТОД СКІНЧЕННИХ ЕЛЕМЕНТІВ З ДВОСТОРОННІМИ АПОСТЕРІОРНИМИ ОЦІНКАМИ ПОХИБОК

Побудовано двосторонні апостеріорні оцінки похибок кусково-лінійних скінченно-елементних апроксимацій для адаптивного методу скінченних елементів. Наведено результати числових експериментів та числові характеристики збіжності.