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ON A BASKAKOV- DURRMEYER TYPE OPERATORS

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Let $C_B[0,+\infty)$ denote the space of all real valued continuous and bounded functions on $[0,+\infty)$. For $f \in C_B[0,+\infty)$, the well-known generalized Baskakov operators are given by

$$B_n^a(f;x) = e^{-\frac{ax}{1+x}} \sum_{k=0}^{\infty} f\left(\frac{k}{n}\right) \frac{P_k(n,a)}{k!} \frac{x^k}{(1+x)^{n+k}}, \qquad x \ge 0, \ n \in N,$$

$P_k(n,a)$ is defined as $e^{\frac{ax}{1+x}} = \sum_{k=0}^{\infty} \frac{P_k(n,a)}{k!} \frac{x^k}{(1+x)^{n+k}}$,

and $P_k(n,a) = \sum_{i=0}^{k} \binom{k}{i} (n)_i a^{k-i}$, $a \ge 0, (n)_0 = 1$, $(n)_i = n(n+1)\ldots(n+i-1), i \ge 1$.

Operator B_n^a was object research in [1-3]. Erencin [4] investigated the Baskakov-Durrmeyer type operators given by

$$L_n(f;x) = \sum_{k=0}^{\infty} W_{n,k}^a(x) \frac{1}{\mathrm{B}(k+1,n)} \int_0^{\infty} \frac{t^k}{(1+t)^{n+k+1}} f(t)dt, \qquad x \ge 0, \ n \in N,$$

$$W_{n,k}^a = e^{-\frac{ax}{1+x}} \frac{P_k(n,a)}{k!} \frac{x^k}{(1+x)^{n+k}}, \quad \mathrm{B}(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \quad \alpha > 0, \quad \beta > 0,$$

$\Gamma(k)$ – gamma function.

In the present paper we will study some approximate properties of Baskakov-Durrmeyer type operators $M_n^{\alpha,a}$. We determine the rate of convergence and prove the Voronowskaya type theorem for those operators.

For $f \in C_B[0,+\infty)$, we define the operators

$$M_n^{\alpha,a}(f;x) = \sum_{k=0}^{\infty} W_{n,k}^a(x) \frac{1}{\Gamma(\alpha+k+1)} \int_0^{\infty} e^{-ns} (ns)^{\alpha+k} f(s)ds, \qquad x \ge 0,$$

where $n \in N$, $\alpha > -1$.

Theorems below are main results of the paper. We establish certain direct theorem via first and second order of the modulus of continuity [5].

Theorem 1. If $f \in C_B$, then

$$\left| M_n^{\alpha,a}(f;x) - f(x) \right| \le \left[2 + \frac{9}{2} \left(\frac{(\alpha+1)(\alpha+2)}{n} + 2x + x^2 + \frac{a^2}{n} + \right. \right.$$

$$\left. \left. + \frac{2(\alpha+2)a}{n} \right) \right] \omega_2\left(f; \frac{1}{\sqrt{n}} \right) + \frac{5}{\sqrt{n}} (\alpha + 1 + a) \omega_1\left(f; \frac{1}{\sqrt{n}} \right), \ x \ge 0, \ n \in N.$$

In proof theorem 1 we will use the Stieklov function [6] and the lemmas from [7, 8].

Also we prove following the Voronowskaya type theorem.

Theorem 2. Let $f \in C_B$ is twice differentiable at some point $x \ge 0$, then

$$\lim_{n \to +\infty} n(M_n^{\alpha,a}(f;x) - f(x)) = \left(\alpha + \frac{ax}{1+x} + 1 \right) f'(x) + \left(\frac{1}{2} x^2 + x \right) f''(x).$$

1. *Mihesan V.* Uniform approximation with positive linear operators generalized Baskakov method // Automat. Comput. Appl. Math. 7 (1) – 1998. – P. 34-37.
2. *Wafi A. Khatoon S.* Direct and inverse theorems for generalized Baskakov operators // Indian J. Pure Appl. Math. 35 (3). – 2004. – P. 347-358.
3. *Wafi A., Khatoon S.* Direct and inverse theorems for generalized Baskakov operators in polynomial weight space // An. Stiint. Univ. Al. I. Cuza Iasi. Mat. (N.S) 50(1), – P. 159-173.
4. *Erencin A*. Durrmeyer type modification of generalized Baskakov operators // Appl. Math. Comput. – 2011. – P. 4384-4390.
5. *Timan A. F.* Theory of Approximation of Functionan of a Real Variable, Moscow. –1960.
6. *Mazar M., Totik V.* Approximation by modifield Szasz operators // Acta Sci. Math. – 1985. – P. 257-269.
7. *Krech G.*, *Wachnicki E.* Direct estimate for some operators of Durrmeyer type in exponential weighted space. – 2012. [preprint].
8. *Malejki R.*, *Wachnicki E.* On a Baskakov-Durrmeyer type operators // Com. Math.– 2014. –Vol. 54.1.2014.

ПРО ОПЕРАТОРИ ТИПУ БАСКАКОВА-ДЮРРМАЙЄРА

У робої досліджено властивості операторів типу Баскакова-Дюррмайєра. Визначено швидкість збіжності та доведено теорему Вороновської для цих операторів.