Theory of the column and row determinants and inverse matrix over a skew field with involution. Manuscript.

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The column and row determinants of square matrix are introduced over a skew field with involution. The skew field is a composition associative non-split algebra over its center - a field of zero characteristic. Such determinants can be expanded along a corresponding column for the column determinants and a corresponding row for the row determinants. The column and row determinants of a Hermitian matrix are equal and accept the value in the field. This value is defined as the determinant of a Hermitian matrix. The properties of the determinant of a Hermitian matrix are investigated. A double determinant of an arbitrary square matrix over a skew field is introduced. The double determinant is correctly defined as it satisfies the axioms of the noncommutative determinant. The double determinant can be expanded along an arbitrary row or column of a matrix by the column and row determinants of its corresponding left or right Hermitian matrices, respectively. The necessary and sufficient existence condition of the inverse matrix and its determinantal representation by analog of an adjoint matrix have been obtained. The solutions of the right and of the left systems of linear equations are represented by formulas, which generalize the Cramer rule.

Keywords: skew field with involution, quaternion algebra, composition algebra, noncommutative determinant, inverse matrix, system of linear equations, Cramer rule